Chapter 26
Relativity

Quick Quizzes

1. (a). Less time will have passed for you in your frame of reference than for your employer back on Earth. Thus, to maximize your paycheck, you should choose to have your pay calculated according to the elapsed time on a clock on Earth.

2. No. Your sleeping cabin is at rest in your reference frame, thus, it will have its proper length. There will be no change in measured lengths of objects within your spacecraft. Another observer, on a spacecraft traveling at a high speed relative to yours, will claim that you are able to fit into a shorter sleeping cabin (if your body is oriented in a direction parallel to your direction of travel relative to the other observer).

3. (a), (e). The outgoing rocket will appear to have a **shorter** length and a **slower** clock. The answers are the same for the incoming rocket. Length contraction and time dilation depend only on the magnitude of the relative velocity, not on the direction.

4. (a) False (b) False (c) True (d) False

A reflected photon does exert a force on the surface. Although a photon has zero mass, a photon does carry momentum. When it reflects from a surface, there is a change in the momentum, just like the change in momentum of a ball bouncing off a wall. According to the momentum interpretation of Newton's second law, a change in momentum results in a force on the surface. This concept is used in theoretical studies of space sailing. These studies propose building non-powered spacecraft with huge reflective sails oriented perpendicularly to the rays from the Sun. The large number of photons from the Sun reflecting from the surface of the sail will exert a force which, although small, will provide a continuous acceleration. This would allow the spacecraft to travel to other planets without fuel.

5. (a). The downstairs clock runs more slowly because it is closer to Earth and hence experiences a stronger gravitational field than the upstairs clock does.

Answers to Even Numbered Conceptual Questions

2. To be strictly correct, the equation should be written as $E = \gamma mc^2$ where $\gamma = 1\Big/\sqrt{1-(v/c)^2}$. When the object is at rest $v = 0$ and $\gamma = 1$, so the equation reverts to the popular form $E = mc^2$. When $v > 0$, $E > E_R = mc^2$ and the difference $KE = E - E_R = (\gamma - 1)mc^2$ accurately accounts for the kinetic energy of the moving mass.

4. The two observers will agree on the speed of light and on the speed at which they move relative to one another.

6. Special relativity describes inertial reference frames -- that is , reference frames that are not accelerating. General relativity describes all reference frames.

8. You would see the same thing that you see when looking at a mirror when at rest. The theory of relativity tells us that all experiments will give the same results in all inertial frames of reference.

10. The clock in orbit will run more slowly. The extra centripetal acceleration of the orbiting clock makes its history fundamentally different from that of the clock on Earth.

12. The 8 lightyears represents the proper length of a rod from Earth to Sirius, measured by an observer seeing both the rod and Sirius nearly at rest. The astronaut sees Sirius coming toward her at 0.8*c* but also sees the distance contracted to

$$d = (8\text{ ly})\sqrt{1-(v/c)^2} = (8\text{ ly})\sqrt{1-(0.8)^2} = 5\text{ ly}$$

So, the travel time measured on her clock is $t = \dfrac{d}{v} = \dfrac{5\text{ ly}}{0.8c} = \dfrac{(5\text{ yr})c}{0.8c} = 6\,\text{yr}$.

Answers to Even Numbered Problems

2. **(a)** 1.9×10^{-15} s **(b)** 2.0 fringe shifts

4. **(a)** 1.38 yr **(b)** 1.31 lightyears

6. **(a)** 70 beats/min **(b)** 31 beats/min

8. **(a)** 65.0/min **(b)** 10.6/min

10. **(a)** a rectangular box
(b) sides perpendicular to the motion are 2.0 m long, sides parallel to the motion are 1.2 m long

12. **(a)** $0.297L_0$ **(b)** 0.088

14. **(a)** 17.4 m **(b)** 3.30° with respect to the direction of motion

16. **(a)** 21.0 yr **(b)** 14.7 ly **(c)** 10.5 ly **(d)** 35.7 yr

18. **(a)** $2.7 \times 10^{-24}\ \text{kg} \cdot \text{m/s}$ **(b)** $1.6 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$
(c) $5.6 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$

20. **(a)** $0.141c$ **(b)** $0.436c$

22. $0.94c$ to the left

24. $0.696c$

26. 0.161 Hz

28. **(a)** 939 MeV **(b)** 3.01 GeV **(c)** 2.07 GeV

30. $0.980c$

32. **(a)** 0.582 MeV **(b)** 2.45 MeV

34. $m_{faster} = 2.51 \times 10^{-28}$ kg, $m_{slower} = 8.84 \times 10^{-28}$ kg

36. **(a)** 3.52 MeV **(b)** 8.50×10^{20} Hz

38. 0.64 MeV, $3.4 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$ for each photon

40. **(a)** 2.50 MeV/c **(b)** 4.60 GeV/c

42. **(a)** 25.0 yr **(b)** 15.0 yr **(c)** 12.0 ly

44. 1.2×10^{13} m

46. $0.614c$ away from Earth

48. **(a)** $\sim 10^{2}$ s **(b)** $\sim 10^{8}$ km

50. **(a)** 4.38×10^{11} J **(b)** 4.38×10^{11} J

52. **(b)** $0.943c$

54. **(a)** $t = \dfrac{2d}{c + v}$ **(b)** $t = \dfrac{2d}{c}\sqrt{\dfrac{c - v}{c + v}}$

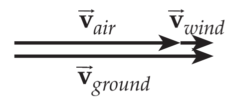
56. 1.47 km

58. **(a)** \$800 **(b)** $\$2.12 \times 10^{9}$

Problem Solutions

26.1 (a) As the plane flies from O to B along path I, $\vec{\mathbf{v}}_{ground} = \vec{\mathbf{v}}_{air} + \vec{\mathbf{v}}_{wind}$ gives

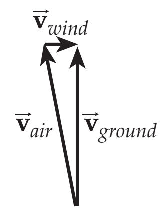
$$v_{ground} = 100\ \frac{\text{m}}{\text{s}} + 20.0\ \frac{\text{m}}{\text{s}} = 120\ \frac{\text{m}}{\text{s}}, \text{ and}$$



$$t_{OB} = \frac{L}{v_{ground}} = \frac{200 \times 10^3\ \text{m}}{120\ \text{m/s}} = \boxed{1.67 \times 10^3\ \text{s}}$$

For the plane following path II from O to A, $\vec{\mathbf{v}}_{ground} = \vec{\mathbf{v}}_{air} + \vec{\mathbf{v}}_{wind}$ with $\vec{\mathbf{v}}_{ground}$ and $\vec{\mathbf{v}}_{wind}$ perpendicular to each other. The Pythagorean theorem then yields

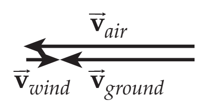
$$v_{ground} = \sqrt{v_{air}^2 - v_{wind}^2} = \sqrt{\left(100\ \frac{\text{m}}{\text{s}}\right)^2 - \left(20.0\ \frac{\text{m}}{\text{s}}\right)^2} = 98.0\ \frac{\text{m}}{\text{s}}$$



and $$t_{OA} = \frac{L}{v_{ground}} = \frac{200 \times 10^3\ \text{m}}{98.0\ \text{m/s}} = \boxed{2.04 \times 10^3\ \text{s}}$$

(b) For the return flight along path I, $\vec{\mathbf{v}}_{ground} = \vec{\mathbf{v}}_{air} + \vec{\mathbf{v}}_{wind}$ gives

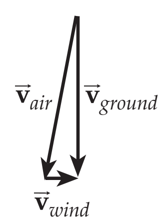
$$v_{ground} = 100\ \frac{\text{m}}{\text{s}} - 20.0\ \frac{\text{m}}{\text{s}} = 80.0\ \frac{\text{m}}{\text{s}}$$



so $$t_{BO} = \frac{L}{v_{ground}} = \frac{200 \times 10^3\ \text{m}}{80.0\ \text{m/s}} = \boxed{2.50 \times 10^3\ \text{s}}$$

As the plane flies from A to O along path II, $\vec{\mathbf{v}}_{ground}$ and $\vec{\mathbf{v}}_{wind}$ are again perpendicular to each other. The Pythagorean theorem gives

$$v_{ground} = \sqrt{v_{air}^2 - v_{wind}^2} = 98.0\ \frac{\text{m}}{\text{s}}$$



and $$t_{AO} = \frac{L}{v_{ground}} = \boxed{2.04 \times 10^3\ \text{s}}$$

(c) The total times of flight are

$$t_{\mathrm{I}} = t_{OB} + t_{BO} = 1.67\times10^{3}\ \mathrm{s} + 2.50\times10^{3}\ \mathrm{s} = 4.17\times10^{3}\ \mathrm{s}$$

and $$t_{\mathrm{II}} = t_{OA} + t_{AO} = 2\left(2.04\times10^{3}\ \mathrm{s}\right) = 4.08\times10^{3}\ \mathrm{s}$$

The difference in total flight times is $\Delta t = t_{\mathrm{I}} - t_{\mathrm{II}} = \boxed{90\ \mathrm{s}}$

26.2 (a) $$\Delta t_{net} = \frac{2Lv^2}{c^3} = \frac{2(28\ \mathrm{m})\left(3.0\times10^{4}\ \mathrm{m/s}\right)^2}{\left(3.0\times10^{8}\ \mathrm{m/s}\right)^3} = \boxed{1.9\times10^{-15}\ \mathrm{s}}$$

(b) Since a fringe shift occurs for every half-wavelength change made in the optical path length, the number of fringe shifts expected is

$$N = \frac{\Delta d_{net}}{\lambda/2} = \frac{2c\left(\Delta t_{net}\right)}{\lambda}$$

$$= \frac{2\left(3.0\times10^{8}\ \mathrm{m/s}\right)\left(1.9\times10^{-15}\ \mathrm{s}\right)}{550\times10^{-9}\ \mathrm{m}} = \boxed{2.0\ \text{fringe shifts}}$$

26.3 $$\Delta t = \gamma\,\Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{3.0\ \mathrm{s}}{\sqrt{1\text{-}(0.80)^2}} = \boxed{5.0\ \mathrm{s}}$$

26.4 (a) Observers on Earth measure the time for the astronauts to reach Alpha Centauri as $\Delta t_E = 4.42\ \mathrm{yr}$. But these observers are moving relative to the astronaut's internal biological clock and hence, experience a dilated version of the proper time interval Δt_p measured on that clock. From $\Delta t_E = \gamma\,\Delta t_p$, we find

$$\Delta t_p = \Delta t_E/\gamma = \Delta t_E\sqrt{1-(v/c)^2} = (4.42\ \mathrm{yr})\sqrt{1-(0.950)^2} = \boxed{1.38\ \mathrm{yr}}$$

(b) The astronauts are moving relative to the span of space separating Earth and Alpha Centauri. Hence, they measure a length contracted version of the proper distance, $L_p = 4.20\ \mathrm{ly}$. The distance measured by the astronauts is

$$L = L_p/\gamma = L_p\sqrt{1-(v/c)^2} = (4.20\ \mathrm{ly})\sqrt{1-(0.950)^2} = \boxed{1.31\ \mathrm{ly}}$$

26.5 (a) Since your ship is identical to his, the proper length of your ship (that is, the length you measure for your ship at rest relative to you) must be the same as the proper length that he measured for his own ship. Thus, both ships must be $\boxed{20\text{ m}}$ meters long when measured by observers at rest relative to them.

(b) His ship moves relative to you at the same speed your ship moves relative to him. Thus, you observe the same length contraction of his ship as he observed for your ship and measure his ship to be $\boxed{19\text{ m}}$ in length.

(c) From $L = L_p/\gamma = L_p\sqrt{1-(v/c)^2}$ with $L_p = 20\text{ m}$ and $L = 19\text{ m}$, we find

$$v = c\sqrt{1-\left(L/L_p\right)^2} = c\sqrt{1-\left(19\text{ m}/20\text{ m}\right)^2} = \boxed{0.31\,c}$$

26.6 (a) The time for 70 beats, as measured by the astronaut and any observer at rest with respect to the astronaut, is $\Delta t_p = 1.0\text{ min}$. The observer in the ship then measures a rate of $\boxed{70\text{ beats/min}}$.

(b) The observer on Earth moves at $v = 0.90c$ relative to the astronaut and measures the time for 70 beats as

$$\Delta t = \gamma\,\Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{1.0\text{ min}}{\sqrt{1\text{-}(0.90)^2}} = 2.3\text{ min}$$

This observer then measures a beat rate of $\dfrac{70\text{ beats}}{2.3\text{ min}} = \boxed{31\text{ beats/min}}$

26.7 (a) $\Delta t = \gamma\,\Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{2.6\times10^{-8}\text{ s}}{\sqrt{1\text{-}(0.98)^2}} = \boxed{1.3\times10^{-7}\text{ s}}$

(b) $d = v(\Delta t) = \left[0.98\left(3.0\times10^{8}\text{ m/s}\right)\right]\left(1.3\times10^{-7}\text{ s}\right) = \boxed{38\text{ m}}$

(c) $d' = v\left(\Delta t_p\right) = \left[0.98\left(3.0\times10^{8}\text{ m/s}\right)\right]\left(2.6\times10^{-8}\text{ s}\right) = \boxed{7.6\text{ m}}$

26.8 (a) As measured by observers in the ship (that is, at rest relative to the astronaut), the time required for 75.0 pulses is $\Delta t_p = 1.00 \text{ min}$.

The time interval required for 75.0 pulses as measured by the Earth observer is $\Delta t = \gamma \Delta t_p = \dfrac{1.00 \text{ min}}{\sqrt{1-(0.500)^2}}$, so the Earth observer measures a pulse rate of

$$rate = \frac{75.0}{\Delta t} = \frac{75.0\sqrt{1-(0.500)^2}}{1.00 \text{ min}} = \boxed{65.0/\text{min}}$$

(b) If $v = 0.990c$, then $\Delta t = \gamma \Delta t_p = \dfrac{1.00 \text{ min}}{\sqrt{1-(0.990)^2}}$

and the pulse rate observed on Earth is

$$rate = \frac{75.0}{\Delta t} = \frac{75.0\sqrt{1-(0.990)^2}}{1.00 \text{ min}} = \boxed{10.6/\text{min}}$$

That is, the life span of the astronaut (reckoned by the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

26.9 As seen by an Earth based observer, the time for the muon to travel 4.6 km is

$$\Delta t = \frac{d}{v} = \frac{4.6\times 10^3 \text{ m}}{0.99\left(3.0\times 10^8 \text{ m/s}\right)}$$

(a) In the rest frame of the muon, this time (the proper lifetime) is

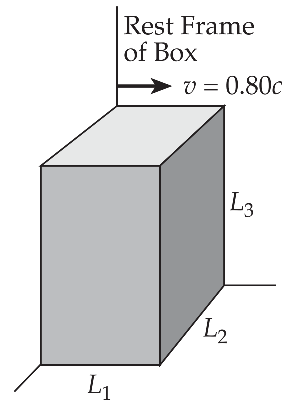
$$\Delta t_p = \frac{\Delta t}{\gamma} = \left[\frac{4.6\times 10^3 \text{ m}}{0.99\left(3.0\times 10^8 \text{ m/s}\right)}\right]\sqrt{1-(0.99)^2} = 2.2\times 10^{-6} \text{ s} = \boxed{2.2\ \mu\text{s}}$$

(b) The muon is at rest in "its frame" and thus travels zero distance as measured in this frame. However, during this time interval, the muon sees Earth move toward it by a distance of

$$d = v\left(\Delta t_p\right) = \left[0.99\left(3.0\times 10^8 \text{ m/s}\right)\right]\left(2.2\times 10^{-6} \text{ s}\right)$$

$$= 6.5\times 10^2 \text{ m} = \boxed{0.65 \text{ km}}$$

26.10 Length contraction occurs only in the dimension parallel to the motion.



(a) The sides labeled L_2 and L_3 in the figure at the right are unaffected, but the side labeled L_1 will appear contracted giving the box a $\boxed{\text{rectangular}}$ shape.

(b) The dimensions of the box, as measured by the observer moving at $v = 0.80c$ relative to it, are

$$L_2 = L_{2p} = \boxed{2.0 \text{ m}},\ L_3 = L_{3p} = \boxed{2.0 \text{ m}}, \text{ and}$$

$$L_1 = L_{1p}\sqrt{1-(v/c)^2} = (2.0 \text{ m})\sqrt{1-(0.80)^2} = \boxed{1.2 \text{ m}}$$

26.11 The proper length of the faster ship is three times that of the slower ship $\left(L_{pf} = 3L_{ps}\right)$, yet they both appear to have the same contracted length, L. Thus,

$$L = L_{ps}\sqrt{1-(v_s/c)^2} = (3L_{ps})\sqrt{1-(v_f/c)^2}, \text{ or } 1-(v_s/c)^2 = 9-9(v_f/c)^2$$

This gives $v_f = \dfrac{c\sqrt{8+(v_s/c)^2}}{3} = \dfrac{\sqrt{8+(0.350)^2}}{3}c = \boxed{0.950c}$

26.12 (a) Observer A measures the proper length, $L_{pA} = L_0$, of the rod that is at rest relative to her. However, observer B is moving at $v = 0.955c$ relative to this rod and will measure the contracted length L_A for it, where

$$L_A = L_{pA}/\gamma = L_0\sqrt{1-(v/c)^2} = L_0\sqrt{1-(0.955)^2} = \boxed{0.297L_0}$$

(b) Observer A is moving at $v = 0.955\,c$ relative to the rod at rest in B's reference frame, and she measures the contracted length $L_B = L_0$ for the length of this rod. This rod's proper length L_{pB} (the length measured by observer B who is at rest relative to this rod) is found from $L_B = L_{pB}\sqrt{1-(v/c)^2}$ as

$$L_{pB} = \frac{L_B}{\sqrt{1-(v/c)^2}} = \frac{L_0}{\sqrt{1-(0.955)^2}} = 3.37L_0$$

Thus, the ratio of the length of A's rod to the length of B's rod, as measured by observer B, is

$$ratio = \frac{L_A}{L_{pB}} = \frac{0.297L_0}{3.37L_0} = \boxed{0.088}$$

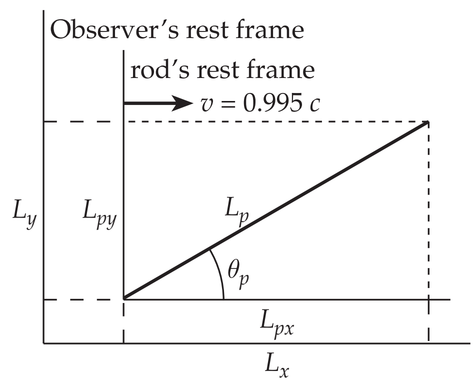
26.13 The trackside observer sees the supertrain length-contracted as

$$L = L_p\sqrt{1-(v/c)^2} = (100\text{ m})\sqrt{1-(0.95)^2} = 31\text{ m}$$

The supertrain $\boxed{\text{appears to fit in the tunnel}}$

with $50\text{ m} - 31\text{ m} = \boxed{19\text{ m to spare}}$

26.14 Note: Excess digits are retained in some steps given below to more clearly illustrate the method of solution.



We are given that $L = 2.00$ m, and $\theta = 30.0°$ (both measured in the observer's rest frame). The components of the rod's length as measured in the observer's rest frame are

$$L_x = L\cos\theta = (2.00\text{ m})\cos 30.0° = 1.732\text{ m}$$

and $$L_y = L\sin\theta = (2.00\text{ m})\sin 30.0° = 1.00\text{ m}$$

The component of length parallel to the motion has been contracted, but the component perpendicular to the motion is unaltered. Thus, $L_{py} = L_y = 1.00$ m and

$$L_{px} = \frac{L_x}{\sqrt{1-(v/c)^2}} = \frac{1.732\text{ m}}{\sqrt{1-(0.995)^2}} = 17.34\text{ m}$$

(a) The proper length of the rod is then

$$L_p = \sqrt{L_{px}^2 + L_{py}^2} = \sqrt{(17.34\text{ m})^2 + (1.00\text{ m})^2} = \boxed{17.4\text{ m}}$$

(b) The orientation angle in the rod's rest frame is

$$\theta_p = \tan^{-1}\left(\frac{L_{py}}{L_{px}}\right) = \tan^{-1}\left(\frac{1.00\text{ m}}{17.34\text{ m}}\right) = \boxed{3.30°}$$

26.15 The centripetal acceleration is provided by the gravitational force, so

$$\frac{mv^2}{r} = \frac{GMm}{r^2}\text{, giving Cooper's speed as } v = \left[\frac{GM}{r}\right]^{1/2} = \left[\frac{GM}{(R_E + h)}\right]^{1/2}$$

or $$v = \left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.38\times10^6 + 0.160\times10^6\right)\ \text{m}}\right]^{\frac{1}{2}} = 7.81\times10^3\ \text{m/s}$$

Then the time period of one orbit is,

$$T = \frac{2\pi(R_E + h)}{v} = \frac{2\pi\left(6.54\times10^6\ \text{m}\right)}{7.81\times10^3\ \text{m/s}} = 5.26\times10^3\ \text{s}$$

(a) The time difference for 22 orbits is

$$\Delta t - \Delta t_p = (\gamma - 1)\Delta t_p = \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right](22T)$$

$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T)$$

$$= \frac{1}{2}\left(\frac{7.81\times10^3\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2 22\left(5.26\times10^3\ \text{s}\right) = \boxed{39.2\ \mu\text{s}}$$

(b) For one orbit, $\Delta t - \Delta t_p = \dfrac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$

The press report is $\boxed{\text{accurate to one digit}}$

26.16 (a) $\Delta t = \gamma\,\Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{15.0\text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\text{ yr}}$

(b) $d = v(\Delta t) = [0.700c](21.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](21.0\text{ yr}) = \boxed{14.7\text{ ly}}$

(c) The astronauts see Earth flying out the back window at $0.700c$:

$$d = v(\Delta t_p) = [0.700c](15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$$

(d) Mission control gets signals for 21.0 yr while the battery is operating and then for 14.7 yr after the battery stops powering the transmitter, 14.7 ly away:
$21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$

26.17 The momentum of the electron is

$$p_e = \gamma m_e v = \frac{(9.11\times10^{-31}\text{ kg})(0.90c)}{\sqrt{1-(0.90)^2}} = (1.9\times10^{-30}\text{ kg})c$$

If the proton has the same momentum, then

$$p_p = \gamma m_p v = \frac{(1.67\times10^{-27}\text{ kg})v}{\sqrt{1-(v/c)^2}} = (1.9\times10^{-30}\text{ kg})c$$

which reduces to $(8.9\times10^2)(v/c) = \sqrt{1-(v/c)^2}$ and yields

$$v = (1.1\times10^{-3})c = (1.1\times10^{-3})(3.0\times10^8\text{ m/s}) = \boxed{3.3\times10^5\text{ m/s}}$$

26.18 The momentum of the electron is $p_e = \gamma m_e v = \dfrac{(9.11\times10^{-31}\text{ kg})v}{\sqrt{1-(v/c)^2}}$.

(a) When $v = 0.010c$,

$$p_e = \frac{(9.11\times10^{-31}\text{ kg})(0.010)(3.0\times10^8\text{ m/s})}{\sqrt{1-(0.010)^2}} = \boxed{2.7\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(b) If $v = 0.50c$, $p_e = \boxed{1.6\times10^{-22}\text{ kg}\cdot\text{m/s}}$, and

(c) When $v = 0.90c$, $p_e = \boxed{5.6\times10^{-22}\ \text{kg}\cdot\text{m/s}}$

26.19 Momentum must be conserved, so the momenta of the two fragments must add to zero. Thus, their magnitudes must be equal, or

$$p_2 = p_1 = \gamma_1 m_1 v_1 = \frac{\left(2.50\times10^{-28}\ \text{kg}\right)(0.893c)}{\sqrt{1-(0.893)^2}} = \left(4.96\times10^{-28}\ \text{kg}\right)c$$

For the heavier fragment, $\dfrac{\left(1.67\times10^{-27}\ \text{kg}\right)v}{\sqrt{1-(v/c)^2}} = \left(4.96\times10^{-28}\ \text{kg}\right)c$

which reduces to $3.37(v/c) = \sqrt{1-(v/c)^2}$ and yields $v = \boxed{0.285c}$

26.20 Using the relativistic form, $p = \dfrac{mv}{\sqrt{1-(v/c)^2}} = \gamma mv$

we find the difference Δp from the classical momentum, mv:

$$\Delta p = \gamma mv - mv = (\gamma-1)mv$$

(a) The difference is 1.00% when $(\gamma-1)mv = 0.010\,0\gamma mv$:

$$\gamma = \frac{1}{0.990} = \frac{1}{\sqrt{1-(v/c)^2}} \quad\Rightarrow\quad 1-(v/c)^2 = (0.990)^2 \qquad \text{or} \qquad v = \boxed{0.141c}$$

(b) The difference is 10.0% when $(\gamma-1)mv = 0.100\gamma mv$:

$$\gamma = \frac{1}{0.900} = \frac{1}{\sqrt{1-(v/c)^2}} \quad\Rightarrow\quad 1-(v/c)^2 = (0.900)^2 \qquad \text{or} \qquad v = \boxed{0.436c}$$

26.21 Taking toward the right as the positive direction, with v_{pL} = velocity of proton relative to laboratory, v_{pe} = velocity of proton relative to electron, and v_{eL} = velocity of electron relative to laboratory, the relativistic velocity addition equation gives

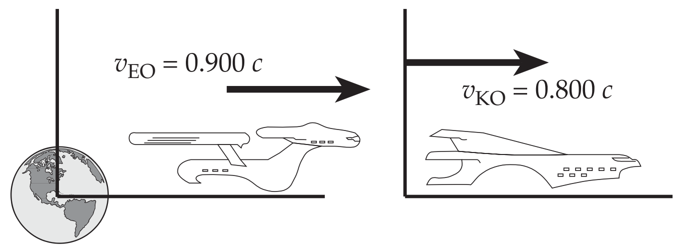
$$v_{\text{pL}} = \frac{v_{\text{pe}} + v_{\text{eL}}}{1+\dfrac{v_{\text{pe}}\cdot v_{\text{eL}}}{c^2}} = \frac{-0.70c + 0.90c}{1+\dfrac{(-0.70c)(0.90c)}{c^2}} = \boxed{+0.54c}$$

26.22 Taking toward the right as positive, with v_{LR} = velocity of L relative to R, v_{LE} = velocity of L relative to Earth, v_{RE} = velocity of R relative to Earth, and $v_{ER} = -v_{RE}$ = velocity of Earth relative to R, the relativistic velocity addition equation gives

$$v_{LR} = \frac{v_{LE} + v_{ER}}{1 + \frac{v_{LE} \cdot v_{ER}}{c^2}} = \frac{-0.70c + (-0.70c)}{1 + \frac{(-0.70c)(-0.70c)}{c^2}} = \boxed{-0.94c}$$

26.23 v_{EO} = velocity of Enterprise relative to Observers on Earth.

v_{KO} = velocity of Klingon ship relative to Observers on Earth.



$v_{OK} = -v_{KO}$ = velocity of Observers on Earth relative to Klingon ship

The relativistic velocity addition equation yields

$$v_{EK} = \frac{v_{EO} + v_{OK}}{1 + \frac{v_{EO} \cdot v_{OK}}{c^2}} = \frac{0.900c + (-0.800c)}{1 + \frac{(0.900c)(-0.800c)}{c^2}} = \boxed{+0.357c}$$

26.24 With v_{RS} = velocity of rocket relative to ship, v_{RE} = velocity of rocket relative to Earth, v_{SE} = velocity of ship relative to Earth, and $v_{ES} = -v_{SE}$ = velocity of Earth relative to ship, the relativistic velocity addition equation gives

$$v_{RS} = \frac{v_{RE} + v_{ES}}{1 + \frac{v_{RE} \cdot v_{ES}}{c^2}} = \frac{0.950c + (-0.750c)}{1 + \frac{(0.950c)(-0.750c)}{c^2}} = \boxed{+0.696c}$$

26.25 Taking toward the right as positive, with v_{BA} = velocity of B relative to A, v_{RA} = velocity of rocket relative to A, v_{RB} = velocity of rocket relative to B, and $v_{BR} = -v_{RB}$ = velocity of B relative to rocket, the relativistic velocity addition equation gives

$$v_{BA} = \frac{v_{BR} + v_{RA}}{1 + \frac{v_{BR} \cdot v_{RA}}{c^2}} = \frac{-(-0.95c) + (0.92c)}{1 + \frac{[-(-0.95c)](0.92c)}{c^2}} = \boxed{+0.998c}$$

26.26 First, determine the velocity of the pulsar relative to the rocket.

Taking toward Earth as positive, with v_{PR} = velocity of pulsar relative to rocket, v_{PE} = velocity of pulsar relative to Earth, v_{RE} = velocity of rocket relative to Earth, and $v_{ER} = -v_{RE}$ = velocity of Earth relative to rocket, the relativistic velocity addition equation gives

$$v_{PR} = \frac{v_{PE} + v_{ER}}{1 + \frac{v_{PE} \cdot v_{ER}}{c^2}} = \frac{0.950c + \left[-(-0.995c)\right]}{1 + \frac{(0.950c)(0.995c)}{c^2}} = 0.999\,87\,c$$

The period of the pulsar in its own reference frame is $\Delta t_p = 0.100$ s, and its period in the rocket's frame of reference is

$$\Delta t = \gamma\left(\Delta t_p\right) = \frac{\Delta t_p}{\sqrt{1-(v_{PR}/c)^2}} = \frac{0.100\text{ s}}{\sqrt{1-(0.999\,87)^2}} = 6.20\text{ s}$$

and the frequency in the rocket's frame is $f = \frac{1}{\Delta t} = \frac{1}{6.20\text{ s}} = \boxed{0.161\text{ Hz}}$

26.27 The instructors measure a proper time of $\Delta t_p = 50$ min on their clock.

(a) First determine the velocity of the students relative to the instructors (and hence relative to the official clock). Taking toward the right in Figure P26.27 as the positive direction, with v_{SI} = velocity of students relative to instructors, v_{SE} = velocity of students relative to Earth, v_{IE} = velocity of instructors relative to Earth, and $v_{EI} = -v_{IE}$ = velocity of Earth relative to instructors, we find

$$v_{SI} = \frac{v_{SE} + v_{EI}}{1 + \frac{v_{SE} \cdot v_{EI}}{c^2}} = \frac{0.60c + (-0.28c)}{1 + \frac{(0.60c)(-0.28c)}{c^2}} = 0.385c$$

The elapsed time on the student's clock is then

$$\Delta t = \gamma\,\Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v_{SI}/c)^2}} = \frac{50\text{ min}}{\sqrt{1-(0.385)^2}} = \boxed{54\text{ min}}$$

(b) The elapsed time on Earth is

$$\Delta t = \gamma\,\Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v_{EI}/c)^2}} = \frac{50\text{ min}}{\sqrt{1-(-0.28)^2}} = \boxed{52\text{ min}}$$

26.28 (a) $E_R = mc^2 = \left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^8\text{ m/s}\right)^2\left(\dfrac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = \boxed{939\text{ MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R = \dfrac{E_R}{\sqrt{1-(v/c)^2}}$

$$= \frac{939\text{ MeV}}{\sqrt{1-(0.950)^2}} = 3.01\times10^3\text{ MeV} = \boxed{3.01\text{ GeV}}$$

(c) $KE = E - E_R = 3.01\times10^3\text{ MeV} - 939\text{ MeV} = 2.07\times10^3\text{ MeV} = \boxed{2.07\text{ GeV}}$

26.29 If $KE = E_R$, then $KE = E - E_R = (\gamma-1)E_R = E_R$ giving $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = 2$

Therefore, $v = c\sqrt{1-1/\gamma^2} = c\sqrt{3/4} = \boxed{c\left(\sqrt{3}/2\right)}$

26.30 The work done accelerating a free particle from rest to speed v equals the kinetic energy given that particle. Thus,

$$KE = E - E_R = \gamma E_R - E_R = 3\,750\text{ MeV}$$

For a proton, $E_R = mc^2 = \left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^8\right)^2\left(\dfrac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 939\text{ MeV}$

Thus, $\gamma - 1 = \dfrac{3\,750\text{ MeV}}{939\text{ MeV}} = 3.99$ giving $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = 4.99$

or $v = c\sqrt{1-(1/4.99)^2} = \boxed{0.980\,c}$

26.31 The nonrelativistic expression for kinetic energy is $KE = \frac{1}{2}mv^2$, while the relativistic expression is $KE = E - E_R = (\gamma - 1)E_R = (\gamma - 1)mc^2$ where $\gamma = 1\Big/\sqrt{1-(v/c)^2}$. Thus, when the relativistic kinetic energy is twice the predicted nonrelativistic value, we have

$$\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right)mc^2 = 2\left(\frac{1}{2}mv^2\right) \qquad \text{or} \qquad 1 = \left[1 + \left(\frac{v}{c}\right)^2\right]\sqrt{1-(v/c)^2}$$

Squaring both sides of the last result and simplifying gives $\left(\frac{v}{c}\right)^2\left[\left(\frac{v}{c}\right)^4 + \left(\frac{v}{c}\right)^2 - 1\right] = 0$

Ignoring the trivial solution $v/c = 0$, we must have $\left(\frac{v}{c}\right)^4 + \left(\frac{v}{c}\right)^2 - 1 = 0$

This is a quadratic equation of the form $x^2 + x - 1 = 0$ with $x = (v/c)^2$. Applying the quadratic formula gives $x = \frac{-1 \pm \sqrt{5}}{2}$

Since $x = (v/c)^2$, we ignore the negative solution and find

$$x = \left(\frac{v}{c}\right)^2 = \frac{-1+\sqrt{5}}{2} = 0.618 \qquad \text{which yields} \qquad v = c\sqrt{0.618} = \boxed{0.786\,c}$$

26.32 The energy input to the electron will be $W = E_f - E_i = (\gamma_f - \gamma_i)E_R$

or $$W = \left(\frac{1}{\sqrt{1-(v_f/c)^2}} - \frac{1}{\sqrt{1-(v_i/c)^2}}\right)E_R \quad \text{where } E_R = 0.511 \text{ MeV}$$

(a) If $v_f = 0.900c$ and $v_i = 0.500c$, then

$$W = \left(\frac{1}{\sqrt{1-(0.900)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)(0.511 \text{ Mev}) = \boxed{0.582 \text{ MeV}}$$

(b) When $v_f = 0.990c$ and $v_i = 0.900c$, we have

$$W = \left(\frac{1}{\sqrt{1-(0.990)^2}} - \frac{1}{\sqrt{1-(0.900)^2}} \right)(0.511 \text{ Mev}) = \boxed{2.45 \text{ MeV}}$$

26.33 When a cubical box moves at speed v, the dimension parallel to the motion is length contracted and other dimensions are unaffected. If all edges of the box had length L_p when at rest, the volume of the moving box is

$$V = (L_p)(L_p)\left(L_p\sqrt{1-(v/c)^2}\right) = V_p\sqrt{1-(v/c)^2}$$

The relativistic density is

$$\rho = \frac{m}{V} = \frac{m}{V_p\sqrt{1-(v/c)^2}} = \frac{8.00 \text{ g}}{(1.00 \text{ cm})^3\sqrt{1-(0.900)^2}} = \boxed{18.4 \text{ g/cm}^3}$$

Note that $\rho = \dfrac{m}{V} = \dfrac{mc^2}{c^2 V} = \dfrac{E_R}{c^2 V}$ as suggested in the problem statement.

26.34 Let m_1 be the mass of the fragment moving at $v_1 = 0.868c$, and m_2 be the mass moving at $v_2 = 0.987c$.

From conservation of mass-energy,

$$E = \frac{m_1c^2}{\sqrt{1-(0.868)^2}} + \frac{m_2c^2}{\sqrt{1-(0.987)^2}} = mc^2$$

giving $2.01m_1 + 6.22m_2 = 3.34 \times 10^{-27}$ kg (1)

The momenta of the two fragments must add to zero, so the magnitudes must be equal, giving $p_1 = p_2$ or $\gamma_1 m_1 v_1 = \gamma_2 m_2 v_2$. This yields

$$2.01m_1(0.868c) = 6.22m_2(0.987c), \text{ or } m_1 = 3.52m_2 \quad (2)$$

Substituting equation (2) into (1) gives

$$(7.07 + 6.22)m_2 = 3.34 \times 10^{-27} \text{ kg}, \text{ or } m_2 = \boxed{2.51 \times 10^{-28} \text{ kg}}$$

Equation (2) then yields $m_1 = 3.52\left(2.51 \times 10^{-28} \text{ kg}\right) = \boxed{8.84 \times 10^{-28} \text{ kg}}$

26.35 The total kinetic energy equals the energy of the original photon minus the total rest energy of the pair, or

$$KE = E_\gamma - 2E_R = 3.00 \text{ MeV} - 2(0.511 \text{ MeV}) = \boxed{1.98 \text{ MeV}}$$

26.36 (a) The energy of the original photon must equal the total energy (total kinetic energy plus total rest energy) of the pair.

Thus, $E_\gamma = KE_{total} + 2E_R = 2.50 \text{ MeV} + 2(0.511 \text{ MeV}) = \boxed{3.52 \text{ MeV}}$

(b) $f = \dfrac{E_\gamma}{h} = \left(\dfrac{3.52 \text{ MeV}}{6.63\times10^{-34} \text{ J}\cdot\text{s}}\right)\left(\dfrac{1.60\times10^{-13} \text{ J}}{1 \text{ MeV}}\right) = \boxed{8.50\times10^{20} \text{ Hz}}$

26.37 To produce minimum energy photons, both members of the proton-antiproton pair should be at rest (that is, have zero kinetic energy) just before annihilation. Then, the total momentum is zero both before and after annihilation. This means that the two photons must have equal magnitude but oppositely directed momenta, and hence, equal energies.

From conservation of energy, $2E_\gamma = 2(E_R + 0)$ or $E_\gamma = E_R$ for each photon. The rest energy of a proton is

$E_R = m_p c^2 = (1.67\times10^{-27} \text{ kg})(3.00\times10^8 \text{ m/s})^2 (1 \text{ MeV}/1.60\times10^{-13} \text{ J}) = 939 \text{ MeV}$

$$f = \frac{E_\gamma}{h} = \frac{939 \text{ MeV}}{6.63\times10^{-34} \text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-13} \text{ J}}{1 \text{ MeV}}\right) = \boxed{2.27\times10^{23} \text{ Hz}}$$

and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{2.27\times10^{23} \text{ Hz}} = 1.32\times10^{-15} \text{ m} = \boxed{1.32 \text{ fm}}$

26.38 The total momentum is zero both before and after the annihilation. Thus, the momenta of the two photons must have equal magnitudes and be oppositely directed. Since $E_\gamma = p_\gamma c$, the photon energies are also equal and conservation of energy gives

$$2E_\gamma = 2(KE + E_R) = 2E = 2E_R\Big/\sqrt{1-(v/c)^2} \qquad \text{or} \qquad E_\gamma = E_R\Big/\sqrt{1-(v/c)^2}$$

$$E_\gamma = \frac{E_R}{\sqrt{1-(v/c)^2}} = \frac{0.511\text{ MeV}}{\sqrt{1-(0.60)^2}} = \boxed{0.64\text{ MeV}}$$

$$p_\gamma = \frac{E_\gamma}{c} = 0.64\ \frac{\text{Mev}}{c}$$

$$= \frac{(0.64\text{ Mev})(1.60\times10^{-13}\text{ J/1 MeV})}{3.00\times10^8\text{ m/s}} = \boxed{3.4\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

26.39 $KE = E - E_R = (\gamma - 1)E_R$

so $\gamma = 1 + \dfrac{KE}{E_R} = \dfrac{1}{\sqrt{1-(v/c)^2}}$ giving $v = c\sqrt{1 - \dfrac{1}{(1+KE/E_R)^2}}$

(a) When $KE = q(\Delta V) = e(500\text{ V}) = 500\text{ eV}$, and $E_R = 939\text{ MeV}$, this yields

$$v = c\sqrt{1-\frac{1}{\left[1+500\text{ eV}/(939\times10^6\text{ eV})\right]^2}} = 1.03\times10^{-3}\,c = \boxed{3.10\times10^5\text{ m/s}}$$

(b) When $KE = q(\Delta V) = e(5.00\times10^8\text{ V}) = 500\text{ MeV}$

$$v = c\sqrt{1-\frac{1}{(1+500\text{ MeV}/939\text{ MeV})^2}} = \boxed{0.758\,c}$$

26.40 From $E^2 = (pc)^2 + E_R^2$ with $E = 5E_R$, we find that $p = \dfrac{E_R\sqrt{24}}{c}$

(a) For an electron, $p = \dfrac{(0.511\text{ MeV})\sqrt{24}}{c} = \boxed{2.50\text{ MeV}/c}$

(b) For a proton, $p = \dfrac{(939\text{ MeV})\sqrt{24}}{c} = 4.60\times10^3\ \dfrac{\text{MeV}}{c} = \boxed{4.60\text{ GeV}/c}$

26.41 $E = KE + E_R = 1.00 \text{ MeV} + 0.511 \text{ MeV} = 1.51 \text{ MeV}$, so $E^2 = (pc)^2 + E_R^2$ gives

$$p = \frac{\sqrt{E^2 - E_R^2}}{c} = \frac{\sqrt{(1.51 \text{ MeV})^2 - (0.511 \text{ MeV})^2}}{c} = \boxed{1.42 \text{ MeV}/c}$$

26.42 (a) The astronomer on Earth measured both the speed and distance in his frame of reference. Thus, the time to impact on his clock is

$$\Delta t = \frac{L_p}{v} = \frac{20.0 \text{ ly}}{0.800c} = \frac{20.0[c(1 \text{ yr})]}{0.800c} = \boxed{25.0 \text{ yr}}$$

(c) The observer on the meteoroid sees Earth rushing at him at speed $0.800c$ but sees a length contracted distance of separation given by

$$L = L_p\sqrt{1-(v/c)^2} = (20.0 \text{ ly})\sqrt{1-(0.800)^2} = \boxed{12.0 \text{ ly}}$$

(b) The time to impact as computed by the observer on the meteoroid is

$$\Delta t = \frac{L}{v} = \frac{12.0 \text{ ly}}{0.800c} = \frac{12.0[c(1 \text{ yr})]}{0.800c} = \boxed{15.0 \text{ yr}}$$

26.43 (a) Since Ted and Mary are in the same frame of reference, they measure the same speed for the ball, namely $u = \boxed{0.80c}$.

(b) $$\Delta t_p = \frac{L_p}{u} = \frac{1.8 \times 10^{12} \text{ m}}{0.80(3.00 \times 10^8 \text{ m/s})} = \boxed{7.5 \times 10^3 \text{ s}}$$

(c) The distance of separation, as measured by Jim, is

$$L = L_p\sqrt{1-(v/c)^2} = (1.8 \times 10^{12} \text{ m})\sqrt{1-(0.60)^2} = \boxed{1.4 \times 10^{12} \text{ m}}$$

Taking toward the right in Figure P26.43 as positive, with v_{BJ} = velocity of ball relative to Jim, v_{BT} = velocity of ball relative to Ted, and v_{TJ} = velocity of Ted relative to Jim, the relativistic velocity addition equation gives

$$v_{\text{BJ}} = \frac{v_{\text{BT}} + v_{\text{TJ}}}{1 + \frac{v_{\text{BT}} \cdot v_{\text{TJ}}}{c^2}} = \frac{-0.80c + 0.60c}{1 + \frac{(-0.80c)(0.60c)}{c^2}} = -0.38c$$

Thus, according to Jim, the ball moves with a *speed* of $\boxed{0.38c}$

26.44 The clock, at rest in the ship's frame of reference, will measure a proper time of $\Delta t_p = 10 \text{ h}$ before sounding. Observers on Earth move at $v = 0.75c$ relative to the clock and measure an elapsed time of

$$\Delta t = \gamma\left(\Delta t_p\right) = \frac{\Delta t_p}{\sqrt{1-\left(v/c\right)^2}} = \frac{10 \text{ h}}{\sqrt{1\text{-}\left(0.75\right)^2}} = 15 \text{ h}$$

The observers on Earth see the clock moving away at $0.75c$ and compute the distance traveled before the alarm sounds as

$$d = v\left(\Delta t\right) = \left[0.75\left(3.0\times10^8 \text{ m/s}\right)\right](15 \text{ h})\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{1.2\times10^{13} \text{ m}}$$

26.45 With $v_{\text{nL}} =$ velocity of nucleus relative to laboratory, $v_{\text{eL}} =$ velocity of electron relative to laboratory, $v_{\text{en}} =$ velocity of electron relative to nucleus, and $v_{\text{ne}} = -v_{\text{en}} =$ velocity of nucleus relative to electron, the relativistic velocity addition equation gives

$$v_{\text{nL}} = \frac{v_{\text{ne}} + v_{\text{eL}}}{1+\dfrac{v_{\text{ne}}\cdot v_{\text{eL}}}{\text{c}^2}} = \frac{-0.70c+0.85c}{1+\dfrac{\left(-0.70c\right)\left(0.85c\right)}{c^2}} = \boxed{+0.37c}$$

26.46 Taking away from Earth as positive, with $v_{\text{JE}} =$ velocity of jet of material relative to Earth, $v_{\text{JQ}} =$ velocity of jet relative to quasar, and $v_{\text{QE}} =$ velocity of quasar relative to Earth, the relativistic velocity addition equation gives

$$v_{\text{JE}} = \frac{v_{\text{JQ}} + v_{\text{QE}}}{1+\dfrac{v_{\text{JQ}}\cdot v_{\text{QE}}}{\text{c}^2}} = \frac{-0.550c+0.870c}{1+\dfrac{\left(-0.550c\right)\left(0.870c\right)}{c^2}} = \boxed{+0.614c}$$

26.47 (a) Observers on Earth measure the distance to Andromeda to be $d = 2.00\times10^6 \text{ ly} = \left(2.00\times10^6 \text{ yr}\right)c$. The time for the trip, in Earth's frame of reference, is

$$\Delta t = \gamma\left(\Delta t_p\right) = \frac{30.0 \text{ yr}}{\sqrt{1-\left(v/c\right)^2}}$$

The required speed is then $v = \frac{d}{\Delta t} = \frac{\left(2.00\times 10^{6}\text{ yr}\right)c}{30.0\text{ yr}\Big/\sqrt{1\text{-}\left(v/c\right)^2}}$

which gives $\left(1.50\times 10^{-5}\right)\frac{v}{c} = \sqrt{1-\left(v/c\right)^2}$

Squaring both sides of this equation and solving for v/c yields

$$\frac{v}{c} = \frac{1}{\sqrt{1+2.25\times 10^{-10}}} \approx 1-\frac{2.25\times 10^{-10}}{2} = \boxed{1-1.12\times 10^{-10}}$$

(b) $KE = (\gamma-1)mc^2$, and

$$\gamma = \frac{1}{\sqrt{1-\left(v/c\right)^2}} \approx \frac{1}{\sqrt{1-\left(1-1.12\times 10^{-10}\right)^2}} = \frac{1}{\sqrt{2.24\times 10^{-10}}} = 6.68\times 10^{4}$$

Thus,

$$KE = \left(6.68\times 10^{4}-1\right)\left(1.00\times 10^{6}\text{ kg}\right)\left(3.00\times 10^{8}\text{ m/s}\right)^2 = \boxed{6.01\times 10^{27}\text{ J}}$$

(c) $cost = KE\times rate$

$$= \left[\left(6.01\times 10^{27}\text{ J}\right)\left(\frac{1\text{ kWh}}{3.60\times 10^{6}\text{ J}}\right)\right](\$0.13/\text{kWh}) = \boxed{\$2.17\times 10^{20}}$$

26.48 The solution to this problem is easier if we start with part (b). First, compute the speed of the protons.

$$KE = (\gamma-1)E_R\text{, so } \gamma = 1+\frac{KE}{E_R} = 1+\frac{10^{13}\text{ MeV}}{939\text{ MeV}} = 1.06\times 10^{10}\text{, or } \gamma \sim 10^{10}$$

Thus, $v = c\sqrt{1-1/\gamma^2} \sim c\sqrt{1-10^{-20}} \approx c$

(b) The diameter of the galaxy, as seen in the proton's frame of reference, is

$$L = L_p\sqrt{1-(v/c)^2} = \frac{L_p}{\gamma} \sim \frac{10^5 \text{ ly}}{10^{10}} = 10^{-5} \text{ ly}$$

Since $1 \text{ ly} = (1 \text{ yr})c = (3.156\times10^7 \text{ s})(3.00\times10^8 \text{ m/s}) \approx 10^{16} \text{ m}$

$$L \approx 10^{-5} \text{ ly}\left(\frac{10^{16} \text{ m}}{1 \text{ ly}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) \sim \boxed{10^8 \text{ km}}$$

(a) The proton sees the galaxy rushing by at $v \approx c$. The time, in the proton's frame of reference for the galaxy to pass is

$$\Delta t = \frac{L}{v} \sim \frac{10^{-5} \text{ ly}}{c} = \frac{(10^{-5} \text{ yr})c}{c} = 10^{-5} \text{ yr}\left(\frac{3.156\times10^7 \text{ s}}{1 \text{ yr}}\right) = 316 \text{ s}$$

or $\Delta t \sim \boxed{10^2 \text{ s}}$

26.49 The length of the space ship, as measured by observers on Earth, is $L = L_p\sqrt{1-(v/c)^2}$. In Earth's frame of reference, the time required for the ship to pass overhead is

$$\Delta t = \frac{L}{v} = \frac{L_p\sqrt{1-(v/c)^2}}{v} = L_p\sqrt{\frac{1}{v^2} - \frac{1}{c^2}}$$

Thus,

$$\frac{1}{v^2} = \frac{1}{c^2} + \left(\frac{\Delta t}{L_p}\right)^2 = \frac{1}{(3.00\times10^8 \text{ m/s})^2} + \left(\frac{0.75\times10^{-6} \text{ s}}{300 \text{ m}}\right)^2 = 1.74\times10^{-17} \ \frac{\text{s}^2}{\text{m}^2}$$

or $$v = \frac{1}{\sqrt{1.74\times10^{-17} \ \frac{\text{s}^2}{\text{m}^2}}} = \left(2.4\times10^8 \ \frac{\text{m}}{\text{s}}\right)\left(\frac{c}{3.00\times10^8 \text{ m/s}}\right) = \boxed{0.80c}$$

26.50 (a) Classically, $KE = \frac{1}{2}mv^2 = \frac{1}{2}(78.0\text{ kg})(106\times10^3\text{ m/s})^2 = \boxed{4.38\times10^{11}\text{ J}}$

(b) $\gamma = \left[1-\left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} = \left[1-\left(\frac{106\times10^3\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2\right]^{-\frac{1}{2}} = \left[1-1.25\times10^{-7}\right]^{-\frac{1}{2}}$

Using the approximation $(1-x)^{-\frac{1}{2}} \approx 1+\frac{x}{2}$ for $x << 1$ gives $\gamma \approx 1+6.24\times10^{-8}$, so

$$KE = (\gamma-1)mc^2 \approx (6.24\times10^{-8})(78.0\text{ kg})(3.00\times10^8\text{ m/s})^2 = \boxed{4.38\times10^{11}\text{ J}}$$

In the limit $v << c$, the classical and relativistic equations yield the same results.

26.51 (a) Taking toward Earth as positive, with $v_{\text{LE}} =$ velocity of lander relative to Earth, $v_{\text{LS}} =$ velocity of lander relative to ship, and $v_{\text{SE}} =$ velocity of ship relative to Earth, the relativistic velocity addition equation gives

$$v_{\text{LE}} = \frac{v_{\text{LS}}+v_{\text{SE}}}{1+\frac{v_{\text{LS}}\cdot v_{\text{SE}}}{c^2}} = \frac{0.800c+0.600c}{1+\frac{(0.800c)(0.600c)}{c^2}} = \boxed{+0.946c}$$

(b) The length contracted distance of separation as measured in the ship's frame of reference is

$$L = L_p\sqrt{1-(v/c)^2} = (0.200\text{ ly})\sqrt{1-(0.600)^2} = \boxed{0.160\text{ ly}}$$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$, and Earth reduces it from the other end at $0.600c$. Thus,

$$\text{time} = \frac{0.160\text{ ly}}{0.800c+0.600c} = \frac{(0.160\text{ yr})c}{1.40c} = \boxed{0.114\text{ yr}}$$

(d) $KE = (\gamma-1)mc^2$

$$= \left(\frac{1}{\sqrt{1-(0.946)^2}}-1\right)(4.00\times10^5\text{ kg})\left(3.00\times10^8\ \frac{\text{m}}{\text{s}}\right)^2 = \boxed{7.50\times10^{22}\text{ J}}$$

26.52 The kinetic energy gained by the electron will equal the loss of potential energy, so

$$KE = q(\Delta V) = e(1.02\text{ MV}) = 1.02\text{ MeV}$$

(a) If Newtonian mechanics remained valid, then $KE = \frac{1}{2}mv^2$, and the speed attained would be

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2(1.02\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{9.11\times10^{-31}\text{ kg}}} = 5.99\times10^{8}\ \frac{\text{m}}{\text{s}} \approx \boxed{2c}$$

(b) $KE = (\gamma - 1)E_R$, so $\gamma = 1 + \frac{KE}{E_R} = 1 + \frac{1.02\text{ MeV}}{0.511\text{ MeV}} = 3.00$

The actual speed attained is

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(3.00)^2} = \boxed{0.943c}$$

26.53 (a) When at rest, muons have a mean lifetime of $\Delta t_p = 2.2\ \mu\text{s}$. In a frame of reference where they move at $v = 0.95c$, the dilated mean lifetime of the muons will be

$$\tau = \gamma(\Delta t_p) = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{2.2\ \mu\text{s}}{\sqrt{1-(0.95)^2}} = \boxed{7.0\ \mu\text{s}}$$

(b) In a frame of reference where the muons travel at $v = 0.95c$, the time required to travel 3.0 km is

$$t = \frac{d}{v} = \frac{3.0\times10^{3}\text{ m}}{0.95(3.00\times10^{8}\text{ m/s})} = 1.05\times10^{-5}\text{ s} = 10.5\ \mu\text{s}$$

If $N_0 = 5.0\times10^4$ muons started the 3.0 km trip, the number remaining at the end is

$$N = N_0 e^{-t/\tau} = (5.0\times10^4)e^{-10.5\ \mu\text{s}/7.0\ \mu\text{s}} = \boxed{1.1\times10^4}$$

26.54 (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt$ is the distance the ship moves toward the mirror in time t. Since this observer agrees that the speed of light is c, the time for it to travel distance D is $t = \frac{D}{c} = \frac{2d - vt}{c}$.

Solving for t yields $\boxed{t = \frac{2d}{c+v}}$

(b) The observer in the rocket sees a length-contracted initial distance to the mirror of $L = d\sqrt{1 - \frac{v^2}{c^2}}$ and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as $D = 2(L - y)$, where $y = vt/2$ is the distance the mirror moves toward the ship before the light reflects off it.

This observer also measures the speed of light to be c, so the time for it to travel distance D is $t = \frac{D}{c} = \frac{2}{c}\left(d\sqrt{1 - \frac{v^2}{c^2}} - \frac{vt}{2}\right)$

Solving for t and simplifying yields $\boxed{t = \frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$

26.55 The students are to measure a proper time of $\Delta t_p = T_0$ on the clock at rest relative to them. The professor will measure a dilated time given by

$$\Delta t = \gamma\left(\Delta t_p\right) = T + t, \qquad \text{or} \qquad \frac{T_0}{\sqrt{1 - (v/c)^2}} = T + t \tag{1}$$

where T is the time she should wait before sending the light signal, and t is the transit time for the signal, both measured in Earth's frame of reference.

The distance, measured in Earth's frame, the signal must travel to reach the receding students is $d = v(T + t)$, and the transit time is

$$t = \frac{d}{c} = \frac{v}{c}(T + t) \qquad \text{or} \qquad t = \frac{(v/c)T}{1 - (v/c)} \tag{2}$$

Substituting equation (2) into (1) yields

$$\frac{T_0}{\sqrt{1-(v/c)^2}} = T\left(1+\frac{v/c}{1-v/c}\right) = \frac{T}{1-v/c}$$

Thus, $T = T_0\left[\frac{1-v/c}{\sqrt{1-(v/c)^2}}\right] = T_0\left[\frac{(1-v/c)^2}{(1-v/c)(1+v/c)}\right]^{\frac{1}{2}}$ or $\boxed{T = T_0\sqrt{\frac{1-v/c}{1+v/c}}}$

26.56 The work required equals the increase in the gravitational potential energy, or $W = \frac{GM_{Sun}m}{R_g}$. If this is to equal the rest energy of the mass removed, then

$$mc^2 = \frac{GM_{Sun}m}{R_g} \qquad \text{or} \qquad R_g = \frac{GM_{Sun}}{c^2}$$

$$R_g = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(3.00\times10^8\ \text{m/s})^2} = 1.47\times10^3\ \text{m} = \boxed{1.47\ \text{km}}$$

26.57 (a) The components of length, measured in the frame moving with the rod, are

$$L_{px} = L_0\cos\theta_0 \quad \text{and} \quad L_{py} = L_0\sin\theta_0$$

The stationary observer will see a length contracted component in the direction parallel to the motion, with the other component unaffected. Therefore,

$$L_x = L_{px}\sqrt{1-(v/c)^2} = L_0\sqrt{1-(v/c)^2}\cos\theta_0 \qquad \text{and} \qquad L_y = L_{py} = L_0\sin\theta_0$$

The length of the rod, as measured by the stationary observer, is

$$L = \sqrt{L_x^2 + L_y^2} = L_0\sqrt{\cos^2\theta_0 - (v/c)^2\cos^2\theta_0 + \sin^2\theta_0}$$

or $\boxed{L = L_0\sqrt{1-(v/c)^2\cos^2\theta_0}}$

(b) The orientation angle seen by the stationary observer is given by

$$\tan\theta = \frac{L_y}{L_x} = \frac{L_0\sin\theta_0}{L_0\sqrt{1-(v/c)^2}\cos\theta_0}, \text{ or } \boxed{\tan\theta = \gamma\tan\theta_0}$$

26.58 The speed of light in a vacuum, expressed in km/h, is

$$c = \left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ km}}{10^{3}\text{ m}}\right)\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = 1.08\times10^{9}\ \text{km/h}$$

At the speed limit, the momentum is

$$p_{\text{lim}} = \frac{mv_{\text{lim}}}{\sqrt{1-\left(v_{\text{lim}}/c\right)^{2}}} = \frac{m(90.0\text{ km/h})}{\sqrt{1-\left(90.0\text{ km/h}/1.08\times10^{9}\text{ km/h}\right)^{2}}} = m(90.0\text{ km/h})$$

Similarly, at $v = 190$ km/h, $p = m(190\text{ km/h})$, and with the fine proportional to the excess momentum $\left[Fine = k\left(p - p_{\text{lim}}\right)\text{ with } k = \text{constant}\right]$, we find that

$$k = \frac{Fine}{p - p_{\text{lim}}} = \frac{\$80.0}{m(190\text{ km/h} - 90.0\text{ km/h})} = \frac{\$80.0}{(100m)\text{ km/h}}$$

(a) When $v = 1\,090$ km/h,

$$p = \frac{m(1\,090\text{ km/h})}{\sqrt{1-(1\,090\text{ km/h})^{2}/\left(1.08\times10^{9}\text{ km/h}\right)^{2}}} = m(1\,090\text{ km/h})$$

and the fine is

$$Fine = k\left(p - p_{\text{lim}}\right) = \left(\frac{\$80.0}{(100m)\text{ km/h}}\right)\left[m(1\,090\text{ km/h} - 90.0\text{ km/h})\right] = \boxed{\$800}$$

(b) When $v = 1\,000\,000\,090$ km/h

$$p = \frac{m(1\,000\,000\,090\text{ km/h})}{\sqrt{1-(1\,000\,000\,090\text{ km/h})^{2}/\left(1.08\times10^{9}\text{ km/h}\right)^{2}}} = m\left(2.65\times10^{9}\text{ km/h}\right)$$

and the fine is

$$Fine = \left(\frac{\$80.0}{(100m)\text{ km/h}}\right)\left[m\left(2.65\times10^{9}\text{ km/h} - 90.0\text{ km/h}\right)\right] = \boxed{\$2.12\times10^{9}}$$

26.59 According to Earth-based observers, the times required for the two trips are

For Speedo: $T_S = \dfrac{L_0}{v_S} = \dfrac{20.0 \text{ yr}}{0.950\, c} = 21.05 \text{ yr}$

For Goslo: $T_G = \dfrac{L_0}{v_G} = \dfrac{20.0 \text{ yr}}{0.750\, c} = 26.67 \text{ yr}$

Thus, after Speedo lands, he must wait and age at the same rate as planet-based observers, for an additional $\Delta T_S = T_G - T_S = (26.67 - 21.05) \text{ yr} = 5.614 \text{ yr}$ before Goslo arrives.

The time required for the trip according to Speedo's internal biological clock (which measures the proper time for his aging process during the trip) is

$$T_{0S} = \frac{T_S}{\gamma} = T_S\sqrt{1-(v_S/c)^2} = (21.05 \text{ yr})\sqrt{1-(0.950)^2} = 6.574 \text{ yr}$$

When Goslo arrives, Speedo has aged a total of

$$\Delta t_S = T_{0S} + \Delta T_S = 6.574 \text{ yr} + 5.614 = 12.19 \text{ yr}$$

The time required for the trip according to Goslo's internal biological clock (and hence the amount he ages) is

$$\Delta t_G = T_{0G} = \frac{T_G}{\gamma} = T_G\sqrt{1-(v_G/c)^2} = (26.67 \text{ yr})\sqrt{1-(0.750)^2} = 17.64 \text{ yr}$$

Thus, we see that when he arrives, $\boxed{\text{Goslo is older}}$ than Speedo, having aged an additional

$$\Delta t_G - \Delta t_S = 17.64 \text{ yr} - 12.19 \text{ yr} = \boxed{5.45 \text{ yr}}$$