Chapter 13
Vibrations and Waves

Quick Quizzes

1. (d). To complete a full cycle of oscillation, the object must travel distance $2A$ to position $x = -A$ and then travel an additional distance $2A$ returning to the original position at $x = +A$.

2. (c). The force producing harmonic oscillation is always directed toward the equilibrium position, and hence, directed opposite to the displacement from equilibrium. The acceleration is in the direction of the force. Thus, it is also always directed opposite to the displacement from equilibrium.

3. (b). In simple harmonic motion, the force (and hence, the acceleration) is directly proportional to the displacement from equilibrium. Therefore, force and acceleration are both at a maximum when the displacement is a maximum.

4. (a). The period of an object-spring system is $T = 2\pi\sqrt{m/k}$. Thus, increasing the mass by a factor of 4 will double the period of oscillation.

5. (c). The total energy of the oscillating system is equal to $\frac{1}{2}kA^2$, where A is the amplitude of oscillation. Since the object starts from rest at displacement A in both cases, it has the same amplitude of oscillation in both cases.

6. (d). The expressions for the total energy, maximum speed, and maximum acceleration are $E = \frac{1}{2}kA^2$, $v_{\text{max}} = A\sqrt{k/m}$, and $a_{\text{max}} = A(k/m)$ where A is the amplitude. Thus, all are changed by a change in amplitude. The period of oscillation is $T = 2\pi\sqrt{m/k}$ and is unchanged by altering the amplitude.

7. (c), (b). An accelerating elevator is equivalent to a gravitational field. Thus, if the elevator is accelerating upward, this is equivalent to an increased effective gravitational field magnitude g, and the period will decrease. Similarly, if the elevator is accelerating downward, the effective value of g is reduced and the period increases. If the elevator moves with constant velocity, the period of the pendulum is the same as that in the stationary elevator.

8. (a). The clock will run *slow*. With a longer length, the period of the pendulum will increase. Thus, it will take longer to execute each swing, so that each second according to the clock will take longer than an actual second.

9. (b). Greater. The value of g on the Moon is about one-sixth the value of g on Earth, so the period of the pendulum on the moon will be greater than the period on Earth.

Answers to Even Numbered Conceptual Questions

2. Each half-spring will have twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant which is twice that of the complete spring.

4. To understand how we might have anticipated this similarity in speeds, consider sound as a motion of air molecules in a certain direction superimposed on the random, high speed, thermal molecular motions predicted by kinetic theory. Individual molecules experience billions of collisions per second with their neighbors, and as a result, do not travel very far in any appreciable time interval. With this interpretation, the energy of a sound wave is carried as kinetic energy of a molecule and transferred to neighboring molecules by collision. Thus, the energy transmitted by a sound wave in, say, a compression, travels from molecule to molecule at about the rms speed, or actually somewhat less, as observed, since multiple collisions slow the process a bit.

6. Friction. This includes both air-resistance and damping within the spring.

8. No. The period of vibration is $T = 2\pi\sqrt{L/g}$ and g is smaller at high altitude. Therefore, the period is longer on the mountain top and the clock will run slower.

10. Shorten the pendulum to decrease the period between ticks.

12. The speed of the pulse is $v = \sqrt{F/\mu}$, so increasing the tension F in the hose increases the speed of the pulse. Filling the hose with water increases the mass per unit length μ, and will decrease the speed of the pulse.

14. Assume that the building has height h and that you wish the jumper to start stretching the bungee cord when he reaches a height of $h/2$ above the ground. The unstretched length of the bungee cord must then be $\ell = h/2$.

Furthermore, assume that you wish the bungee cord to bring the jumper to rest just as he reaches ground level (that is, when the cord is stretched a distance of $\Delta\ell = h/2$). For this to occur, the elastic potential energy in the cord at this point must equal the total gravitational potential energy the jumper has lost, leaving him with zero kinetic energy when he reaches ground level. This means that

$$\frac{1}{2}k(\Delta\ell)^2_{\max} = \frac{1}{2}k\left(\frac{h}{2}\right)^2 = mgh$$

where mg is the weight of the jumper. The required spring constant of the elastic would then be $k = 8mg/h$. You must be careful to check that the cord can withstand a maximum tension of $F_{\max} = k(\Delta\ell)_{\max} = \left(\frac{8mg}{h}\right)\left(\frac{h}{2}\right) = 4mg$ without breaking. Also realize that the jumper will experience a net upward force of $3mg$ when the cord has this maximum tension. Thus, the jumper must withstand a $3g$ upward acceleration just as he is brought to rest at ground level.

16. If the tension remains the same, the speed of a wave on the string does not change. This means, from $v = \lambda f$, that if the frequency is doubled, the wavelength must decrease by a factor of two.

18. The speed of a wave on a string is given by $v = \sqrt{F/\mu}$. This says the speed is independent of the frequency of the wave. Thus, doubling the frequency leaves the speed unaffected.

20. **(a)** The wall exerts a force to the left, and the cart exerts an equal magnitude force to the right. The total force acting on the spring is zero. **(b)** The experimenter exerts a force to the right, and the spring exerts an equal magnitude force to the left. The total force acting on the cart is zero. **(c)** After the cart is released, the only force acting on the cart is the spring force. The cart then undergoes simple harmonic motion along the surface, with amplitude equal to the original displacement from the equilibrium position.

Answers to Even Numbered Problems

2. **(a)** 1.1×10^{2} N
(b) The graph is a straight line passing through the origin with slope equal to $k=1.0\times10^{3}$ N/m .

4. **(a)** 8.00 s **(b)** No, the force is not of Hooke's law form.

6. **(a)** 327 N **(b)** 1.25×10^{3} N/m

8. **(a)** 575 N/m **(b)** 46.0 J

10. 2.23 m/s

12. **(a)** 2.61 m/s **(b)** 2.38 m/s

14. **(a)** 11 cm/s **(b)** 6.3 cm/s **(c)** 3.0 N

16. **(a)** 0.15 J **(b)** 0.78 m/s **(c)** 18 m/s^2

18. 3.06 m/s

20. **(a)** 0.628 m/s **(b)** 0.500 Hz **(c)** 3.14 rad/s

22. 3.95 N/m

24. 2.2 Hz

26. **(a)** 0.30 m, 0.24 m **(b)** 0.30 m **(c)** 1/6 Hz
(d) 6.0 s

28. **(a)** 250 N/m **(b)** $T=0.281$ s, $f=3.56$ Hz, $\omega=22.4$ rad/s
(c) 0.313 J **(d)** 5.00 cm **(e)** 1.12 m/s, 25.0 m/s^2
(f) 0.919 cm

30. **(a)** 59.6 m **(b)** 37.5 s

32. **(a)** gain time **(b)** 1.1 s

34. **(a)** 3.65 s **(b)** 6.41 s **(c)** 4.24 s

36. 58.8 s

38. 5.67 mm

40. 0.800 m/s

42. **(a)** 0.20 Hz **(b)** 0.25 Hz

44. 219 N

46. 1.64 m/s^2

48. 7.07 m/s

50. 586 m/s

52. **(a)** 0 **(b)** 0.30 m

54. **(a)** 0.25 m **(b)** 0.47 N/m **(c)** 0.23 m
(d) 0.12 m/s

56. 0.990 m

58. **(a)** 100 m/s **(b)** 374 J

60. **(a)** 19.8 m/s **(b)** 8.94 m

64. 32.9 ms

66. **(a)** 6.93 m/s **(b)** 1.14 m

68. **(a)** 28.0 J **(b)** 0.446 m

70. 1.3 cm/s

Problem Solutions

13.1 (a) The force exerted on the block by the spring is

$$F_s = -kx = -(160\ \text{N/m})(-0.15\ \text{m}) = +24\ \text{N}$$

or $F_s = \boxed{\text{24 N directed toward equilibrium position}}$

(b) From Newton's second law, the acceleration is

$$a = \frac{F_s}{m} = \frac{+24\ \text{N}}{0.40\ \text{kg}} = +60\ \frac{\text{m}}{\text{s}^2} = \boxed{60\ \frac{\text{m}}{\text{s}^2}\ \text{toward equilibrium position}}$$

13.2 (a) The spring constant is $k = \dfrac{|F_s|}{x} = \dfrac{mg}{x} = \dfrac{50\ \text{N}}{5.0\times10^{-2}\ \text{m}} = 1.0\times10^{3}\ \text{N/m}$

$$F = |F_s| = kx = (1.0\times10^{3}\ \text{N/m})(0.11\ \text{m}) = \boxed{1.1\times10^{2}\ \text{N}}$$

(b) The graph will be a $\boxed{\text{straight line passing through the origin}}$ with a slope equal to $k = 1.0\times10^{3}\ \text{N/m}$.

13.3 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m before coming to rest momentarily. It will then repeat this motion over and over again with a regular period.

(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = 0$, the time required for the ball to reach the ground is $t = \sqrt{\dfrac{2(\Delta y)}{a_y}} = \sqrt{\dfrac{2(-4.00\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.904\ \text{s}$. This is one-half of the time for a complete cycle of the motion. Thus, the period is $T = \boxed{1.81\ \text{s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the object is a constant given by $F = -mg$ (except when it is contact with the ground). This is not in the form of Hooke's law.

13.4 (a) The motion is periodic since the ball continuously repeats its back and forth motion between the walls with no loss of energy (because of perfectly elastic collisions). The time for the ball to travel from one wall to the other is one-half period and is given by

$$\frac{T}{2} = \frac{\Delta x}{v} = \frac{12.0\ \text{m}}{3.00\ \text{m/s}} = 4.00\ \text{s}$$

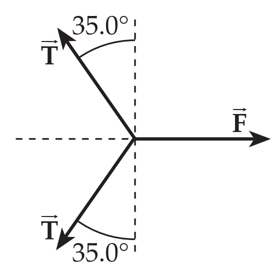
The period is then $T = \boxed{8.00\ \text{s}}$

(b) The $\boxed{\text{motion is not simple harmonic}}$ since the force acting on the ball is not of the form $F = -kx$. In fact, here $F = 0$ everywhere except when the ball is in contact with a wall.

13.5 When the system is in equilibrium, the tension in the spring $F = k|x|$ must equal the weight of the object. Thus,

$$k|x| = mg \text{ giving } m = \frac{k|x|}{g} = \frac{(47.5\ \text{N})(5.00 \times 10^{-2}\ \text{m})}{9.80\ \text{m/s}^2} = \boxed{0.242\ \text{kg}}$$

13.6 (a) The free-body diagram of the point in the center of the string is given at the right. From this, we see that



$$\Sigma F_x = 0 \Rightarrow F - 2T \sin 35.0° = 0$$

$$\text{or } T = \frac{F}{2 \sin 35.0°} = \frac{375\ \text{N}}{2 \sin 35.0°} = \boxed{327\ \text{N}}$$

(b) Since the bow requires an applied horizontal force of 375 N to hold the string at 35.0° from the vertical, the tension in the spring must be 375 N when the spring is stretched 30.0 cm. Thus, the spring constant is

$$k = \frac{F}{x} = \frac{375\ \text{N}}{0.300\ \text{m}} = \boxed{1.25 \times 10^3\ \text{N/m}}$$

13.7 (a) Assume the rubber bands obey Hooke's law. Then, the force constant of each band is

$$k = \frac{F_s}{x} = \frac{15\ \text{N}}{1.0\times 10^{-2}\ \text{m}} = 1.5\times 10^3\ \text{N/m}$$

Thus, when both bands are stretched 0.20 m, the total elastic potential energy is

$$PE_s = 2\left(\frac{1}{2}kx^2\right) = \left(1.5\times 10^3\ \text{N/m}\right)\left(0.20\ \text{m}\right)^2 = \boxed{60\ \text{J}}$$

(b) Conservation of mechanical energy gives $\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i$, or

$$\frac{1}{2}mv^2 + 0 = 0 + 60\ \text{J}\text{, so } v = \sqrt{\frac{2(60\ \text{J})}{50\times 10^{-3}\ \text{kg}}} = \boxed{49\ \text{m/s}}$$

13.8 (a) $k = \dfrac{F_{\max}}{x_{\max}} = \dfrac{230\ \text{N}}{0.400\ \text{m}} = \boxed{575\ \text{N/m}}$

(b) $\textit{work done} = PE_s = \dfrac{1}{2}kx^2 = \dfrac{1}{2}\left(575\ \text{N/m}\right)\left(0.400\right)^2 = \boxed{46.0\ \text{J}}$

13.9 From conservation of mechanical energy,

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i \text{ or } 0 + mgh_f + 0 = 0 + 0 + \frac{1}{2}kx_i^2$$

giving

$$k = \frac{2mgh_f}{x_i^2} = \frac{2\left(0.100\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(0.600\ \text{m}\right)}{\left(2.00\times 10^{-2}\ \text{m}\right)^2} = \boxed{2.94\times 10^3\ \text{N/m}}$$

13.10 Conservation of mechanical energy, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$,

gives $\dfrac{1}{2}mv_i^2 + 0 + 0 = 0 + 0 + \dfrac{1}{2}kx_f^2$,

or $$v_i = \sqrt{\frac{k}{m}}\,x_i = \sqrt{\frac{5.00\times 10^6\ \text{N/m}}{1000\ \text{kg}}}\left(3.16\times 10^{-2}\ \text{m}\right) = \boxed{2.23\ \text{m/s}}$$

13.11 At $x = A$, $v = 0$ and conservation of energy gives

$$E = KE + PE_s = 0 + \frac{1}{2}kA^2 \text{ or } A^2 = \frac{2E}{k}$$

(a) At $x = A/2$, the elastic potential energy is

$$PE_s = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{k}{8}A^2 = \frac{k}{8}\left(\frac{2E}{k}\right) = \boxed{\frac{E}{4}}$$

From the energy conservation equation, the kinetic energy is then

$$KE = E - PE_s = E - \frac{E}{4} = \boxed{\frac{3E}{4}}$$

(b) When $KE = PE_s$, conservation of energy yields $E = KE + PE_s = 2PE_s$ or $PE_s = E/2$. Since we also have $PE_s = kx^2/2$, this yields

$$x = \sqrt{\frac{2PE_s}{k}} = \sqrt{\frac{2(E/2)}{k}} = \sqrt{\frac{E}{k}} = \sqrt{\frac{(kA^2/2)}{k}} = \boxed{\frac{A}{\sqrt{2}}}$$

13.12 (a) From the work-energy theorem,

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

or $F \cdot x_f = \frac{1}{2}mv_f^2 + 0 + \frac{1}{2}kx_f^2$

This yields

$$v_f = \sqrt{\frac{2F \cdot x - kx_f^2}{m}}$$

$$= \sqrt{\frac{2(20.0\text{ N})(0.300\text{ m}) - 19.6\text{ N/m}(0.300\text{ m})^2}{1.50\text{ kg}}} = \boxed{2.61\text{ m/s}}$$

(b) The work-energy theorem now contains one more nonzero term, giving

$$v_f = \sqrt{\frac{2\left(F - \mu_k mg\right)\cdot x - kx_f^2}{m}}$$

$$= \sqrt{\frac{2\left[20.0\text{ N} - (0.200)(1.50\text{ kg})\left(9.80\text{ m/s}^2\right)\right](0.300\text{ m}) - 19.6\text{ N/m}(0.300\text{ m})^2}{1.50\text{ kg}}}$$

$$v_f = \boxed{2.38\text{ m/s}}$$

13.13 An unknown quantity of mechanical energy is converted into internal energy during the collision. Thus, we apply conservation of momentum from just before to just after the collision and obtain $mv_i + M(0) = (M+m)V$, or the speed of the block and embedded bullet just after collision is

$$V = \left(\frac{m}{M+m}\right)v_i = \left(\frac{10.0\times10^{-3}\text{ kg}}{2.01\text{ kg}}\right)(300\text{ m/s}) = 1.49\text{ m/s}$$

Now, we use conservation of mechanical energy from just after collision until the block comes to rest. This gives $0 + \frac{1}{2}kx_f^2 = \frac{1}{2}(M+m)V^2$, or

$$x_f = V\sqrt{\frac{M+m}{k}} = (1.49\text{ m/s})\sqrt{\frac{2.01\text{ kg}}{19.6\text{ N/m}}} = \boxed{0.478\text{ m}}$$

13.14 (a) In the absence of friction, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$ gives

$$\frac{1}{2}mv_f^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_i^2$$

or $v_f = x_i\sqrt{\frac{k}{m}} = (0.30\text{ cm})\sqrt{\frac{2\,000\text{ N/m}}{1.5\text{ kg}}} = \boxed{11\text{ cm/s}}$

(b) When friction is present, $W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$ gives

$$-f \cdot x_i = \left(\frac{1}{2}mv_f^2 + 0 + 0\right) - \left(0 + 0 + \frac{1}{2}kx_i^2\right)$$

or

$$v_f = \sqrt{\frac{kx_i^2 - 2f \cdot x_i}{m}}$$

$$= \sqrt{\frac{(2\,000 \text{ N/m})\left(3.0\times10^{-3} \text{ m}\right)^2 - 2(2.0 \text{ N})\left(3.0\times10^{-3} \text{ m}\right)}{1.5 \text{ kg}}}$$

$$v_f = 0.063 \text{ m/s} = \boxed{6.3 \text{ cm/s}}$$

(c) If $v_f = 0$ at $x = 0$, then $W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$

becomes $-f \cdot x_i = (0) - \left(0 + 0 + \frac{1}{2}kx_i^2\right)$

or $f = \dfrac{k\,x_i}{2} = \dfrac{(2\,000 \text{ N/m})\left(3.0\times10^{-3} \text{ m}\right)}{2} = \boxed{3.0 \text{ N}}$

13.15 From conservation of mechanical energy,

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$$

we have $\frac{1}{2}mv^2 + 0 + \frac{1}{2}kx^2 = 0 + 0 + \frac{1}{2}kA^2$, or $v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$

(a) The speed is a maximum at the equilibrium position, $x = 0$.

$$v_{\max} = \sqrt{\frac{k}{m}A^2} = \sqrt{\frac{(19.6 \text{ N/m})}{(0.40 \text{ kg})}(0.040 \text{ m})^2} = \boxed{0.28 \text{ m/s}}$$

(b) When $x = -0.015$ m,

$$v = \sqrt{\frac{(19.6 \text{ N/m})}{(0.40 \text{ kg})}\left[(0.040 \text{ m})^2 - (-0.015 \text{ m})^2\right]} = \boxed{0.26 \text{ m/s}}$$

(c) When $x = +0.015\text{ m}$,

$$v = \sqrt{\frac{(19.6\text{ N/m})}{(0.40\text{ kg})}\left[(0.040\text{ m})^2 - (+0.015\text{ m})^2\right]} = \boxed{0.26\text{ m/s}}$$

(d) If $v = \frac{1}{2}v_{\max}$, then $\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \frac{1}{2}\sqrt{\frac{k}{m}A^2}$

This gives $A^2 - x^2 = \frac{A^2}{4}$, or $x = A\frac{\sqrt{3}}{2} = (4.0\text{ cm})\frac{\sqrt{3}}{2} = \boxed{3.5\text{ cm}}$

13.16 (a) $KE = 0$ at $x = A$, so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$, or the total energy is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(250\text{ N/m})(0.035\text{ m})^2 = \boxed{0.15\text{ J}}$$

(b) The maximum speed occurs at the equilibrium position where $PE_s = 0$. Thus, $E = \frac{1}{2}mv_{\max}^2$, or

$$v_{\max} = \sqrt{\frac{2E}{m}} = A\sqrt{\frac{k}{m}} = (0.035\text{ m})\sqrt{\frac{250\text{ N/m}}{0.50\text{ kg}}} = \boxed{0.78\text{ m/s}}$$

(c) The acceleration is $a = \frac{\Sigma F}{m} = \frac{-kx}{m}$. Thus, $a = a_{\max}$ at $x = -x_{\max} = -A$.

$$a_{\max} = \frac{-k(-A)}{m} = \left(\frac{k}{m}\right)A = \left(\frac{250\text{ N/m}}{0.50\text{ kg}}\right)(0.035\text{ m}) = \boxed{18\text{ m/s}^2}$$

13.17 The maximum speed occurs at the equilibrium position and is

$$v_{\max} = \sqrt{\frac{k}{m}}A\text{. Thus, } m = \frac{kA^2}{v_{\max}^2} = \frac{(16.0\text{ N/m})(0.200\text{ m})^2}{(0.400\text{ m/s})^2} = 4.00\text{ kg}\text{, and}$$

$$F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{39.2\text{ N}}$$

13.18 $$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\left(\frac{10.0\text{ N/m}}{50.0\times 10^{-3}\text{ kg}}\right)\left[(0.250\text{ m})^2 - (0.125\text{ m})^2\right]} = \boxed{3.06\text{ m/s}}$$

13.19 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the uniform circular motion of the "bump" projected on a plane perpendicular to the tire.

(b) Note that the tangential speed of a point on the rim of a rolling tire is the same as the translational speed of the axle. Thus, $v_t = v_{car} = 3.00\ \text{m/s}$ and the angular velocity of the tire is

$$\omega = \frac{v_t}{r} = \frac{3.00\ \text{m/s}}{0.300\ \text{m}} = 10.0\ \text{rad/s}$$

Therefore, the period of the motion is

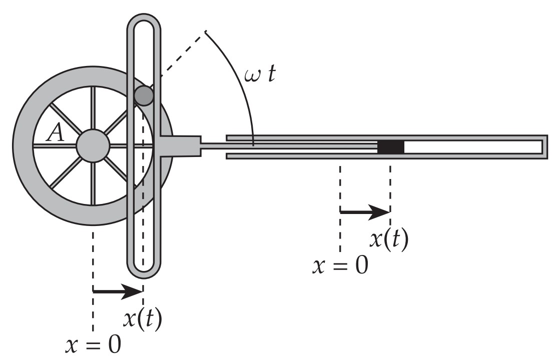
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{10.0\ \text{rad/s}} = \boxed{0.628\ \text{s}}$$

13.20 (a) $v_t = \frac{2\pi r}{T} = \frac{2\pi(0.200\ \text{m})}{2.00\ \text{s}} = \boxed{0.628\ \text{m/s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.00\ \text{s}} = \boxed{0.500\ \text{Hz}}$

(c) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.00\ \text{s}} = \boxed{3.14\ \text{rad/s}}$

13.21 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $x = A\cos\theta = A\cos\omega t$ where A is the distance from the center of the wheel to the crank pin. This is of the correct form to describe simple harmonic motion. Hence, one must conclude that the motion is indeed simple harmonic.



13.22 The period of oscillations of a mass-spring system is given by $T = 2\pi\sqrt{m/k}$ and the frequency is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus, $k = 4\pi^2 f^2 m = 4\pi^2\left(5.00\ \text{s}^{-1}\right)^2\left(4.00\times10^{-3}\ \text{kg}\right) = \boxed{3.95\ \text{N/m}}$

13.23 The spring constant is found from

$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010\ \text{kg})(9.80\ \text{m/s}^2)}{3.9 \times 10^{-2}\ \text{m}} = 2.5\ \text{N/m}$$

When the object attached to the spring has mass $m = 25\ \text{g}$, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{2.5\ \text{N/m}}} = \boxed{0.63\ \text{s}}$$

13.24 The springs compress 0.80 cm when supporting an additional load of $m = 320\ \text{kg}$. Thus, the spring constant is

$$k = \frac{mg}{x} = \frac{(320\ \text{kg})(9.80\ \text{m/s}^2)}{0.80 \times 10^{-2}\ \text{m}} = 3.9 \times 10^5\ \text{N/m}$$

When the empty car, $M = 2.0 \times 10^3\ \text{kg}$, oscillates on the springs, the frequency will be

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{3.9 \times 10^5\ \text{N/m}}{2.0 \times 10^3\ \text{kg}}} = \boxed{2.2\ \text{Hz}}$$

13.25 (a) The period of oscillation is $T = 2\pi\sqrt{m/k}$ where k is the spring constant and m is the mass of the object attached to the end of the spring. Hence,

$$T = 2\pi\sqrt{\frac{0.250\ \text{kg}}{9.5\ \text{N/m}}} = \boxed{1.0\ \text{s}}$$

(b) If the cart is released from rest when it is 4.5 cm from the equilibrium position, the amplitude of oscillation will be $A = 4.5\ \text{cm} = 4.5 \times 10^{-2}\ \text{m}$. The maximum speed is then given by

$$v_{\text{max}} = A\omega = A\sqrt{\frac{k}{m}} = (4.5 \times 10^{-2}\ \text{m})\sqrt{\frac{9.5\ \text{N/m}}{0.250\ \text{kg}}} = \boxed{0.28\ \text{m/s}}$$

(c) When the cart is 14 cm from the left end of the track, it has a displacement of $x = 14\ \text{cm} - 12\ \text{cm} = 2.0\ \text{cm}$ from the equilibrium position. The speed of the cart at this distance from equilibrium is

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\frac{9.5\ \text{N/m}}{0.250\ \text{kg}}\left[(0.045\ \text{m})^2 - (0.020\ \text{m})^2\right]} = \boxed{0.25\ \text{m/s}}$$

13.26 (a) At $t = 0$, $x = (0.30\text{ m})\cos(0) = \boxed{0.30\text{ m}}$, and at $t = 0.60\text{ s}$,

$$x = (0.30\text{ m})\cos\left[\left(\frac{\pi}{3}\text{ rad/s}\right)(0.60\text{ s})\right] = (0.30\text{ m})\cos(0.20\pi\text{ rad}) = \boxed{0.24\text{ m}}$$

(b) $A = x_{max} = (0.30\text{ m})(1) = \boxed{0.30\text{ m}}$

(c) $x = (0.30\text{ m})\cos\left(\frac{\pi}{3}t\right)$ is of the form $x = A\cos(\omega t)$ with an angular frequency of $\omega = \frac{\pi}{3}\text{ rad/s}$. Thus, $f = \frac{\omega}{2\pi} = \frac{\pi/3}{2\pi} = \boxed{\frac{1}{6}\text{ Hz}}$

(d) The period is $T = \frac{1}{f} = \boxed{6.0\text{ s}}$

13.27 (a) At $t = 3.50\text{ s}$,

$$F = -kx = -\left(5.00\ \frac{\text{N}}{\text{m}}\right)(3.00\text{ m})\cos\left[\left(1.58\ \frac{\text{rad}}{\text{s}}\right)(3.50\text{ s})\right] = -11.0\text{ N},$$

or $F = \boxed{11.0\text{ N directed to the left}}$

(b) The angular frequency is $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\text{ N/m}}{2.00\text{ kg}}} = 1.58\text{ rad/s}$ and the period of oscillation is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.58\text{ rad/s}} = 3.97\text{ s}$. Hence the number of oscillations made in 3.50 s is $N = \frac{\Delta t}{T} = \frac{3.50\text{ s}}{3.97\text{ s}} = \boxed{0.881}$

13.28 (a) $k = \frac{F}{x} = \frac{7.50\text{ N}}{3.00\times 10^{-2}\text{ m}} = \boxed{250\text{ N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250\text{ N/m}}{0.500\text{ kg}}} = \boxed{22.4\text{ rad/s}}$, $f = \frac{\omega}{2\pi} = \frac{22.4\text{ rad/s}}{2\pi} = \boxed{3.56\text{ Hz}}$,

and $T = \frac{1}{f} = \frac{1}{3.56\text{ Hz}} = \boxed{0.281\text{ s}}$

(c) At $t = 0$, $v = 0$ and $x = 5.00 \times 10^{-2}$ m, so the total energy of the oscillator is

$$E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$= 0 + \frac{1}{2}(250\ \text{N/m})(5.00 \times 10^{-2}\ \text{m})^2 = \boxed{0.313\ \text{J}}$$

(d) When $x = A$, $v = 0$ so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$.

Thus, $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.313\ \text{J})}{250\ \text{N/m}}} = 5.00 \times 10^{-2}\ \text{m} = \boxed{5.00\ \text{cm}}$

(e) At $x = 0$, $KE = \frac{1}{2}mv_{max}^2 = E$,

or $v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.313\ \text{J})}{0.500\ \text{kg}}} = \boxed{1.12\ \text{m/s}}$

$$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(250\ \text{N/m})(5.00 \times 10^{-2}\ \text{m})}{0.500\ \text{kg}} = \boxed{25.0\ \text{m/s}^2}$$

(f) At $t = 0.500$ s,

$$x = A\cos(\omega t) = (5.00\ \text{cm})\cos[(22.4\ \text{rad/s})(0.500\ \text{s})] = \boxed{0.919\ \text{cm}}$$

13.29 From Equation 13.6, $v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\sqrt{\omega^2(A^2 - x^2)}$

Hence, $v = \pm\omega\sqrt{A^2 - A^2\cos^2(\omega t)} = \pm\omega A\sqrt{1 - \cos^2(\omega t)} = \boxed{\pm\omega A\sin(\omega t)}$

From Equation 13.2, $a = -\frac{k}{m}x = -\omega^2[A\cos(\omega t)] = \boxed{-\omega^2 A\cos(\omega t)}$

13.30 (a) The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(15.5\ \text{s})^2}{4\pi^2} = \boxed{59.6\ \text{m}}$$

(b) On the Moon, where $g = 1.67\ \text{m/s}^2$, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{37.5\ \text{s}}$$

13.31 The period of a simple pendulum is $T = 2\pi\sqrt{L/g}$ where L is its length. The number of complete oscillations per second (that is, the frequency) for this pendulum is then given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{2.00\ \text{m}}} = 0.352\ \text{s}^{-1}$$

Hence, the number of oscillations in a time $\Delta t = 5.00\ \text{min} = 300\ \text{s}$ is

$$N = f(\Delta t) = (0.352\ \text{s}^{-1})(300\ \text{s}) = 105.7 \text{ or } \boxed{105 \text{ complete oscillations}}$$

13.32 (a) The lower temperature will cause the pendulum to contract. The shorter length will produce a smaller period, so the clock will run faster or $\boxed{\text{gain time}}$.

(b) The period of the pendulum is $T_0 = 2\pi\sqrt{\frac{L_0}{g}}$ at 20°C,

and at –5.0°C it is $T = 2\pi\sqrt{\frac{L}{g}}$. The ratio of these periods is $\frac{T_0}{T} = \sqrt{\frac{L_0}{L}}$.

From Chapter 10, the length at –5.0°C is $L = L_0 + \alpha_{\text{Al}} L_0 (\Delta T)$, so

$$\frac{L_0}{L} = \frac{1}{1 + \alpha_{\text{Al}}(\Delta T)} = \frac{1}{1 + \left[24 \times 10^{-6}\ (°\text{C})^{-1}\right][-5.0°\text{C} - 20°\text{C}]} = \frac{1}{0.999\,4} = 1.000\,6$$

This gives $\frac{T_0}{T} = \sqrt{\frac{L_0}{L}} = \sqrt{1.000\,6} = 1.000\,3$. Thus in one hour (3 600 s), the chilled pendulum will gain $(1.000\,3 - 1)(3\,600\ \text{s}) = \boxed{1.1\ \text{s}}$.

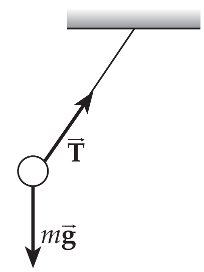
13.33 (a) The period of the pendulum is $T = 2\pi\sqrt{L/g}$. Thus, on the Moon where the free-fall acceleration is smaller, the period will be longer and the clock will run $\boxed{\text{slow}}$.

(b) The ratio of the pendulum's period on the Moon to that on Earth is

$$\frac{T_{Moon}}{T_{Earth}} = \frac{2\pi\sqrt{L/g_{Moon}}}{2\pi\sqrt{L/g_{Earth}}} = \sqrt{\frac{g_{Earth}}{g_{Moon}}} = \sqrt{\frac{9.80}{1.63}} = 2.45$$

Hence, the pendulum of the clock on Earth makes 2.45 "ticks" while the clock on the Moon is making 1.00 "tick". After the Earth clock has ticked off 24.0 h and again reads 12:00 midnight, the Moon clock will have ticked off $\frac{24.0\text{ h}}{2.45} = 9.79\text{ h}$ and will read $\boxed{9:47\text{ AM}}$.

13.34 The apparent free-fall acceleration is the vector sum of the actual free-fall acceleration and the negative of the elevator's acceleration. To see this, consider an object that is suspended by a string in the elevator and that appears to be at rest to the elevator passengers. These passengers believe the tension in the string is the negative of the object's weight, or $\vec{\mathbf{T}} = -m\vec{\mathbf{g}}_{app}$ where $\vec{\mathbf{g}}_{app}$ is the apparent free-fall acceleration in the elevator.



An observer located outside the elevator applies Newton's second law to this object by writing $\Sigma\vec{\mathbf{F}} = \vec{\mathbf{T}} + m\vec{\mathbf{g}} = m\vec{\mathbf{a}}_e$ where $\vec{\mathbf{a}}_e$ is the acceleration of the elevator and all its contents. Thus, $\vec{\mathbf{T}} = m\vec{\mathbf{a}}_e - m\vec{\mathbf{g}} = -m\vec{\mathbf{g}}_{app}$, which gives $\vec{\mathbf{g}}_{app} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$.

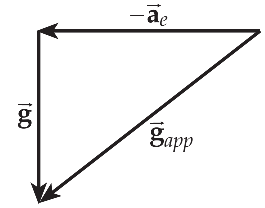
(a) If we choose downward as the positive direction, then $\vec{\mathbf{a}}_e = -5.00\text{ m/s}^2$ in this case and $\vec{\mathbf{g}}_{app} = (9.80 + 5.00)\text{ m/s}^2 = +14.8\text{ m/s}^2$ (downward). The period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\text{ m}}{14.8\text{ m/s}^2}} = \boxed{3.65\text{ s}}$$

(b) Again choosing downward as positive, $\vec{\mathbf{a}}_e = 5.00\text{ m/s}^2$ and $\vec{\mathbf{g}}_{app} = (9.80 - 5.00)\text{ m/s}^2 = +4.80\text{ m/s}^2$ (downward) in this case. The period is now given by

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\text{ m}}{4.80\text{ m/s}^2}} = \boxed{6.41\text{ s}}$$

(c) If $\vec{\mathbf{a}}_e = 5.00\ \text{m/s}^2$ horizontally, the vector sum $\vec{\mathbf{g}}_{app} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$ is as shown in the sketch at the right. The magnitude is



$$g_{app} = \sqrt{\left(5.00\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = 11.0\ \text{m/s}^2\ ,$$

and the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{app}}} = 2\pi\sqrt{\frac{5.00\ \text{m}}{11.0\ \text{m/s}^2}} = \boxed{4.24\ \text{s}}$$

13.35 (a) From $T = 2\pi\sqrt{L/g}$, the length of a pendulum with period T is $L = \dfrac{gT^2}{4\pi^2}$.

On Earth, with $T = 1.0\ \text{s}$, $L = \dfrac{\left(9.80\ \text{m/s}^2\right)\left(1.0\ \text{s}\right)^2}{4\pi^2} = 0.25\ \text{m} = \boxed{25\ \text{cm}}$

If $T = 1.0\ \text{s}$ on Mars, $L = \dfrac{\left(3.7\ \text{m/s}^2\right)\left(1.0\ \text{s}\right)^2}{4\pi^2} = 0.094\ \text{m} = \boxed{9.4\ \text{cm}}$

(b) The period of an object on a spring is $T = 2\pi\sqrt{m/k}$, which is independent of the local free-fall acceleration. Thus, the same mass will work on Earth and on Mars. This mass is

$$m = \frac{kT^2}{4\pi^2} = \frac{\left(10\ \text{N/m}\right)\left(1.0\ \text{s}\right)^2}{4\pi^2} = \boxed{0.25\ \text{kg}}$$

13.36 The frequency of the wave (that is, the number of crests passing the cork each second) is $f = 2.00\ \text{s}^{-1}$ and the wavelength (distance between successive crests) is $\lambda = 8.50\ \text{cm}$. Thus, the wave speed is

$$v = \lambda f = \left(8.50\ \text{cm}\right)\left(2.00\ \text{s}^{-1}\right) = 17.0\ \text{cm/s} = 0.170\ \text{m/s}$$

and the time required for the ripples to travel 10.0 m over the surface of the water is

$$\Delta t = \frac{\Delta x}{v} = \frac{10.0\ \text{m}}{0.170\ \text{m/s}} = \boxed{58.8\ \text{s}}$$

13.37 (a) The amplitude, A, is the maximum displacement from equilibrium. Thus, from Figure P13.37, $A = \frac{1}{2}(18.0\text{ cm}) = \boxed{9.00\text{ cm}}$

(b) The wavelength, λ, is the distance between successive crests (or successive troughs). From Figure P13.37, $\lambda = 2(10.0\text{ cm}) = \boxed{20.0\text{ cm}}$

(c) The period is $T = \frac{1}{f} = \frac{1}{25.0\text{ Hz}} = 4.00 \times 10^{-2}\text{ s} = \boxed{40.0\text{ ms}}$

(d) The speed of the wave is $v = \lambda f = (0.200\text{ m})(25.0\text{ Hz}) = \boxed{5.00\text{ m/s}}$

13.38 From $v = \lambda f$, the wavelength (and size of smallest detectable insect) is

$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{60.0 \times 10^{3}\text{ Hz}} = 5.67 \times 10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$$

13.39 (a) $T = \frac{1}{f} = \frac{1}{88.0 \times 10^{6}\text{ Hz}} = 1.14 \times 10^{-8}\text{ s} = \boxed{11.4\text{ ns}}$

(b) $\lambda = \frac{v}{f} = \frac{3.00 \times 10^{8}\text{ m/s}}{88.0 \times 10^{6}\text{ Hz}} = \boxed{3.41\text{ m}}$

13.40 The distance between successive maxima in a transverse wave is the wavelength of that wave. Hence, $\lambda = 1.20\text{ m}$. The frequency (number of crests passing a fixed point each second) is

$$f = \frac{8}{12.0\text{ s}} = 0.667\text{ s}^{-1} = 0.667\text{ Hz}$$

Therefore, the wave speed is $v = \lambda f = (1.20\text{ m})(0.667\text{ s}^{-1}) = \boxed{0.800\text{ m/s}}$

13.41 The speed of the wave is $v = \frac{\Delta x}{\Delta t} = \frac{425\text{ cm}}{10.0\text{ s}} = 42.5\text{ cm/s}$

and the frequency is $f = \frac{40.0\text{ vib}}{30.0\text{ s}} = 1.33\text{ Hz}$

Thus, $\lambda = \frac{v}{f} = \frac{42.5\text{ cm/s}}{1.33\text{ Hz}} = \boxed{31.9\text{ cm}}$

13.42 (a) When the boat is at rest in the water, the speed of the wave relative to the boat is the same as the speed of the wave relative to the water, $v = 4.0\ \text{m/s}$. The frequency detected in this case is

$$f = \frac{v}{\lambda} = \frac{4.0\ \text{m/s}}{20\ \text{m}} = \boxed{0.20\ \text{Hz}}$$

(b) Taking westward as positive, $\vec{\mathbf{v}}_{boat,water} = \vec{\mathbf{v}}_{boat,wave} + \vec{\mathbf{v}}_{wave,water}$ gives

$$\vec{\mathbf{v}}_{boat,wave} = \vec{\mathbf{v}}_{boat,water} - \vec{\mathbf{v}}_{wave,water} = +1.0\ \text{m/s} - (-4.0\ \text{m/s}) = +5.0\ \text{m/s}$$

Thus, $$f = \frac{v_{boat,wave}}{\lambda} = \frac{5.0\ \text{m/s}}{20\ \text{m}} = \boxed{0.25\ \text{Hz}}$$

13.43 The down and back distance is $4.00\ \text{m} + 4.00\ \text{m} = 8.00\ \text{m}$.

The speed is then $v = \frac{d_{total}}{t} = \frac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{F/\mu}$

Now, $\mu = \frac{m}{L} = \frac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$, so

$$F = \mu v^2 = (5.00 \times 10^{-2}\ \text{kg/m})(40.0\ \text{m/s})^2 = \boxed{80.0\ \text{N}}$$

13.44 The speed of the wave is $v = \frac{\Delta x}{\Delta t} = \frac{20.0\ \text{m}}{0.800\ \text{s}} = 25.0\ \text{m/s}$, and the mass per unit length of the rope is $\mu = \frac{m}{L} = 0.350\ \text{kg/m}$. Thus, from $v = \sqrt{F/\mu}$, we obtain

$$F = v^2\mu = (25.0\ \text{m/s})^2(0.350\ \text{kg/m}) = \boxed{219\ \text{N}}$$

13.45 (a) The mass per unit length is $\mu = \frac{m}{L} = \frac{0.0600\ \text{kg}}{5.00\ \text{m}} = 0.0120\ \text{kg/m}$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (50.0\ \text{m/s})^2(0.0120\ \text{kg/m}) = \boxed{30.0\ \text{N}},$$

(b) $$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{8.00\ \text{N}}{0.0120\ \text{kg/m}}} = \boxed{25.8\ \text{m/s}}$$

13.46 The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.00 \times 10^{-3}\ \text{kg}}{1.60\ \text{m}} = 2.50 \times 10^{-3}\ \text{kg/m}\,,$$

and the speed of the pulse is $v = \frac{L}{\Delta t} = \frac{1.60\ \text{m}}{0.0361\ \text{s}} = 44.3\ \text{m/s}$.

Thus, the tension in the wire is

$$F = v^2 \mu = (44.3\ \text{m/s})^2 (2.50 \times 10^{-3}\ \text{kg/m}) = 4.91\ \text{N}$$

But, the tension in the wire is the weight of a 3.00-kg object on the Moon. Hence, the local free-fall acceleration is

$$g = \frac{F}{m} = \frac{4.91\ \text{N}}{3.00\ \text{kg}} = \boxed{1.64\ \text{m/s}^2}$$

13.47 The period of the pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$, so the length of the string is

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(2.00\ \text{s})^2}{4\pi^2} = 0.993\ \text{m}$$

Then mass per unit length of the string is then

$$\mu = \frac{m}{L} = \frac{0.060\,0\ \text{kg}}{0.993\ \text{m}} = 0.060\,4\ \frac{\text{kg}}{\text{m}}$$

When the pendulum is vertical and stationary, the tension in the string is

$$F = M_{ball}\, g = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}\,,$$

and the speed of transverse waves in it is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{49.0\ \text{N}}{0.060\,4\ \text{kg/m}}} = \boxed{28.5\ \text{m/s}}$$

13.48 If $\mu_1 = m_1/L$ is the mass per unit length for the first string, then $\mu_2 = m_2/L = m_1/2L = \mu_1/2$ is that of the second string. Thus, with $F_2 = F_1 = F$, the speed of waves in the second string is

$$v_2 = \sqrt{\frac{F}{\mu_2}} = \sqrt{\frac{2F}{\mu_1}} = \sqrt{2}\left(\sqrt{\frac{F}{\mu_1}}\right) = \sqrt{2}\,v_1 = \sqrt{2}\left(5.00\ \text{m/s}\right) = \boxed{7.07\ \text{m/s}}$$

13.49 (a) The tension in the string is $F = mg = (3.0\ \text{kg})(9.80\ \text{m/s}^2) = 29\ \text{N}$. Then, from $v = \sqrt{F/\mu}$, the mass per unit length is

$$\mu = \frac{F}{v^2} = \frac{29\ \text{N}}{(24\ \text{m/s})^2} = \boxed{0.051\ \text{kg/m}}$$

(b) When $m = 2.00\ \text{kg}$, the tension is

$$F = mg = (2.0\ \text{kg})(9.80\ \text{m/s}^2) = 20\ \text{N}$$

and the speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20\ \text{N}}{0.051\ \text{kg/m}}} = \boxed{20\ \text{m/s}}$$

13.50 If the tension in the wire is F, the tensile stress is $Stress = F/A$, so the speed of transverse waves in the wire may be written as

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{A \cdot Stress}{m/L}} = \sqrt{\frac{Stress}{m/(A \cdot L)}}$$

But, $A \cdot L = V$ = volume, so $m/(A \cdot L) = \rho$ = density. Thus, $v = \sqrt{\dfrac{Stress}{\rho}}$.

When the stress is at its maximum, the speed of waves in the wire is

$$v_{\text{max}} = \sqrt{\frac{(Stress)_{\text{max}}}{\rho}} = \sqrt{\frac{2.70 \times 10^9\ \text{Pa}}{7.86 \times 10^3\ \text{kg/m}^3}} = \boxed{586\ \text{m/s}}$$

13.51 From $v = \sqrt{F/\mu}$, the tension in the string is $F = v^2\mu$. Thus, the ratio of the new tension to the original is

$$\frac{F_2}{F_1} = \frac{v_2^2}{v_1^2}\text{, giving } F_2 = \left(\frac{v_2}{v_1}\right)^2 F_1 = \left(\frac{30.0\ \text{m/s}}{20.0\ \text{m/s}}\right)^2 (6.00\ \text{N}) = \boxed{13.5\ \text{N}}$$

13.52 (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free there is no inversion on reflection. When they meet the amplitude is $A' = 2A = 2(0.15\ \text{m}) = \boxed{0.30\ \text{m}}$.

13.53 (a) $\boxed{\text{Constructive interference}}$ produces the maximum amplitude

$$A'_{\text{max}} = A_1 + A_2 = \boxed{0.50\ \text{m}}$$

(b) $\boxed{\text{Destructive interference}}$ produces the minimum amplitude

$$A'_{\text{min}} = A_1 - A_2 = \boxed{0.10\ \text{m}}$$

13.54 We are given that $x = A\cos(\omega t) = (0.25\ \text{m})\cos(0.4\pi t)$.

(a) By inspection, the amplitude is seen to be $A = \boxed{0.25\ \text{m}}$

(b) The angular frequency is $\omega = 0.4\pi\ \text{rad/s}$. But $\omega = \sqrt{k/m}$, so the spring constant is

$$k = m\omega^2 = (0.30\ \text{kg})(0.4\pi\ \text{rad/s})^2 = \boxed{0.47\ \text{N/m}}$$

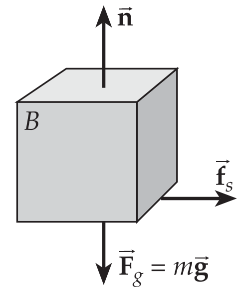
(c) At $t = 0.30\ \text{s}$, $x = (0.25\ \text{m})\cos\left[(0.4\pi\ \text{rad/ s})(0.30\ \text{s})\right] = \boxed{0.23\ \text{m}}$

(d) From conservation of mechanical energy, the speed at displacement x is given by $v = \omega\sqrt{A^2 - x^2}$. Thus, at $t = 0.30\ \text{s}$, when $x = 0.23\ \text{m}$, the speed is

$$v = (0.4\pi\ \text{rad/s})\sqrt{(0.25\ \text{m})^2 - (0.23\ \text{m})^2} = \boxed{0.12\ \text{m/s}}$$

13.55 The maximum acceleration of the oscillating system is

$$a_{max} = \omega^2 A = (2\pi f)^2 A$$



The friction force exerted between the two blocks must be capable of accelerating block B at this rate. When block B is on the verge of slipping, $f_s = (f_s)_{max} = \mu_s n = \mu_s mg = ma_{max}$ and we must have

$$a_{max} = (2\pi f)^2 A = \mu_s g$$

Thus, $A = \dfrac{\mu_s g}{(2\pi f)^2} = \dfrac{(0.600)(9.80\ \text{m/s}^2)}{[2\pi(1.50\ \text{Hz})]^2} = 6.62\times 10^{-2}\ \text{m} = \boxed{6.62\ \text{cm}}$

13.56 Since the spring is "light", we neglect any small amount of energy lost in the collision with the spring, and apply conservation of mechanical energy from when the block first starts until it comes to rest again. This gives

$$(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i \text{, or } 0 + 0 + \frac{1}{2}kx_{max}^2 = 0 + 0 + mgh_i$$

Thus, $x_{max} = \sqrt{\dfrac{2mgh_i}{k}} = \sqrt{\dfrac{2(0.500\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})}{20.0\ \text{N/m}}} = \boxed{0.990\ \text{m}}$

13.57 Choosing $PE_g = 0$ at the initial height of the block, conservation of mechanical energy gives $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$, or

$$\frac{1}{2}mv^2 + mg(-x) + \frac{1}{2}kx^2 = 0,$$

where v is the speed of the block after falling distance x.

(a) When $v = 0$, the non-zero solution to the energy equation from above gives

$$\frac{1}{2}kx_{max}^2 = mgx_{max},$$

or $k = \dfrac{2mg}{x_{max}} = \dfrac{2(3.00\ \text{kg})(9.80\ \text{m/s}^2)}{0.100\ \text{m}} = \boxed{588\ \text{N/m}}$

(b) When $x = 5.00\text{ cm} = 0.050\,0\text{ m}$, the energy equation gives

$$v = \sqrt{2gx - \frac{kx^2}{m}}\text{, or}$$

$$v = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(0.050\,0\text{ m}\right) - \frac{\left(588\text{ N/m}\right)\left(0.050\,0\text{ m}\right)^2}{3.00\text{ kg}}} = \boxed{0.700\text{ m/s}}$$

13.58 (a) We apply conservation of mechanical energy from *just after* the collision until the block comes to rest.

$$\left(KE + PE_s\right)_f = \left(KE + PE_s\right)_i \text{ gives } 0 + \frac{1}{2}kx_f^2 = \frac{1}{2}MV^2 + 0$$

or the speed of the block just after the collision is

$$V = \sqrt{\frac{kx_f^2}{M}} = \sqrt{\frac{\left(900\text{ N/m}\right)\left(0.050\,0\text{ m}\right)^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

Now, we apply conservation of momentum from just before impact to immediately after the collision. This gives

$$m\left(v_{bullet}\right)_i + 0 = m\left(v_{bullet}\right)_f + MV$$

or $\left(v_{bullet}\right)_f = \left(v_{bullet}\right)_i - \left(\frac{M}{m}\right)V$

$$= 400\text{ m/s} - \left(\frac{1.00\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)\left(1.5\text{ m/s}\right) = \boxed{100\text{ m/s}}$$

(b) The mechanical energy converted into internal energy during the collision is

$$\Delta E = KE_i - \Sigma KE_f = \frac{1}{2}m\left(v_{bullet}\right)_i^2 - \frac{1}{2}m\left(v_{bullet}\right)_f^2 - \frac{1}{2}MV^2$$

or

$$\Delta E = \frac{1}{2}\left(5.00\times10^{-3}\text{ kg}\right)\left[\left(400\text{ m/s}\right)^2 - \left(100\text{ m/s}\right)^2\right] - \frac{1}{2}\left(1.00\text{ kg}\right)\left(1.50\text{ m/s}\right)^2$$

$$\Delta E = \boxed{374\text{ J}}$$

13.59 Choose $PE_g = 0$ when the blocks start from rest. Then, using conservation of mechanical energy from when the blocks are released until the spring returns to its unstretched length gives $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$, or

$$\frac{1}{2}\left(m_1 + m_2\right)v_f^2 + \left(m_1 g\, x \sin 40° - m_2 g\, x\right) + 0 = 0 + 0 + \frac{1}{2}k\, x^2$$

$$\frac{1}{2}\left[(25 + 30)\ \text{kg}\right]v_f^2 + (25\ \text{kg})\left(9.80\ \text{m/s}^2\right)\left[(0.200\ \text{m})\sin 40°\right]$$
$$-(30\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.200\ \text{m}) = \frac{1}{2}(200\ \text{N/m})(0.200\ \text{m})^2$$

yielding $v_f = \boxed{1.1\ \text{m/s}}$

13.60 (a) When the gun is fired, the energy initially stored as elastic potential energy in the spring is transformed into kinetic energy of the bullet. Assuming no loss of energy, we have $\frac{1}{2}mv^2 = \frac{1}{2}kx_i^2$, or

$$v = x_i\sqrt{\frac{k}{m}} = (0.200\ \text{m})\sqrt{\frac{9.80\ \text{N/m}}{1.00\times 10^{-3}\ \text{kg}}} = \boxed{19.8\ \text{m/s}}$$

(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time required for the pellet to drop 1.00 m to the floor, starting with $v_{0y} = 0$, is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.00\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.452\ \text{s}$$

The range (horizontal distance traveled during the flight) is then

$$\Delta x = v_{0x}t = (19.8\ \text{m/s})(0.452\ \text{s}) = \boxed{8.94\ \text{m}}$$

13.61 (a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500\ \text{N/m}}{2.00\ \text{kg}}} = \boxed{15.8\ \text{rad/s}}$

(b) Apply Newton's second law to the block while the elevator is accelerating:

$$\Sigma F_y = F_s - mg = ma_y$$

With $F_s = kx$ and $a_y = g/3$, this gives $kx = m\left(g + g/3\right)$, or

$$x = \frac{4mg}{3k} = \frac{4\left(2.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{3\left(500\text{ N/m}\right)} = 5.23\times10^{-2}\text{ m} = \boxed{5.23\text{ cm}}$$

13.62 (a) When the block is given some small upward displacement, the net restoring force exerted on it by the rubber bands is

$$F_{net} = \Sigma F_y = -2F\sin\theta,\text{ where }\tan\theta = \frac{y}{L}$$

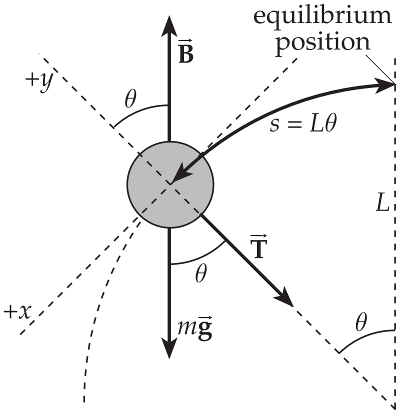
For small displacements, the angle θ will be very small. Then $\sin\theta \approx \tan\theta = \frac{y}{L}$, and the net restoring force is

$$F_{net} = -2F\left(\frac{y}{L}\right) = \boxed{-\left(\frac{2F}{L}\right)y}$$

(b) The net restoring force found in part (a) is in the form of Hooke's law $F = -ky$, with $k = \frac{2F}{L}$. Thus, the motion will be simple harmonic, and the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{2F}{mL}}}$$

13.63 The free-body diagram at the right shows the forces acting on the balloon when it is displaced distance $s = L\theta$ along the circular arc it follows. The net force tangential to this path is



$$F_{net} = \Sigma F_x = -B\sin\theta + mg\sin\theta = -(B - mg)\sin\theta$$

For small angles, $\sin\theta \approx \theta = \dfrac{s}{L}$

Also, $mg = (\rho_{\text{He}}V)g$

and the buoyant force is $B = (\rho_{\text{air}}V)g$. Thus, the net restoring force

acting on the balloon is $F_{net} \approx -\left[\dfrac{(\rho_{air} - \rho_{\text{He}})Vg}{L}\right]s$

Observe that this is in the form of Hooke's law, $F = -k\,s$,

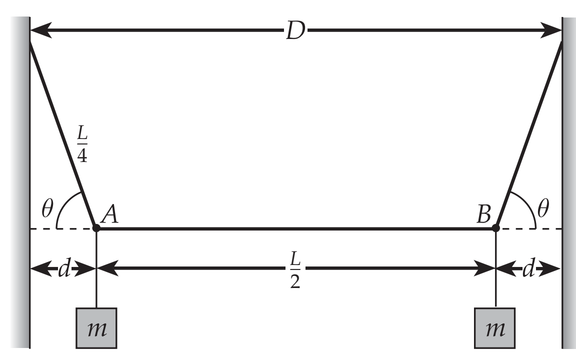
with $k = (\rho_{air} - \rho_{\text{He}})Vg/L$

Thus, the motion will be simple harmonic and the period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}}V}{(\rho_{air} - \rho_{\text{He}})Vg/L}} = 2\pi\sqrt{\left(\frac{\rho_{\text{He}}}{\rho_{air} - \rho_{\text{He}}}\right)\frac{L}{g}}$$

This yields $T = 2\pi\sqrt{\left(\dfrac{0.180}{1.29 - 0.180}\right)\dfrac{(3.00\text{ m})}{(9.80\text{ m/s}^2)}} = \boxed{1.40\text{ s}}$

13.64 Observe in the sketch at the right that $2d + L/2 = D$, or

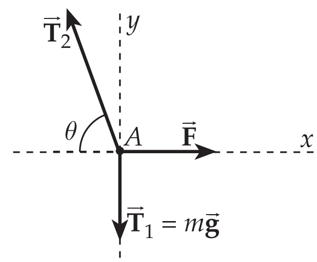


$$d = \frac{D - L/2}{2} = \frac{2.00\text{ m} - 1.50\text{ m}}{2} = 0.250\text{ m}$$

Thus,

$$\theta = \cos^{-1}\left(\frac{d}{L/4}\right) = \cos^{-1}\left(\frac{0.250\text{ m}}{0.750\text{ m}}\right) = 70.5°$$

Now, consider a free body diagram of point A:



$$\Sigma F_x = 0 \Rightarrow F = T_2 \cos(70.5°)$$

and $$\Sigma F_y = 0 \Rightarrow T_2 \sin(70.5°) = mg = 19.6 \text{ N}$$

Hence, the tension in the section between A and B is

$$F = \frac{19.6 \text{ N}}{\tan(70.5°)} = 6.93 \text{ N}$$

The mass per unit length of the string is

$$\mu = \frac{10.0 \times 10^{-3} \text{ kg}}{3.00 \text{ m}} = 3.33 \times 10^{-3} \text{ kg/m}$$

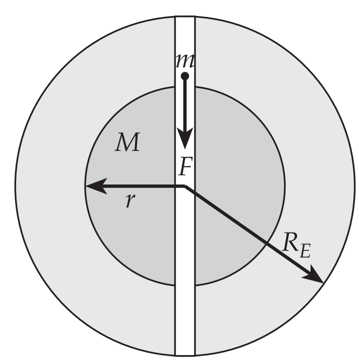
so the speed of transverse waves in the string between points A and B is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{6.93 \text{ N}}{3.33 \times 10^{-3} \text{ kg/m}}} = 45.6 \text{ m/s}$$

The time for the pulse to travel from A to B is

$$t = \frac{L/2}{v} = \frac{1.50 \text{ m}}{45.6 \text{ m/s}} = 3.29 \times 10^{-2} \text{ s} = \boxed{32.9 \text{ ms}}$$

13.65 Newton's law of gravitation is



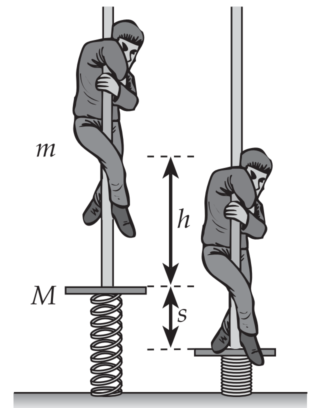
$$F = -\frac{GMm}{r^2}, \text{ where } M = \rho\left(\frac{4}{3}\pi r^3\right)$$

Thus, $$F = -\left(\frac{4}{3}\pi\rho G m\right) r$$

which is of Hooke's law form, $F = -k\,r$, with

$$k = \frac{4}{3}\pi\rho G m$$

13.66 (a) Apply the work-energy theorem from the instant the firefighter starts from rest until just before contact with the platform.



$$W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i \text{ gives}$$

$$-f \cdot h = \left(\frac{1}{2}mv^2 + 0\right) - \left(0 + mgh\right), \text{ or } v = \sqrt{2\left(g - \frac{f}{m}\right)h}$$

$$v = \sqrt{2\left(9.80 \text{ m/s}^2 - \frac{300 \text{ N}}{60 \text{ kg}}\right)(5.00 \text{ m})} = \boxed{6.93 \text{ m/s}}$$

(b) Next, apply conservation of momentum to find the speed V of the firefighter and platform immediately after the perfectly inelastic collision. This gives

$$(m + M)V = mv + M(0)$$

or $V = \left(\frac{m}{m+M}\right)v = \left(\frac{60.0}{60.0+20.0}\right)(6.93 \text{ m/s}) = 5.20 \text{ m/s}$

Finally, apply the work-energy theorem from just after the collision until the firefighter comes to rest.

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i \text{ gives}$$

$$-f \cdot s = \left(0 + 0 + \frac{1}{2}ks^2\right) - \left(\frac{1}{2}(m+M)V^2 + (m+M)gs + 0\right)$$

or $s^2 - \frac{2}{k}\left[(m+M)g - f\right]s - \left(\frac{m+M}{k}\right)V^2 = 0$

Using the given data, we obtain $s^2 - (0.387 \text{ m})s - 0.865 \text{ m}^2 = 0$, and the quadratic formula gives a positive solution of $s = \boxed{1.14 \text{ m}}$

13.67 (a) Using conservation of mechanical energy, $(KE + PE_s)_f = (KE + PE_s)_i$, from the moment of release to the instant of separation gives

$$\frac{1}{2}(m_1 + m_2)v^2 + 0 = 0 + \frac{1}{2}kA^2$$

or $v = A\sqrt{\dfrac{k}{m_1 + m_2}} = (0.20\ \text{m})\sqrt{\dfrac{100\ \text{N/m}}{(9.0 + 7.0)\ \text{kg}}} = \boxed{0.50\ \text{m/s}}$

(b) After the two blocks separate, m_1 oscillates with new amplitude A' found by applying $(KE + PE_s)_f = (KE + PE_s)_i$ to the $m_1 + \text{spring}$ system from the moment of separation until the spring is fully stretched the first time.

$$0 + \frac{1}{2}kA'^2 = \frac{1}{2}m_1 v^2$$

or $A' = v\sqrt{\dfrac{m_1}{k}} = (0.50\ \text{m/s})\sqrt{\dfrac{9.0\ \text{kg}}{100\ \text{N/m}}} = 0.15\ \text{m}$

The period of this oscillation is $T = 2\pi\sqrt{\dfrac{m_1}{k}} = 2\pi\sqrt{\dfrac{9.0\ \text{kg}}{100\ \text{N/m}}} = 1.9\ \text{s}$,

so the spring is fully stretched for the first time at $t = \dfrac{T}{4} = 0.47\ \text{s}$ after separation. During this time, m_2 has moved distance $x = vt$ from the point of separation. Thus, the distance separating the two blocks at this instant is

$$D = vt - A' = (0.50\ \text{m/s})(0.47\ \text{s}) - 0.15\ \text{m} = 0.086\ \text{m} = \boxed{8.6\ \text{cm}}$$

13.68 (a) Apply the work-energy theorem from the instant before the block contacts the spring until the instant the block leaves the spring.

$$W_{nc} = KE_f - KE_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$
$$= \frac{1}{2}(8.00\ \text{kg})\left[(3.00\ \text{m/s})^2 - (4.00\ \text{m/s})^2\right] = -28.0\ \text{J}$$

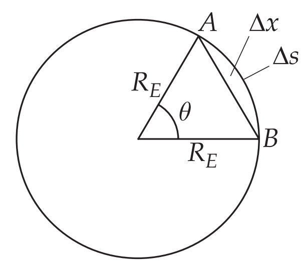
or the mechanical energy lost is $|W_{nc}| = \boxed{28.0\ \text{J}}$

(b) The energy spent overcoming the friction force while the block is in contact with the spring is $f \cdot s = |W_{nc}|$, where $s = 2x_{max}$ with x_{max} being the maximum distance the spring was compressed. Hence,

$$x_{max} = \frac{|W_{nc}|}{2f} = \frac{|W_{nc}|}{2\mu_k n} = \frac{|W_{nc}|}{2\mu_k mg}$$

$$= \frac{28.0 \text{ J}}{2(0.400)(8.00 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.446 \text{ m}}$$

13.69 (a) As seen in the sketch at the right, the length of the chord Δx followed by the longitudinal wave is shorter than the arc length Δs traveled by the transverse wave. Also, the longitudinal wave travels faster than does the transverse wave. Thus, $\boxed{\text{the longitudinal wave}}$ will arrive first.



(b) With $\theta = 60.0° = \pi/3$ radians, the arc length, Δs, traveled by the transverse wave is

$$\Delta s = R_E\theta = R_E\left(\frac{\pi}{3} \text{ rad}\right)$$

and the time for this wave to travel from A to B is: $\Delta t_t = \dfrac{\Delta s}{v_t} = \dfrac{R_E(\pi/3)}{v_t}$

From the sketch, observe that the chord labeled Δx is one side of an isosceles triangle. Thus, $\Delta x = R_E$ and the travel time for the longitudinal wave is

$$\Delta t_\ell = \frac{\Delta x}{v_\ell} = \frac{R_E}{v_\ell}$$

The difference in the arrival times of the two waves is then

$$\Delta t = \Delta t_t - \Delta t_\ell = R_E\left(\frac{\pi}{3v_t} - \frac{1}{v_\ell}\right) = (6.37 \times 10^3 \text{ km})\left[\frac{\pi}{3(4.50 \text{ km/s})} - \frac{1}{7.80 \text{ km/s}}\right]$$

or $\Delta t = (666 \text{ s})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{11.1 \text{ min}}$

13.70 The inner tip of the wing is attached to the end of the spring and always moves with the same speed as the end of the vibrating spring. Thus, its maximum speed is

$$v_{inner,\,\max} = v_{spring,\,\max} = A\sqrt{\frac{k}{m}} = (0.20\text{ cm})\sqrt{\frac{4.7\times10^{-4}\text{ N/m}}{0.30\times10^{-3}\text{ kg}}} = 0.25\text{ cm/s}$$

Treating the wing as a rigid bar, all points in the wing have the same angular velocity at any instant in time. As the wing rocks on the fulcrum, the inner tip and outer tips follow circular paths of different radii. Since the angular velocities of the tips are always equal, we may write $\omega = \frac{v_{outer}}{r_{outer}} = \frac{v_{inner}}{r_{inner}}$. The maximum speed of the outer tip is then

$$v_{outer,\,\max} = \left(\frac{r_{outer}}{r_{inner}}\right)v_{inner,\,\max} = \left(\frac{15.0\text{ mm}}{3.00\text{ mm}}\right)(0.25\text{ cm/s}) = \boxed{1.3\text{ cm/s}}$$

13.71 (a) If the surface is frictionless, the total mechanical energy of the system is conserved. Thus, taking the initial position at the point when the block starts from rest and the final position at the equilibrium position, we have

$$KE_f + PE_f = KE_i + PE_i \quad\Rightarrow\quad \frac{1}{2}mv_f^2 + 0 = 0 + \frac{1}{2}kx_i^2$$

or $$v_f = |x_i|\sqrt{\frac{k}{m}} = (2.0\times10^{-2}\text{ m})\sqrt{\frac{1.0\times10^{3}\text{ N/m}}{1.6\text{ kg}}} = \boxed{0.50\text{ m/s}}$$

(b) When a constant friction force retards the motion of the block, the work-energy theorem gives

$$W_{nc} = (KE+PE)_f - (KE+PE)_i \text{ or } -f\cdot s = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 - \frac{1}{2}mv_i^2 - \frac{1}{2}kx_i^2$$

where s is the total distance the block moves between the initial and final states. If the block is initially at rest at $|x_i| = 2.0$ cm, and the final state is when the block makes its first pass through the equilibrium position, then $s = |x_i|$ and we have

$$-(4.0\text{ N})\cdot(2.0\times10^{-2}\text{ m}) = \frac{1}{2}(1.6\text{ kg})v_f^2 + 0 - 0 - \frac{1}{2}(1.0\times10^{3}\text{ N/m})(2.0\times10^{-2}\text{ m})^2$$

or $$v_f = \sqrt{\frac{0.20\text{ J} - 0.080\text{ J}}{0.80\text{ kg}}} = \boxed{0.39\text{ m/s}}$$

(c) When a friction force is present, the system is continuously spending energy as the friction force does negative work on it. The block will cease to move when all of the original energy (initially stored as elastic potential energy) has been used doing work to overcome friction. That is, until

$$f \cdot s = \frac{1}{2}kx_i^2 \text{ or } s = \frac{kx_i^2}{2f} = \frac{\left(1.0\times10^3\text{ N/m}\right)\left(2.0\times10^{-2}\text{ m}\right)^2}{2\left(4.0\text{ N}\right)} = 5.0\times10^{-2}\text{ m} = \boxed{5.0\text{ cm}}$$