Chapter 29
Nuclear Physics

Quick Quizzes

1. (c). At the end of the first half-life interval, half of the original sample has decayed and half remains. During the second half-life interval, half of the remaining portion of the sample decays. The total fraction of the sample that has decayed during the two half-lives is $\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)=\frac{3}{4}$.

2. (c). The half-life of a radioactive material is $T_{1/2}=\frac{\ln 2}{\lambda}$, where λ is the decay constant for that material. Thus, if $\lambda_A=2\lambda_B$, we have $\left(T_{1/2}\right)_A=\frac{\ln 2}{\lambda_A}=\frac{\ln 2}{2\lambda_B}=\frac{1}{2}\left(T_{1/2}\right)_B=\frac{1}{2}(4\text{ h})=2\text{ h}$.

3. (a). Conservation of momentum requires the momenta of the two fragments be equal in magnitude and oppositely directed. Thus, from $KE=p^2/2m$, the lighter alpha particle has more kinetic energy that the more massive daughter nucleus.

4. (a) and (b). Reactions (a) and (b) both conserve total charge and total mass number as required. Reaction (c) violates conservation of mass number with the sum of the mass numbers being 240 before reaction and being only 223 after reaction.

5. (b). In an endothermic reaction, the threshold energy exceeds the magnitude of the Q value by a factor of $(1+m/M)$, where m is the mass of the incident particle, and M is the mass of the target nucleus.

Answers to Even Numbered Conceptual Questions

2. Since the nucleus was at rest before decay, the total linear momentum will be zero both before and after decay. This means that the alpha particle and the daughter nucleus must recoil in opposite directions with equal magnitude momenta, $m_\alpha v_\alpha = m_D v_D$. The kinetic energy of the alpha particle is then

$$(KE)_\alpha = \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}m_\alpha\left(\frac{m_D^2}{m_\alpha^2}v_D^2\right) = \frac{m_D}{m_\alpha}\left(\frac{1}{2}m_D v_D^2\right) = \frac{m_D}{m_\alpha}(KE)_D$$

Since the mass of the daughter nucleus will be much larger than that of the alpha particle, the kinetic energy of the alpha particle, $(KE)_\alpha$, is considerably larger than that of the daughter nucleus, $(KE)_D$.

4. An alpha particle is a doubly positive charged helium nucleus, is very massive and does not penetrate very well. A beta particle is a singly negative charged electron and is very light and only slightly more difficult to shield from. A gamma ray is a high energy photon, or high frequency electromagnetic wave, and has high penetrating ability.

6. Beta particles have greater penetrating ability than do alpha particles.

8. The much larger mass of the alpha particle as compared to that of the beta particle ensures that it will not deflect as much as does the beta, which has a mass about 7000 times smaller.

10. Consider the reaction ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + \bar{\nu}$. We have six positive charges before the event on the carbon-14 nucleus. After the decay, we still have a net of six positive charges, as $+7$ from the nitrogen and -1 from the electron. Thus, in order to have conservation of charge, the neutrino must be uncharged.

12. We would have to revise our age values upward for ancient materials. That is, we would conclude that the materials were older than we had thought because the greater cosmic ray intensity would have left the samples with a larger percentage of carbon-14 when they died, and a longer time would have been necessary for it to decay to the percentage found at present.

14. The amount of carbon-14 left in very old materials is extremely small, and detection cannot be accomplished with a high degree of accuracy.

16. **(a)** The nucleus loses two protons and two neutrons, which are ejected in the form of an alpha particle. **(b)** In beta decay, the nucleus loses 1 neutron, but gains 1 proton, leaving the total number of nucleons unchanged.

Answers to Even Numbered Problems

2. $\sim 10^{28}$ protons, $\sim 10^{28}$ neutrons, $\sim 10^{28}$ electrons

4. $\rho_n/\rho_a = 8.6\times 10^{13}$

6. **(a)** 7.89 cm for ^{12}C, 8.21 cm for ^{13}C **(b)** $\dfrac{7.89\text{ cm}}{8.21\text{ cm}} = \sqrt{\dfrac{12}{13}} = 0.961$

8. **(a)** 1.9×10^{-15} m **(b)** 7.4×10^{-15} m

10. **(a)** 1.11 MeV/nucleon **(b)** 7.07 MeV/nucleon
(c) 8.79 MeV/nucleon **(d)** 7.57 MeV/nucleon

12. $E_b/A = 8.765$ MeV/nucleon for ^{55}Mn;
$E_b/A = 8.786$ MeV/nucleon for ^{56}Fe;
$E_b/A = 8.768$ MeV/nucleon for ^{59}Co

14. 7.93 MeV

16. 8.7×10^{3} Bq

18. **(a)** 8.06 d **(b)** It is probably $^{131}_{53}I$.

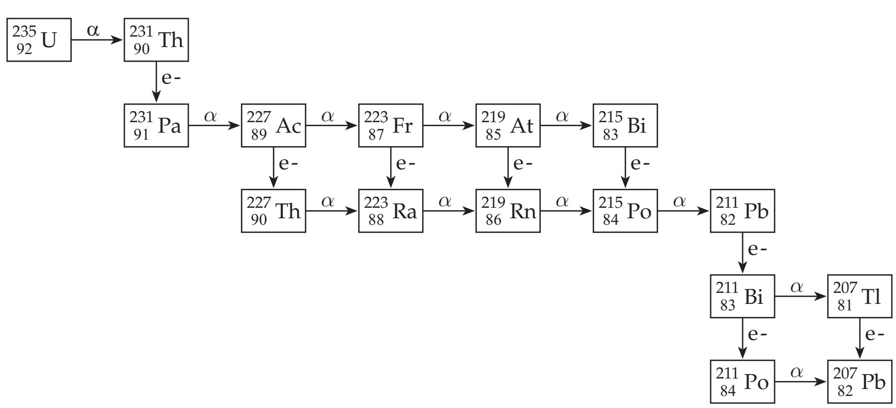
20. 2.29 g

22. 1.72×10^{4} yr

24. 1.66×10^{3} yr

26. $^{12}_{6}C$, $^{4}_{2}He$, $^{14}_{6}C$

28.



30. 4.28 MeV

32. **(a)** $^{66}_{28}\text{Ni} \rightarrow {}^{66}_{29}\text{Cu} + {}^{0}_{-1}\text{e} + \bar{\nu}$ **(b)** 186 keV

34. 1.67×10^4 yr

36. 9.96×10^3 yr

38. $^{4}_{2}\text{He}$, $^{4}_{2}\text{He}$

40. 17.3 MeV

42. **(a)** $^{10}_{5}\text{B}$ **(b)** –2.79 MeV

44. 22.0 MeV

46. 5.0 rad

48. **(a)** 2.00 J/kg **(b)** 4.78×10^{-4} °C

50. **(a)** 2.5×10^{-3} rem/x-ray **(b)** ≈ 38 times background levels

52. **(a)** 10 h **(b)** 3.2 m

54. 24 decays/min

56. fraction decayed $= 0.003\,5$ (0.35%)

58. **(a)** 2.7 fm **(b)** 1.5×10^2 N **(c)** 2.6 MeV
(d) 7.4 fm, 3.8×10^2 N, 18 MeV

60. 12 mg

62. 6×10^9 yr

64. **(a)** 60.5 Bq/L **(b)** 40.6 d

66. **(a)** 4.28×10^{-12} J **(b)** 1.19×10^{57} **(c)** 107 billion years

Problem Solutions

29.1 The average nuclear radii are $r = r_0 A^{1/3}$, where $r_0 = 1.2 \times 10^{-15}$ m $= 1.2$ fm, and A is the mass number.

For ${}^{2}_{1}\text{H}$, $r = (1.2 \text{ fm})(2)^{1/3} = \boxed{1.5 \text{ fm}}$

For ${}^{60}_{27}\text{Co}$, $r = (1.2 \text{ fm})(60)^{1/3} = \boxed{4.7 \text{ fm}}$

For ${}^{197}_{79}\text{Au}$, $r = (1.2 \text{ fm})(197)^{1/3} = \boxed{7.0 \text{ fm}}$

For ${}^{239}_{94}\text{Pu}$, $r = (1.2 \text{ fm})(239)^{1/3} = \boxed{7.4 \text{ fm}}$

29.2 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons. So protons and neutrons are nearly equally numerous in your body, each contributing (say) 35 kg out of a total body mass of 70 kg.

$$N = 35 \text{ kg}\left(\frac{1 \text{ nucleon}}{1.67 \times 10^{-27} \text{ kg}}\right) \boxed{\sim 10^{28} \text{ protons}} \text{ and } \boxed{\sim 10^{28} \text{ neutrons}}$$

The electron number is precisely equal to the proton number,

$$N_e = \boxed{\sim 10^{28} \text{ electrons}}$$

29.3 From $M_E = \rho_n V = \rho_n \left(\frac{4}{3}\pi r^3\right)$, we find

$$r = \left(\frac{3 M_E}{4\pi \rho_n}\right)^{1/3} = \left[\frac{3\left(5.98 \times 10^{24} \text{ kg}\right)}{4\pi\left(2.3 \times 10^{17} \text{ kg/m}^3\right)}\right]^{1/3} = \boxed{1.8 \times 10^{2} \text{ m}}$$

29.4 The mass of the hydrogen atom is approximately equal to that of the proton, 1.67×10^{-27} kg. If the radius of the atom is $r=0.53\times10^{-10}$ m, then

$$\rho_a=\frac{m}{V}=\frac{m}{(4/3)\pi r^3}=\frac{3\left(1.67\times10^{-27}\text{ kg}\right)}{4\pi\left(0.53\times10^{-10}\text{ m}\right)^3}=2.7\times10^{3}\text{ kg/m}^3$$

The ratio of the nuclear density to this atomic density is

$$\frac{\rho_n}{\rho_a}=\frac{2.3\times10^{17}\text{ kg/m}^3}{2.7\times10^{3}\text{ kg/m}^3}=\boxed{8.6\times10^{13}}$$

29.5 (a) $$F_{\max}=\frac{k_e q_1 q_2}{r_{\min}^2}=\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left[(2)(6)\left(1.60\times10^{-19}\text{ C}\right)^2\right]}{\left(1.00\times10^{-14}\text{ m}\right)^2}=\boxed{27.6\text{ N}}$$

(b) $$a_{\max}=\frac{F_{\max}}{m_\alpha}=\frac{27.6\text{ N}}{6.64\times10^{-27}\text{ kg}}=\boxed{4.16\times10^{27}\text{ m/s}^2}$$

(c) $$PE_{\max}=\frac{k_e q_1 q_2}{r_{\min}}=F_{\max}\cdot r_{\min}=(27.6\text{ N})\left(1.00\times10^{-14}\text{ m}\right)$$

$$=2.76\times10^{-13}\text{ J}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)=\boxed{1.73\text{ MeV}}$$

29.6 (a) From conservation of energy, $\Delta KE = -\Delta PE$, or $\frac{1}{2}mv^2 = q(\Delta V)$. Also, the centripetal acceleration is supplied by the magnetic force,

so $\frac{mv^2}{r} = qvB$, or $v = qBr/m$

The energy equation then yields $r = \sqrt{2m(\Delta V)/qB^2}$

For ^{12}C, m=12 u

And $$r = \sqrt{\frac{2\left[12\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(1\,000\text{ V}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.200\text{ T}\right)^2}} = \boxed{7.89\text{ cm}}$$

For ^{13}C, m=13 u

and $$r = \sqrt{\frac{2\left[13\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(1\,000\text{ V}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.200\text{ T}\right)^2}} = \boxed{8.21\text{ cm}}$$

(b) $$\frac{r_1}{r_2} = \frac{\sqrt{2m_1(\Delta V)/qB^2}}{\sqrt{2m_2(\Delta V)/qB^2}} = \sqrt{\frac{m_1}{m_2}}$$

$$\boxed{\frac{r_{12}}{r_{13}} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961 \quad\text{and}\quad \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961}$$, so they do agree.

29.7 (a) At the point of closest approach, $PE_f = KE_i$, so $\frac{k_e(2e)(79e)}{r_{\min}} = \frac{1}{2}m_\alpha v^2$

or $v = \sqrt{\frac{2k_e(2e)(79e)}{m_\alpha r_{\min}}}$

$$= \sqrt{\frac{316\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(6.64\times10^{-27}\text{ kg}\right)\left(3.2\times10^{-14}\text{ m}\right)}} = \boxed{1.9\times10^7\text{ m/s}}$$

(b) $KE_i = \frac{1}{2} m_\alpha v^2$

$$= \frac{1}{2}\left(6.64\times10^{-27}\text{ kg}\right)\left(1.85\times10^{7}\text{ m/s}\right)^2\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = \boxed{7.1\text{ MeV}}$$

29.8 Using $r = r_0 A^{1/3}$, with $r_0 = 1.2\times10^{-15}\text{ m} = 1.2\text{ fm}$, gives:

(a) For ${}^{4}_{2}\text{He}$, $A = 4$, and $r = (1.2\text{ fm})(4)^{1/3} = 1.9\text{ fm} = \boxed{1.9\times10^{-15}\text{ m}}$

(b) For ${}^{238}_{92}\text{He}$, $A = 238$, and $r = (1.2\text{ fm})(238)^{1/3} = 7.4\text{ fm} = \boxed{7.4\times10^{-15}\text{ m}}$

29.9 For ${}^{93}_{41}\text{Nb}$,

$$\Delta m = 41 m_{\text{H}} + 52 m_n - m_{\text{Nb}}$$

$$= 41(1.007\ 825\text{u}) + 52(1.008\ 665\text{u}) - (92.906\ 376\ 8\text{u}) = 0.865\ 028\text{u}$$

Thus, $\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.865\ 028\text{u})(931.5\text{ MeV/u})}{93} = \boxed{8.66\text{ MeV/nucleon}}$

For ${}^{197}_{79}\text{Au}$,

$$\Delta m = 79(1.007\ 825\text{u}) + 118(1.008\ 665\text{u}) - (196.966\ 543\text{u}) = 1.674\ 102\text{u}$$

and $\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(1.674\ 102\text{u})(931.5\text{ MeV/u})}{197} = \boxed{7.92\text{ MeV/nucleon}}$

29.10 (a) For ${}^{2}_{1}\text{H}$,

$$\Delta m = 1(1.007\ 825\text{u}) + 1(1.008\ 665\text{u}) - (2.014\ 102\text{u}) = 0.002\ 388\text{u}$$

and $\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.002\ 388\text{u})(931.5\text{ MeV/u})}{2} = \boxed{1.11\text{ MeV/nucleon}}$

(b) For ${}^{4}_{2}\text{He}$,

$$\Delta m = 2(1.007\ 825\,\text{u}) + 2(1.008\ 665\,\text{u}) - (4.002\ 602\,\text{u}) = 0.030\ 378\,\text{u}$$

and $\dfrac{E_b}{A} = \dfrac{(\Delta m)c^2}{A} = \dfrac{(0.030\,378\,\text{u})(931.5\ \text{MeV/u})}{4} = \boxed{7.07\ \text{MeV/nucleon}}$

(c) For ${}^{56}_{26}\text{Fe}$,

$$\Delta m = 26(1.007\ 825\,\text{u}) + 30(1.008\ 665\,\text{u}) - (55.934\ 940) = 0.528\ 460\,\text{u}$$

and $\dfrac{E_b}{A} = \dfrac{(\Delta m)c^2}{A} = \dfrac{(0.528\,460\,\text{u})(931.5\ \text{MeV/u})}{56} = \boxed{8.79\ \text{MeV/nucleon}}$

(d) For ${}^{238}_{92}\text{U}$,

$$\Delta m = 92(1.007\ 825\,\text{u}) + 146(1.008\ 665\,\text{u}) - (238.050\ 784) = 1.934\ 206\,\text{u}$$

and $\dfrac{E_b}{A} = \dfrac{(\Delta m)c^2}{A} = \dfrac{(1.934\,206\,\text{u})(931.5\ \text{MeV/u})}{238} = \boxed{7.57\ \text{MeV/nucleon}}$

29.11 For ${}^{15}_{8}\text{O}$,

$$\Delta m = 8(1.007\ 825\,\text{u}) + 7(1.008\ 665\,\text{u}) - (15.003\ 065) = 0.120\ 190\ \text{u}$$

and $\quad E_b\big|_{^{15}\text{O}} = (\Delta m)c^2 = (0.120\,190\,\text{u})(931.5\ \text{MeV/u}) = 111.957\ \text{MeV}$

For ${}^{15}_{7}\text{N}$,

$$\Delta m = 7(1.007\ 825\,\text{u}) + 8(1.008\,665\,\text{u}) - (15.000\,108) = 0.123\,987\ \text{u}$$

and $\quad E_b\big|_{^{15}\text{N}} = (\Delta m)c^2 = (0.123\,987\,\text{u})(931.5\ \text{MeV/u}) = 115.494\ \text{MeV}$

Therefore, $\quad \Delta E_b = E_b\big|_{^{15}\text{N}} - E_b\big|_{^{15}\text{O}} = \boxed{3.54\ \text{MeV}}$

29.12 $\Delta m = Z m_{\mathrm{H}} + (A-Z) m_n - m$ and $E_b/A = \Delta m(931.5 \text{ MeV/u})/A$

Nucleus	Z	$(A-Z)$	m (in u)	Δm (in u)	E_b/A (in MeV)
$^{55}_{25}\mathrm{Mn}$	25	30	54.938 048	0.517 527	8.765
$^{56}_{26}\mathrm{Fe}$	26	30	55.934 940	0.528 460	8.786
$^{59}_{27}\mathrm{Co}$	27	32	58.933 198	0.555 357	8.768

Therefore, $^{56}_{26}\mathrm{Fe}$ has a greater binding energy per nucleon than its neighbors. This gives us finer detail than is shown in Figure 29.4.

29.13 For $^{23}_{11}\mathrm{Na}$,

$$\Delta m = 11(1.007\ 825\,\mathrm{u}) + 12(1.008\ 665\,\mathrm{u}) - (22.989\ 770\,\mathrm{u}) = 0.200\ 285\,\mathrm{u}$$

and $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.200\ 285\,\mathrm{u})(931.5 \text{ MeV/u})}{23} = 8.111 \text{ MeV/nucleon}$$

For $^{23}_{12}\mathrm{Mg}$,

$$\Delta m = 12(1.007\ 825\,\mathrm{u}) + 11(1.008\ 665\,\mathrm{u}) - (22.994\ 127\,\mathrm{u}) = 0.195\ 088\,\mathrm{u}$$

so $$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.195\ 088\,\mathrm{u})(931.5 \text{ MeV/u})}{23} = 7.901 \text{ MeV/nucleon}$$

The binding energy per nucleon is

greater for $^{23}_{11}\mathrm{Na}$ by $\boxed{0.210 \text{ MeV/nucleon}}$

This is attributable to $\boxed{\text{less proton repulsion in } ^{23}_{11}\mathrm{Na}}$

29.14 The sum of the mass of $^{42}_{20}\mathrm{Ca}$ plus the mass of a neutron exceeds the mass of $^{43}_{20}\mathrm{Ca}$. This difference in mass must represent the mass equivalence of the energy spent removing a the last neutron from $^{43}_{20}\mathrm{Ca}$ to produce $^{42}_{20}\mathrm{Ca}$ plus a free neutron. Thus,

$$E = \left(m_{^{42}_{20}\mathrm{Ca}} + m_n - m_{^{43}_{20}\mathrm{Ca}}\right)c^2 = (41.958\ 622 \text{ u} + 1.008\ 665 \text{ u} - 42.958\ 770 \text{ u})c^2$$

or $$E = (0.008\ 517 \text{ u})(931.5 \text{ MeV/u}) = \boxed{7.93 \text{ MeV}}$$

29.15 The decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}}$, so the activity is

$$R = \lambda N = \frac{N \ln 2}{T_{1/2}} = \frac{\left(3.0\times 10^{16}\right)\ln 2}{(14\text{ d})\left(8.64\times 10^{4}\text{ s/d}\right)} = 1.7\times 10^{10}\text{ decays/s}$$

or $$R = \left(1.7\times 10^{10}\text{ decays/s}\right)\left(\frac{1\text{ Ci}}{3.7\times 10^{10}\text{ decays/s}}\right) = \boxed{0.46\text{ Ci}}$$

29.16 The activity is $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{T_{1/2}}$. Thus,

$$R = R_0 e^{-\left(t \ln 2/T_{1/2}\right)} = \left(1.1\times 10^{4}\text{ Bq}\right)e^{-\frac{(2.0\text{ h})\ln 2}{6.05\text{ h}}} = \boxed{8.7\times 10^{3}\text{ Bq}}$$

29.17 (a) The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(8.04\text{ d})\left(8.64\times 10^{4}\text{ s/d}\right)} = \boxed{9.98\times 10^{-7}\text{ s}^{-1}}$$

(b) $R = \lambda N$, so the required number of nuclei is

$$N = \frac{R}{\lambda} = \frac{0.50\times 10^{-6}\text{ Ci}}{9.98\times 10^{-7}\text{ s}^{-1}}\left(\frac{3.7\times 10^{10}\text{ decays/s}}{1\text{ Ci}}\right) = \boxed{1.9\times 10^{10}\text{ nuclei}}$$

29.18 (a) From $R = R_0 e^{-\lambda t}$, where $\lambda = \dfrac{\ln 2}{T_{1/2}}$, and $R = 0.842 R_0$ when $t = 2.00\text{ d}$, we find

$$0.842 = e^{-\lambda(2.00\text{ d})} \qquad \text{or} \qquad \lambda = -\frac{\ln(0.842)}{2.00\text{ d}} = 8.60\times 10^{-2}\text{ d}^{-1}$$

Thus, $$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{8.60\times 10^{-2}\text{ d}^{-1}} = \boxed{8.06\text{ d}}$$

(b) Comparing the computed half-life to the measured half-lives of the radioactive materials in Appendix B shows that this is probably $\boxed{{}^{131}_{53}\text{I} \text{ or Iodine-131}}$

29.19 Recall that the activity of a radioactive sample is directly proportional to the number of radioactive nuclei present, and hence, to the mass of the radioactive material present.

Thus, $\frac{R}{R_0}=\frac{N}{N_0}=\frac{m}{m_0}=\frac{0.25\times10^{-3}\text{ g}}{1.0\times10^{-3}\text{ g}}=0.25$ when $t=2.0\text{ h}$

From $R=R_0e^{-\lambda t}$, we obtain $0.25=e^{-\lambda(2.0\text{ h})}$ and $\lambda=-\frac{\ln(0.25)}{2.0\text{ h}}=0.693\text{ h}^{-1}$

Then, the half-life is $T_{1/2}=\frac{\ln 2}{\lambda}=\frac{\ln 2}{0.693\text{ h}^{-1}}=\boxed{1.0\text{ h}}$

29.20 Recall that the activity of a radioactive sample is directly proportional to the number of radioactive nuclei present and hence, to the mass of the radioactive material present.

Thus, $\frac{R}{R_0}=\frac{N}{N_0}=\frac{m}{m_0}$ and $R=R_0e^{-\lambda t}$ becomes $m=m_0e^{-\lambda t}$

The decay constant is $\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{3.83\text{ d}}=0.181\text{ d}^{-1}$

If $m_0=3.00\text{ g}$ and the elapsed time is $t=1.50\text{ d}$, the mass of radioactive material remaining is

$$m=m_0e^{-\lambda t}=(3.00\text{ g})e^{-(0.181\text{ d}^{-1})(1.50\text{ d})}=\boxed{2.29\text{ g}}$$

29.21 From $R=R_0\,e^{-\lambda t}$, with $R=(1.00\times10^{-3})R_0$, we find $e^{-\lambda t}=\frac{R}{R_0}$

and $t=-\frac{\ln(R/R_0)}{\lambda}=-T_{1/2}\left[\frac{\ln(R/R_0)}{\ln 2}\right]$

$$=-(432\text{ yr})\left[\frac{\ln(1.00\times10^{-3})}{\ln 2}\right]=\boxed{4.31\times10^{3}\text{ yr}}$$

29.22 Using $R=R_0\,e^{-\lambda t}$, with $R/R_0=0.125$, gives $\lambda t=-\ln(R/R_0)$

or $t=-\frac{\ln(R/R_0)}{\lambda}=-T_{1/2}\left[\frac{\ln(R/R_0)}{\ln 2}\right]=-(5\,730\text{ yr})\left[\frac{\ln(0.125)}{\ln 2}\right]=\boxed{1.72\times10^{4}\text{ yr}}$

29.23 (a) The initial activity is $R_0 = 10.0\text{ mCi}$, and at $t = 4.00\text{ h}$, $R = 8.00\text{ mCi}$. Then, from $R = R_0\, e^{-\lambda t}$, the decay constant is

$$\lambda = -\frac{\ln\left(R/R_0\right)}{t} = -\frac{\ln(0.800)}{4.00\text{ h}} = \boxed{5.58\times10^{-2}\text{ h}^{-1}}$$

and the half-life is $T_{1/2} = \dfrac{\ln 2}{\lambda} = \dfrac{\ln 2}{5.58\times10^{-2}\text{ h}^{-1}} = \boxed{12.4\text{ h}}$

(b) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{\left(10.0\times10^{-3}\text{ Ci}\right)\left(3.70\times10^{10}\text{ s}^{-1}/1\text{ Ci}\right)}{\left(5.58\times10^{-2}\text{ h}^{-1}\right)\left(1\text{ h}/3\,600\text{ s}\right)} = \boxed{2.39\times10^{13}\text{ nuclei}}$

(c) $R = R_0\, e^{-\lambda t} = (10.0\text{ mCi})e^{-\left(5.58\times10^{-2}\text{ h}^{-1}\right)(30\text{ h})} = \boxed{1.9\text{ mCi}}$

29.24 The number of ${}^{90}_{38}\text{Sr}$ nuclei initially present is

$$N_0 = \frac{\text{total mass}}{\text{mass per nucleus}} = \frac{5.00\text{ kg}}{(89.907\,7\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)} = 3.35\times10^{25}$$

The half-life of ${}^{90}_{38}\text{Sr}$ is $T_{1/2} = 29.1\text{ yr}$ (Appendix B), so the initial activity is

$$R_0 = \lambda N_0 = \frac{N_0 \ln 2}{T_{1/2}} = \frac{\left(3.35\times10^{25}\right)\ln 2}{(29.1\text{ yr})\left(3.156\times10^{7}\text{ s/yr}\right)} = 2.53\times10^{16}\text{ counts/s}$$

The time when the activity will be $R = 10.0\text{ counts/min}$ is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -\left(T_{1/2}\right)\frac{\ln\left(R/R_0\right)}{\ln 2}$$

$$= -(29.1\text{ yr})\frac{\ln\left[\dfrac{10.0\text{ min}^{-1}}{2.53\times10^{16}\text{ s}^{-1}}\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\right]}{\ln 2} = \boxed{1.66\times10^{3}\text{ yr}}$$

29.25 ${}^{212}_{83}\text{Bi} \to \boxed{{}^{208}_{81}\text{Tl}} + {}^{4}_{2}\text{He}$

${}^{95}_{36}\text{Kr} \to \boxed{{}^{95}_{37}\text{Rb}} + {}^{0}_{-1}\text{e}$

$\boxed{{}^{144}_{60}\text{Nd}} \to {}^{4}_{2}\text{He} + {}^{140}_{58}\text{Ce}$

29.26 ${}^{12}_{5}\text{B} \rightarrow \boxed{{}^{12}_{6}\text{C}} + {}^{0}_{-1}\text{e}$

${}^{234}_{90}\text{Th} \rightarrow {}^{230}_{88}\text{Ra} + \boxed{{}^{4}_{2}\text{He}}$

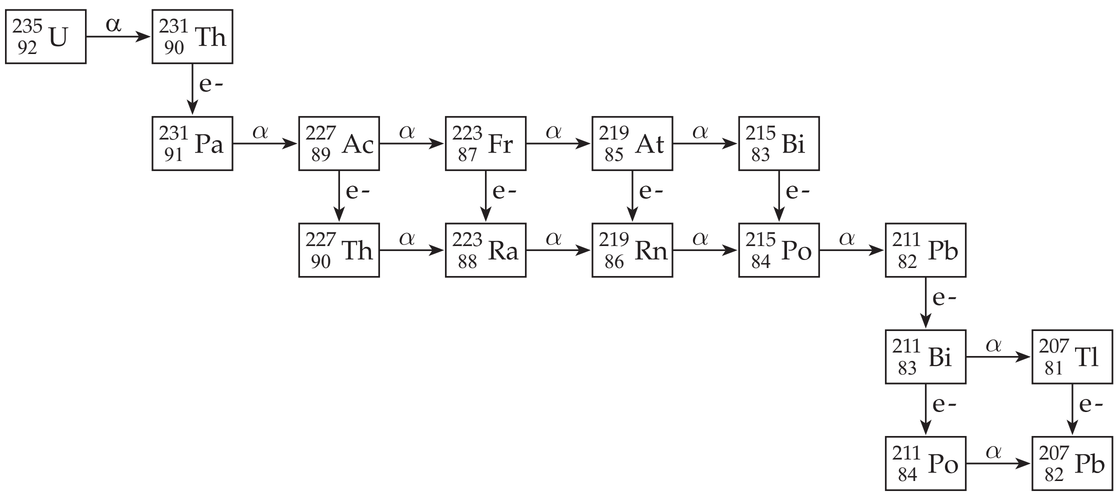
$\boxed{{}^{14}_{6}\text{C}} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}$

29.27 $\boxed{{}^{40}_{20}\text{Ca}} \rightarrow {}^{0}_{+1}\text{e} + {}^{40}_{19}\text{K}$

${}^{98}_{44}\text{Ru} \rightarrow {}^{4}_{2}\text{He} + \boxed{{}^{94}_{42}\text{Mo}}$

${}^{144}_{60}\text{Nd} \rightarrow \boxed{{}^{4}_{2}\text{He}} + {}^{140}_{58}\text{Ce}$

29.28



29.29 The more massive ${}^{56}_{27}\text{Co}$ decays into the less massive ${}^{56}_{26}\text{Fe}$. To conserve charge, the charge of the emitted particle must be +1e. Since the parent and the daughter have the same mass number, the emitted particle must have essentially zero mass. Thus, the decay must be positron emission or $\boxed{e^{+}\text{ decay}}$. The decay equation is

$$\boxed{{}^{56}_{27}\text{Co} \rightarrow {}^{56}_{26}\text{Fe} + e^{+} + \nu}$$

29.30 The energy released in the decay ${}^{238}_{92}\text{U} \rightarrow {}^{4}_{2}\text{He} + {}^{234}_{90}\text{Th}$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^{238}\text{U}} - \left(m_{{}^{4}\text{He}} + m_{{}^{234}\text{Th}}\right)\right]c^2$$

$$= \left[238.050\,784\text{ u} - \left(4.002\,602\text{ u} + 234.043\,583\text{ u}\right)\right]\left(931.5\text{ MeV/u}\right)$$

$$= \boxed{4.28\text{ MeV}}$$

29.31 The Q value of a decay, $Q = (\Delta m)c^2$, is the amount of energy released in the decay. Here, Δm is the difference between the mass of the original nucleus and the total mass of the decay products. If $Q > 0$, the decay may occur spontaneously.

(a) For the decay ${}^{40}_{20}\text{Ca} \rightarrow e^+ + {}^{40}_{19}\text{K}$, the masses of the electrons do not automatically cancel. Thus, we add 20 electrons to each side of the decay to yield neutral atoms and obtain

$$\left({}^{40}_{20}\text{Ca} + 20e^-\right) \rightarrow e^+ + \left({}^{40}_{19}\text{K} + 19e^-\right) + e^- \qquad \text{or} \qquad {}^{49}\text{Ca}_{\text{atom}} \rightarrow e^+ + {}^{40}\text{K}_{\text{atom}} + e^-$$

Then,

$$Q = \left(m_{{}^{40}_{20}\text{Ca}_{\text{atom}}} - m_{{}^{40}_{19}\text{K}_{\text{atom}}} - 2m_e\right)c^2 = \left[39.962\,591\text{ u} - 39.964\,000\text{ u} - 2\left(0.000\,549\text{ u}\right)\right]c^2$$

or $Q = \left(-0.002\,507\text{ u}\right)c^2 < 0$ so the decay $\boxed{\text{cannot occur spontaneously}}$

(b) In the decay ${}^{144}_{60}\text{Nd} \rightarrow {}^{4}_{2}\text{He} + {}^{140}_{58}\text{Ce}$, we may add 60 electrons to each side forming all neutral atoms and use masses from Appendix B to find

$$Q = \left(m_{{}^{144}_{60}\text{Nd}} - m_{{}^{4}_{2}\text{He}} - m_{{}^{140}_{58}\text{Ce}}\right)c^2 = \left(143.910\,082\text{ u} - 4.002\,602\text{ u} - 139.905\,434\text{ u}\right)c^2$$

or $Q = \left(+0.002\,046\text{ u}\right)c^2 > 0$ so the decay $\boxed{\text{can occur spontaneously}}$

29.32 (a) $\boxed{{}^{66}_{28}\text{Ni} \rightarrow {}^{66}_{29}\text{Cu} + {}^{0}_{-1}e + \bar{\nu}}$

(b) Because of the mass differences, neglect the kinetic energy of the recoiling daughter nucleus in comparison to that of the other decay products. Then, the maximum kinetic energy of the beta particle occurs when the neutrino is given zero energy. That maximum is

$$KE_{max} = \left(m_{^{66}\text{Ni}} - m_{^{66}\text{Cu}}\right)c^2 = \left(65.929\,1 \text{ u} - 65.928\,9 \text{ u}\right)\left(931.5 \text{ MeV/u}\right)$$

$$= 0.186 \text{ MeV} = \boxed{186 \text{ keV}}$$

29.33 In the decay ${}^3_1\text{H} \rightarrow {}^3_2\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$, the anti-neutrino is massless. Adding 1 electron to each side of the decay gives $\left({}^3_1\text{H} + \text{e}^-\right) \rightarrow \left({}^3_2\text{He} + 2\text{ e}^-\right) + \bar{\nu}$, or ${}^3_1\text{H}_{\text{atom}} \rightarrow {}^3_2\text{He}_{\text{atom}} + \bar{\nu}$. Therefore, using neutral atomic masses from Appendix B, the energy released is

$$E = (\Delta m)c^2 = \left(m_{^3\text{H}} - m_{^3\text{He}}\right)c^2 = \left(3.016\,049 \text{ u} - 3.016\,029 \text{ u}\right)\left(931.5 \text{ MeV/u}\right)$$

$$= 0.018\,6 \text{ MeV} = \boxed{18.6 \text{ keV}}$$

29.34 The initial activity of the 1.00-kg carbon sample would have been

$$R_0 = \left(1.00 \times 10^3 \text{ g}\right)\left(\frac{15.0 \text{ counts/min}}{1.00 \text{ g}}\right) = 1.50 \times 10^4 \text{ min}^{-1}$$

From $R = R_0 e^{-\lambda t}$, and $T_{1/2} = 5\,730 \text{ yr}$ for ${}^{14}\text{C}$ (Appendix B), the age of the sample is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2}\frac{\ln\left(R/R_0\right)}{\ln 2}$$

$$= -\left(5\,730 \text{ yr}\right)\frac{\ln\left(\dfrac{2.00 \times 10^3 \text{ min}^{-1}}{1.50 \times 10^4 \text{ min}^{-1}}\right)}{\ln 2} = \boxed{1.67 \times 10^4 \text{ yr}}$$

29.35 From $R = R_0 e^{-\lambda t}$, and $T_{1/2} = 5\,730 \text{ y}$ for ${}^{14}\text{C}$ (Appendix B), the age of the sample is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2}\frac{\ln\left(R/R_0\right)}{\ln 2} = -\left(5\,730 \text{ yr}\right)\frac{\ln(0.600)}{\ln 2} = \boxed{4.22 \times 10^3 \text{ yr}}$$

29.36 The total number of carbon nuclei in the sample is

$$N = \left(\frac{21.0 \times 10^{-3} \text{ g}}{12.0 \text{ g/mol}} \right) \left(6.02 \times 10^{23} \text{ nuclei/mol} \right) = 1.05 \times 10^{21}$$

Of these, one in every 7.70×10^{11} was originally ^{14}C. Hence, the initial number of ^{14}C nuclei present was $N_0 = 1.05 \times 10^{21} / 7.70 \times 10^{11} = 1.37 \times 10^9$, and the initial activity was

$$R_0 = \lambda N_0 = \frac{N_0 \ln 2}{T_{1/2}} = \frac{\left(1.37 \times 10^9\right) \ln 2}{\left(5\,730 \text{ yr}\right)} \left(\frac{1 \text{ yr}}{365.24 \text{ d}} \right) \left(\frac{7 \text{ d}}{1 \text{ week}} \right) = 3.17 \times 10^3 \text{ week}^{-1}$$

The current activity, accounting for counter efficiency, is

$$R = \frac{\text{count rate}}{\text{efficiency}} = \frac{837 \text{ week}^{-1}}{0.880} = 951 \text{ week}^{-1}$$

Therefore, from $R = R_0 e^{-\lambda t}$, the age of the sample is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2} \frac{\ln\left(R/R_0\right)}{\ln 2}$$

$$= -\left(5\,730 \text{ yr}\right) \frac{\ln\left(\dfrac{951 \text{ week}^{-1}}{3.17 \times 10^3 \text{ week}^{-1}} \right)}{\ln 2} = \boxed{9.96 \times 10^3 \text{ yr}}$$

29.37 (a) $^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \rightarrow {}^{1}_{0}\text{n} + \boxed{^{30}_{15}\text{P}}$

(b) $Q = (\Delta m)c^2 = \left(m_{^{27}\text{Al}} + m_{^{4}\text{He}} - m_{\text{n}} - m_{^{30}\text{P}} \right) c^2$

$$Q = \left[26.981\,538 \text{ u} + 4.002\,602 \text{ u} - 1.008\,665 \text{ u} - 29.978\,310 \text{ u} \right] \left(931.5 \text{ MeV/u} \right)$$

$$= \boxed{-2.64 \text{ MeV}}$$

29.38 $\boxed{^{4}_{2}\text{He}} + {}^{14}_{7}\text{N} \rightarrow {}^{1}_{1}\text{H} + {}^{17}_{8}\text{O}$

$^{7}_{3}\text{Li} + {}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + \boxed{^{4}_{2}\text{He}}$

29.39 (a) $\boxed{{}^{21}_{10}\text{Ne}} + {}^{4}_{2}\text{He} \rightarrow {}^{24}_{12}\text{Mn} + {}^{1}_{0}\text{n}$

(b) ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{90}_{38}\text{Sr} + \boxed{{}^{144}_{54}\text{Xe}} + 2\,{}^{1}_{0}\text{n}$

(c) $2\,{}^{1}_{1}\text{H} \rightarrow {}^{2}_{1}\text{H} + \boxed{\text{e}^{+}} + \boxed{\nu}$

29.40 $Q = (\Delta m)c^2 = \left(m_{{}^1\text{H}} + m_{{}^7\text{Li}} - 2m_{{}^4\text{He}}\right)c^2$

$$= \left[1.007\ 825\,\text{u} + 7.016\ 003\,\text{u} - 2\left(4.002\ 602\,\text{u}\right)\right]\left(931.5\ \text{MeV/u}\right) = \boxed{17.3\ \text{MeV}}$$

29.41 (a) ${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{1}_{1}\text{H} + \boxed{{}^{13}_{6}\text{C}}$

(b) ${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + \boxed{{}^{10}_{5}\text{B}}$

29.42 (a) ${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He} \rightarrow \boxed{{}^{10}_{5}\text{B}} + {}^{1}_{0}\text{n}$

(b) $Q = (\Delta m)c^2 = \left(m_{{}^7\text{Li}} + m_{{}^4\text{He}} - m_{{}^{10}\text{B}} - m_{\text{n}}\right)c^2$

$$= \left[7.016\ 003\,\text{u} + 4.002\ 602\,\text{u} - 10.012\ 936\,\text{u} - 1.008\ 665\,\text{u}\right]\left(931.5\ \text{MeV/u}\right)$$

$$= \boxed{-2.79\ \text{MeV}}$$

29.43 (a) Requiring that both charge and the number of nucleons (atomic mass number) be conserved, the reaction is found to be

$$\boxed{{}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}}$$

Note that the antineutrino has been included to conserve electron-lepton number which will be discussed in the next chapter.

(b) We add 79 electrons to both sides of the reaction equation given above to produce neutral atoms so we may use mass values from Appendix B. This gives

$$^{197}_{79}\text{Au}_{\text{atom}} + ^{1}_{0}\text{n} \rightarrow ^{198}_{80}\text{Hg}_{\text{atom}} + \bar{\nu}$$

and, remembering that the neutrino is massless, the Q value is found to be

$$Q = (\Delta m)c^2 = \left(m_{^{197}_{79}\text{Au}} + m_n - m_{^{198}_{80}\text{Hg}}\right)c^2 = (196.966\,543\text{ u} + 1.008\,665\text{ u} - 197.966\,750\text{ u})c^2$$

or $Q = (+0.008\,458\text{ u})(931.5\text{ MeV/u}) = 7.88\text{ MeV}$

The kinetic energy carried away by the daughter nucleus is negligible. Thus, the energy released may be split in any manner between the electron and neutrino, with the maximum kinetic energy of the electron being $\boxed{7.88\text{ MeV}}$

29.44 The energy released in the reaction $^{1}_{0}\text{n} + ^{4}_{2}\text{He} \rightarrow ^{2}_{1}\text{H} + ^{3}_{1}\text{H}$ is

$$Q = (\Delta m)c^2 = \left(m_{\text{n}} + m_{^4\text{He}} - m_{^2\text{H}} - m_{^3\text{H}}\right)c^2$$

$$= [1.008\,665\text{ u} + 4.002\,602\text{ u} - 2.014\,102\text{ u} - 3.016\,049\text{ u}](931.5\text{ MeV/u})$$

$$= -17.6\text{ MeV}$$

The threshold energy of the incident neutron is then

$$KE_{\text{min}} = \left(1 + \frac{m_{\text{n}}}{m_{^4\text{He}}}\right)|Q| = \left(1 + \frac{1.008\,665\text{ u}}{4.002\,602\text{ u}}\right)|-17.6\text{ MeV}| = \boxed{22.0\text{ MeV}}$$

29.45 (a) $^{18}_{8}\text{O} + ^{1}_{1}\text{H} \rightarrow ^{18}_{9}\text{F} + \boxed{^{1}_{0}\text{n}}$

(b) The energy released in this reaction is

$$Q = (\Delta m)c^2 = \left(m_{^{18}\text{O}} + m_{^1\text{H}} - m_{^{18}\text{F}} - m_{\text{n}}\right)c^2$$

$$= \left[17.999\,160\text{ u} + 1.007\,825\text{ u} - m_{^{18}\text{F}} - 1.008\,665\text{ u}\right](931.5\text{ MeV/u})$$

Since we know that $Q = -2.453\text{ MeV}$, we find that $m_{^{18}\text{F}} = \boxed{18.000\,953\text{ u}}$

29.46 For equivalent doses, it is necessary that

$$(\text{heavy ion dose in rad}) \times \text{RBE}_{\text{heavy ions}} = (\text{x-ray dose in rad}) \times \text{RBE}_{\text{x-rays}}$$

or

$$\text{ion dose in rad} = \frac{(\text{x-ray dose in rad}) \times \text{RBE}_{\text{x-rays}}}{\text{RBE}_{\text{heavy ions}}} = \frac{(100\text{ rad})(1.0)}{20} = \boxed{5.0\text{ rad}}$$

29.47 For each rad of radiation, 10^{-2} J of energy is delivered to each kilogram of absorbing material. Thus, the total energy delivered in this whole body dose to a 75.0-kg person is

$$E = (25.0\text{ rad})\left(10^{-2}\ \frac{\text{J/kg}}{\text{rad}}\right)(75.0\text{ kg}) = \boxed{18.8\text{ J}}$$

29.48 (a) Each rad of radiation delivers 10^{-2} J of energy to each kilogram of absorbing material. Thus, the energy delivered per unit mass with this dose is

$$\frac{E}{m} = (200\text{ rad})\left(10^{-2}\ \frac{\text{J/kg}}{\text{rad}}\right) = \boxed{2.00\text{ J/kg}}$$

(b) From $E = Q = mc(\Delta T)$, the expected temperature rise with this dosage is

$$\Delta T = \frac{E/m}{c} = \frac{2.00\text{ J/kg}}{4\,186\text{ J/kg}\cdot{}^\circ\text{C}} = \boxed{4.78\times10^{-4}\ {}^\circ\text{C}}$$

29.49 The rate of delivering energy to each kilogram of absorbing material is

$$\left(\frac{E/m}{\Delta t}\right) = (10\text{ rad/s})\left(10^{-2}\ \frac{\text{J/kg}}{\text{rad}}\right) = 0.10\ \frac{\text{J/kg}}{\text{s}}$$

The total energy needed per unit mass is

$$E/m = c(\Delta T) = \left(4\,186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(50^\circ\text{C}) = 2.1\times10^{5}\ \text{J/kg}$$

so the required time will be

$$\Delta t = \frac{\text{energy needed}}{\text{delivery rate}} = \frac{2.1\times10^{5}\ \text{J/kg}}{0.10\ \text{J/kg}\cdot\text{s}} = 2.1\times10^{6}\ \text{s}\left(\frac{1\text{ d}}{8.64\times10^{4}\ \text{s}}\right) = \boxed{24\text{ d}}$$

29.50 (a) The number of x-rays taken per year is

$$production = (8 \text{ x-ray/d})(5 \text{ d/week})(50 \text{ weeks/yr}) = 2.0\times10^{3} \text{ x-ray/yr}$$

so the exposure per x-ray taken is

$$exposure\ rate = \frac{exposure}{production} = \frac{5.0 \text{ rem/yr}}{2.0\times10^{3} \text{ x-ray/yr}} = \boxed{2.5\times10^{-3} \text{ rem/x-ray}}$$

(b) The exposure due to background radiation is 0.13 rem/yr. Thus, the work-related exposure of 5.0 rem/yr is

$$\frac{5.0 \text{ rem/yr}}{0.13 \text{ rem/yr}} \approx \boxed{38 \text{ times background levels}}$$

29.51 (a) From $N = \dfrac{R}{\lambda} = \dfrac{R_0 e^{-\lambda t}}{\lambda} = \left(\dfrac{T_{1/2} R_0}{\ln 2}\right) e^{-t\ln 2/T_{1/2}}$, the number of decays occurring during the 10-day period is

$$\Delta N = N_0 - N = \left(\frac{T_{1/2} R_0}{\ln 2}\right)\left(1 - e^{-t\ln 2/T_{1/2}}\right)$$

$$= \left[\frac{(14.3 \text{ d})(1.31\times10^{6} \text{ decay/s})}{\ln 2}\left(\frac{8.64\times10^{4} \text{ s}}{1 \text{ d}}\right)\left(1 - e^{-(10.0 \text{ d})\ln 2/14.3 \text{ d}}\right)\right]$$

$= \boxed{8.97\times10^{11} \text{ decays}}$, and one electron is emitted per decay.

(b) The total energy deposited is found to be

$$E = \left(700 \frac{\text{keV}}{\text{decay}}\right)(8.97\times10^{11} \text{ decays})\left(\frac{1.60\times10^{-16} \text{ J}}{1 \text{ keV}}\right) = \boxed{0.100 \text{ J}}$$

(c) The total absorbed dose (measured in rad) is given by

$$dose = \frac{\text{energy deposited per unit mass}}{\text{energy deposition per rad}}$$

$$= \frac{(0.100 \text{ J}/0.100 \text{ kg})}{\left(10^{-2} \dfrac{\text{J/kg}}{\text{rad}}\right)} = \boxed{100 \text{ rad}}$$

29.52 (a) The dose (in rem) received in time Δt is given by

$$dose = (dose\ in\ rad) \times RBE$$

$$= \left[\left(100 \times 10^{-3}\ \frac{\text{rad}}{\text{h}}\right)\Delta t\right] \times (1.00) = \left(0.100\ \frac{\text{rem}}{\text{h}}\right)\Delta t$$

If this dose is to be 1.0 rem, the required time is

$$\Delta t = \frac{1.0\ \text{rem}}{0.100\ \text{rem/h}} = \boxed{10\ \text{h}}$$

(b) Assuming the radiation is emitted uniformly in all directions, the intensity of the radiation is given by $I = I_0/4\pi r^2$

Therefore, $\dfrac{I_r}{I_1} = \dfrac{I_0/4\pi r^2}{I_0/4\pi(1.0\ \text{m})^2} = \dfrac{(1.0\ \text{m})^2}{r^2}$

and $r = (1.0\ \text{m})\sqrt{\dfrac{I_1}{I_r}} = (1.0\ \text{m})\sqrt{\dfrac{100\ \text{mrad/h}}{10\ \text{mrad/h}}} = \boxed{3.2\ \text{m}}$

29.53 From $R = R_0 e^{-\lambda t}$, the elapsed time is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -T_{1/2}\frac{\ln(R/R_0)}{\ln 2} = -(14.0\ \text{d})\frac{\ln(20.0\ \text{mCi}/200\ \text{mCi})}{\ln 2} = \boxed{46.5\ \text{d}}$$

29.54 Assuming that the carbon-14 activity of a living organism 20 000 years ago was the same as today, the original activity of the sample was

$$R_0 = (15.0\ \text{decays/min}\cdot\text{g})(18\ \text{g}) = 2.7 \times 10^2\ \text{decays/min}$$

The decay constant for carbon-14 is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5\,730\ \text{yr}} = 1.21 \times 10^{-4}\ \text{yr}^{-1}$$

so the expected current activity is

$$R = R_0 e^{-\lambda t} = (2.7 \times 10^2\ \text{decay/min})\, e^{-(1.21\times 10^{-4}\ \text{yr}^{-1})(20\,000\ \text{yr})} = \boxed{24\ \text{decays/min}}$$

29.55 The energy released in the reaction ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0^1\text{n}$ is

$$Q = (\Delta m)c^2 = \left(2m_{{}^2\text{H}} - m_{{}^3\text{He}} - m_\text{n}\right)c^2$$

$$= \left[2(2.014\,102\,\text{u}) - 3.016\,029\,\text{u} - 1.008\,665\,\text{u}\right](931.5\ \text{MeV/u})$$

$$= \boxed{+3.27\ \text{MeV}}$$

Since $Q > 0$, $\boxed{\text{no threshold energy is required}}$

29.56 The speed of a neutron having a kinetic energy of

$$KE = (0.040\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right) = 6.4\times10^{-21}\ \text{J}$$

is $$v = \sqrt{\frac{2(KE)}{m_n}} = \sqrt{\frac{2\left(6.4\times10^{-21}\ \text{J}\right)}{1.675\times10^{-27}\ \text{kg}}} = 2.8\times10^3\ \text{m/s}$$

and the time required to travel 10.0 km is $t = \dfrac{\Delta x}{v} = \dfrac{10.0\times10^3\ \text{m}}{2.8\times10^2\ \text{m/s}} = 3.6\ \text{s}$

The decay constant for free neutrons is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{12\ \text{min}}\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = 9.6\times10^{-4}\ \text{s}^{-1}$

Since the number remaining after time *t* is $N = N_0 e^{-\lambda t}$, the fraction having decayed in this time is

$$\text{fraction decayed} = \frac{N_0 - N}{N_0} = 1 - e^{-\lambda t} = 1 - e^{-\left(9.6\times10^{-4}\ \text{s}^{-1}\right)(3.6\ \text{s})} = \boxed{0.003\,5\ \text{or}\ 0.35\%}$$

29.57 (a) $N_0 = \dfrac{\text{mass of sample}}{\text{mass per atom}} = \dfrac{1.00\ \text{kg}}{(239.05\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)} = \boxed{2.52\times10^{24}}$

(b) $R_0 = \lambda N_0 = \dfrac{N_0 \ln 2}{T_{1/2}} = \dfrac{\left(2.52\times10^{24}\right)\ln 2}{(24\,120\ \text{yr})\left(3.156\times10^7\ \text{s/yr}\right)} = \boxed{2.29\times10^{12}\ \text{Bq}}$

(c) From $R = R_0 e^{-\lambda t}$, the required time is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -\frac{T_{1/2}\ln(R/R_0)}{\ln 2}$$

$$= -\frac{(24\,120 \text{ yr})\ln(0.100/2.29\times 10^{12})}{\ln 2} = \boxed{1.07\times 10^{6} \text{ yr}}$$

29.58 (a) $r = r_0 A^{1/3} = (1.2\times 10^{-15} \text{ m})(12)^{1/3} = 2.7\times 10^{-15} \text{ m} = \boxed{2.7 \text{ fm}}$

(b) With $Z = 6$,

$$F = \frac{k_e e[(Z-1)e]}{r^2} = \frac{5(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19} \text{ C})^2}{(2.7\times 10^{-15} \text{ m})^2}$$

or $F = \boxed{1.5\times 10^{2} \text{ N}}$

(c) The work done is the increase in the electrical potential energy, or

$$W = PE|_r - PE|_{r=\infty} = \frac{k_e e[(Z-1)e]}{r} - 0 = \frac{5(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^{22})(1.60\times 10^{-19} \text{ C})^2}{2.7\times 10^{-15} \text{ m}}$$

$$= 4.2\times 10^{-13} \text{ J}\left(\frac{1 \text{ MeV}}{1.60\times 10^{-13} \text{ J}}\right) = \boxed{2.6 \text{ MeV}}$$

(d) Repeating the previous calculations for ${}^{238}_{92}\text{U}$ (with $Z = 92$ and $A = 238$) gives

$$r = r_0(238)^{1/3} = \boxed{7.4 \text{ fm}}, \quad F = \frac{k_e e[(91)e]}{r^2} = \boxed{3.8\times 10^{2} \text{ N}}$$

and $$W = \frac{k_e e[(91)e]}{r} = \boxed{18 \text{ MeV}}$$

29.59 (a) If we assume all the ^{87}Sr nuclei came from the decay of ^{87}Rb nuclei, the original number of ^{87}Rb nuclei was

$$N_0 = 1.82\times10^{10} + 1.07\times10^{9} = 1.93\times10^{10}$$

Then, from $N = N_0 e^{-\lambda t}$, the elapsed time is

$$t = -\frac{\ln(N/N_0)}{\lambda} = -\frac{T_{1/2}\ln(N/N_0)}{\ln 2}$$

$$= -\frac{(4.8\times10^{10}\ \text{yr})\ln\left(\dfrac{1.82\times10^{10}}{1.93\times10^{10}}\right)}{\ln 2} = \boxed{4.0\times10^{9}\ \text{yr}}$$

(b) It could be $\boxed{\text{no older}}$. It could be younger $\boxed{\text{if some }^{87}\text{Sr were initially present}}$

29.60 From $R = \lambda N$, the number of ^{60}Co nuclei present in a $10\ \text{Ci}$ source is

$$N = \frac{R}{\lambda} = \frac{R(T_{1/2})}{\ln 2}$$

$$= (10\ \text{Ci})\left(3.7\times10^{10}\ \frac{\text{decay/s}}{\text{Ci}}\right)\frac{(5.2\ \text{yr})}{\ln 2}\left(\frac{3.156\times10^{7}\ \text{s}}{1\ \text{yr}}\right) = 8.8\times10^{19}$$

The number that was present 30 months (2.5 years) ago must have been

$$N_0 = \frac{N}{e^{-\lambda t}} = N e^{t\cdot\ln 2/T_{1/2}} = (8.8\times10^{19})e^{(2.5\ \text{yr})\ln 2/5.2\ \text{yr}} = 1.2\times10^{20}$$

and the initial mass of ^{60}Co was

$$m = N_0\, m_{atom} = (1.2\times10^{20})\left[(59.93\ \text{u})(1.66\times10^{-27}\ \text{kg/u})\right]$$

$$= 1.2\times10^{-5}\ \text{kg} = \boxed{12\ \text{mg}}$$

29.61 The total activity of the working solution at $t=0$ was

$$(R_0)_{total} = (2.5 \text{ mCi/mL})(10 \text{ mL}) = 25 \text{ mCi}$$

Therefore, the initial activity of the 5.0-mL sample which will be drawn from the 250-mL working solution was

$$(R_0)_{sample} = (R_0)_{total}\left(\frac{5.0 \text{ mL}}{250 \text{ mL}}\right) = (25 \text{ mCi})\left(\frac{5.0 \text{ mL}}{250 \text{ mL}}\right) = 0.50 \text{ mCi} = 5.0\times10^{-4} \text{ Ci}$$

With a half-life of 14.96 h for ^{24}Na (Appendix B), the activity of the sample after 48 h is

$$R = R_0 e^{-\lambda t} = R_0 e^{-t\ln 2/T_{1/2}} = \left(5.0\times10^{-4} \text{ Ci}\right)e^{-(48 \text{ h})\ln 2/(14.96 \text{ h})}$$

$$= 5.4\times10^{-5} \text{ Ci} = \boxed{54 \ \mu\text{Ci}}$$

29.62 The decay constants for the two uranium isotopes are

$$\lambda_1 = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{0.70\times10^{9} \text{ yr}} = 9.9\times10^{-10} \text{ yr}^{-1} \ \left(\text{for } ^{235}_{92}\text{U}\right)$$

and $$\lambda_2 = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47\times10^{9} \text{ yr}} = 1.55\times10^{-10} \text{ yr}^{-1} \ \left(\text{for } ^{238}_{92}\text{U}\right)$$

If there were N_0 nuclei of each isotope present at $t=0$, the ratio of the number remaining would be

$$\frac{N_{235}}{N_{238}} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}} = e^{(\lambda_2-\lambda_1)t}, \text{ and the elapsed time is } t = \frac{\ln\left(N_{235}/N_{238}\right)}{\lambda_2-\lambda_1}$$

With a measured value of 0.007 for this ratio, the estimated age is

$$t = \frac{\ln(0.007)}{(1.55-9.9)\times10^{-10} \text{ yr}^{-1}} = \boxed{6\times10^{9} \text{ yr}}$$

29.63 The original activity per unit area is

$$R_0 = \frac{5.0\times10^6 \text{ Ci}}{1.0\times10^4 \text{ km}^2}\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right)^2 = 5.0\times10^{-4} \text{ Ci/m}^2 = 5.0\times10^2 \ \mu\text{Ci/m}^2$$

From $R = R_0 e^{-\lambda t}$, the required time is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -\frac{T_{1/2}\ln(R/R_0)}{\ln 2}$$

$$= -\frac{(28.7 \text{ yr})\ln(2.0 \ \mu\text{Ci}/5.0\times10^2 \ \mu\text{Ci})}{\ln 2} = \boxed{2.3\times10^2 \text{ yr}}$$

29.64 (a) The mass of a single ^{40}K atom is

$$m = (39.964 \text{ u})(1.66\times10^{-27} \text{ kg/u}) = 6.63\times10^{-26} \text{ kg} = 6.63\times10^{-23} \text{ g}$$

Therefore, the number of ^{40}K nuclei in a liter of milk is

$$N = \frac{\text{total mass of } ^{40}\text{K present}}{\text{mass per atom}} = \frac{(2.00 \text{ g/L})(0.0117/100)}{6.63\times10^{-23} \text{ g}} = 3.53\times10^{18}/\text{L}$$

and the activity due to potassium is

$$R = \lambda N = \frac{N\ln 2}{T_{1/2}} = \frac{(3.53\times10^{18}/\text{L})\ln 2}{(1.28\times10^9 \text{ yr})(3.156\times10^7 \text{ s/yr})} = \boxed{60.5 \text{ Bq/L}}$$

(b) Using $R = R_0 e^{-\lambda t}$, the time required for the ^{131}I activity to decrease to the level of the potassium is given by

$$t = -\frac{\ln(R/R_0)}{\lambda} = -\frac{T_{1/2}\ln(R/R_0)}{\ln 2} = -\frac{(8.04 \text{ d})\ln(60.5/2\,000)}{\ln 2} = \boxed{40.6 \text{ d}}$$

29.65 The total activity due to ^{59}Fe at the end of the 1 000-h test will be

$$R = R_0 e^{-\lambda t} = R_0 e^{-t \cdot \ln 2 / T_{1/2}} = (20.0\ \mu\text{Ci}) e^{-\left[(10^3\ \text{h}) \cdot \ln 2 / (45.1\ \text{d})(24\ \text{h/d})\right]} = 10.5\ \mu\text{Ci}$$

The total activity in the oil at the end of the test is

$$R_{oil} = \left(\frac{800\ \text{min}^{-1}}{\text{L}}\right)(6.5\ \text{L})$$

$$= 5.2 \times 10^3\ \text{min}^{-1} \left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \mu\text{Ci}}{3.7 \times 10^4\ \text{s}^{-1}}\right) = 2.3 \times 10^{-3}\ \mu\text{Ci}$$

Therefore, the fraction of the iron that was worn away during the test is

$$fraction = \frac{R_{oil}}{R} = \frac{2.3 \times 10^{-3}\ \mu\text{Ci}}{10.5\ \mu\text{Ci}} = 2.2 \times 10^{-4}$$

This represents a mass of

$$m_{worn\ away} = (fraction) \cdot (\text{total mass of iron})$$

$$= (2.2 \times 10^{-4})(0.20\ \text{kg}) = 4.4 \times 10^{-5}\ \text{kg}$$

so the wear rate was $\dfrac{4.4 \times 10^{-5}\ \text{kg}}{1\,000\ \text{h}} = \boxed{4.4 \times 10^{-8}\ \text{kg/h}}$

29.66 (a) Adding 2 electrons to each side of the reaction

$$4\ {}_1^1\text{H} + 2\text{e}^- \rightarrow {}_2^4\text{He} + 2\nu + \gamma$$

gives $4\left({}_1^1\text{H} + \text{e}^-\right) \rightarrow \left({}_2^4\text{He} + 2\,\text{e}^-\right) + 2\nu + \gamma$, or $4\,{}_1^1\text{H}_{\text{atom}} \rightarrow {}_2^4\text{He}_{\text{atom}} + 2\nu + \gamma$

Since the neutrinos and the photon are massless, the available energy is

$$Q = (\Delta m)c^2 = \left[4 m_{^1\text{H}} - m_{^4\text{He}}\right]c^2$$

$$= \left[4(1.007\ 825\ \text{u}) - 4.002\ 602\ \text{u}\right](931.5\ \text{MeV/u})$$

$$= 26.7\ \text{MeV}\left(1.60 \times 10^{-13}\ \text{J/MeV}\right) = \boxed{4.28 \times 10^{-12}\ \text{J}}$$

(b) $N = \dfrac{m_{Sun}}{m_{^1\mathrm{H}}} = \dfrac{1.99 \times 10^{30}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{1.19 \times 10^{57}}$

(c) With 4 hydrogen nuclei consumed per reaction, the total energy available is

$$E = \left(\frac{N}{4}\right)Q = \left(\frac{1.19 \times 10^{57}}{4}\right)\left(4.28 \times 10^{-12}\ \text{J}\right) = 1.27 \times 10^{45}\ \text{J}$$

At a power output of $\mathcal{P} = 3.76 \times 10^{26}\ \text{W}$, this supply would last for

$$t = \frac{E}{\mathcal{P}} = \frac{1.27 \times 10^{45}\ \text{J}}{3.76 \times 10^{26}\ \text{J/s}} = 3.39 \times 10^{18}\ \text{s}\left(\frac{1\ \text{yr}}{3.156 \times 10^{7}\ \text{s}}\right) = 1.07 \times 10^{11}\ \text{yr}$$

$$= 107 \times 10^{9}\ \text{yr} = \boxed{107\ \text{billion years}}$$