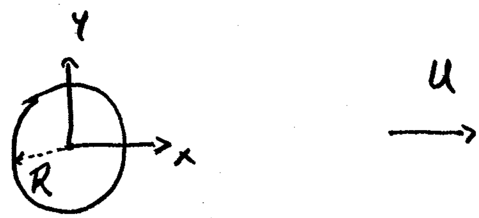
Final Project

FLOW AROUND CYLINDER

Consider the incompressible Navier-Stokes flow around a cylinder which is very long compared with its radius. To a very good approximation, the flow is two-dimensional. In the experiments of D.J. Tritton, the length was more than 15 times the diameter and the flow is well described by 2D fluid dynamics:



Far away from the flow the asymptotic velocity is u. Define the Reynolds number

$$R_e = \frac{2R\rho u}{\eta}$$

In Oseen's approximation, at small Reynolds numbers the drag force is given by

$$F_x = \frac{4\pi\eta u}{\log\left(3.703\,\frac{\nu}{uR}\right)}$$ drag per unit length

Define the drag coefficient:

$$C_D = \frac{F_x}{\frac{1}{2}\rho u^2 2R} = \frac{8\pi}{Re \log\left(\frac{7.407}{Re}\right)}$$

Tritton's experimental resuts for Re between 0.5 and 120 are shown below:

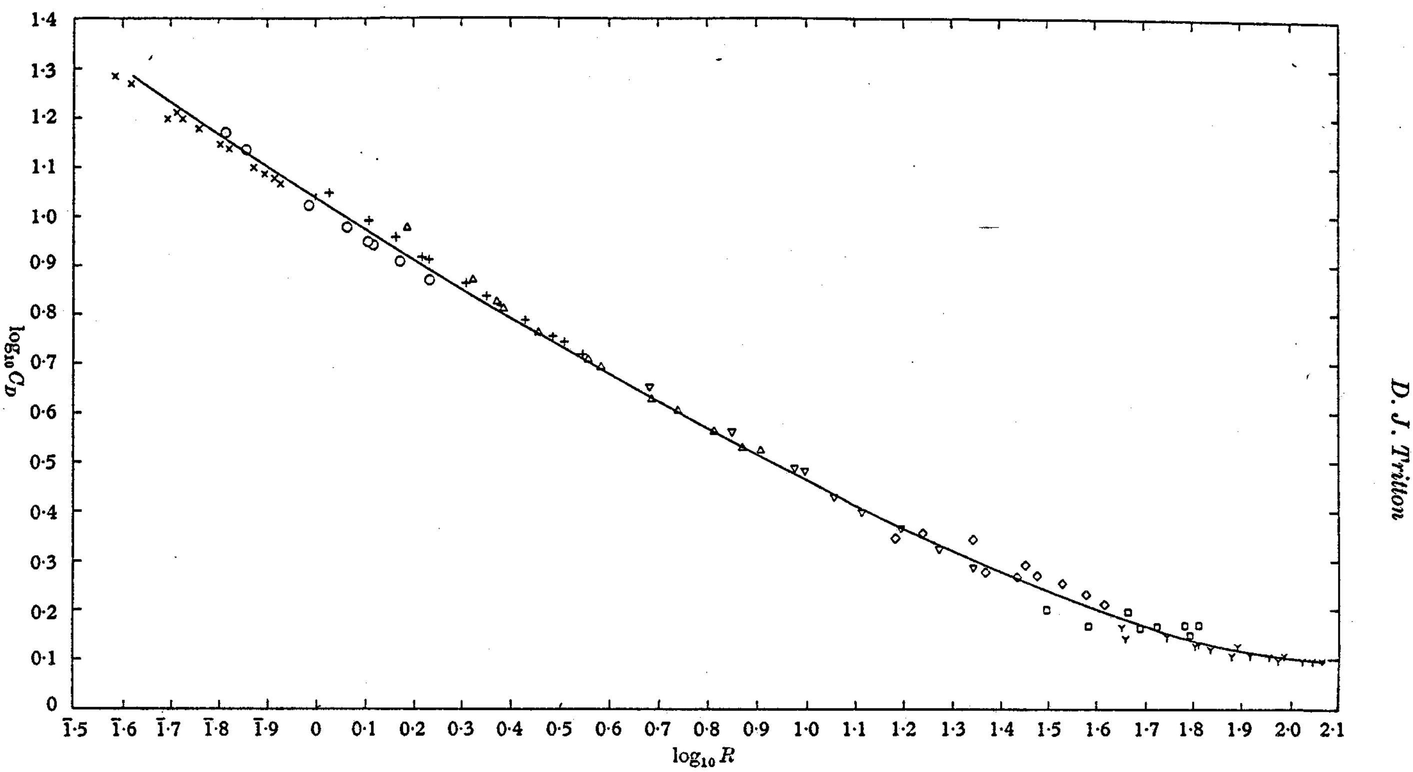
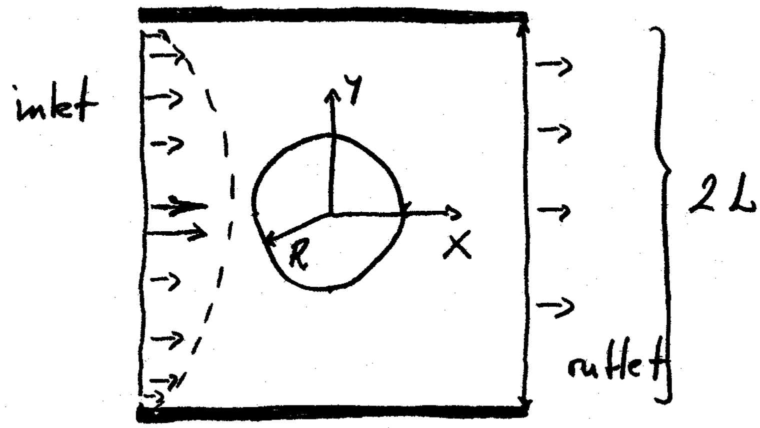


FIGURE 1. Plot of $\log C_D$ against $\log R$ showing all the observations listed in table 1. The line is an estimated mean curve, bearing in mind that the slope of the line for each fibre is known with greater accuracy than its position. This line is used for comparison purposes in figures 2 and 3. Absolute measurements: ×, fibre 1; ○, fibre 2; +, fibre 3; ◇, fibre 6; □, fibre 7; Y, fibre 8. 'Fitted' points: △, fibre 4; ▽, fibre 5.

Comparison with theory is shown on page 8 of lecture 16

The geometry of the computational layout:



Parabolic velocity profile at inlet:

$$u_x = \frac{3}{2}\frac{U}{L^2}\left[2L(y+L) - (y+L)^2\right], \quad u_y = 0$$

mean value U

$L = 16$, $R = 1$, $\rho = 1$, $U = 1$ in the original setup

Three grid levels

Normalization of c_D: U is not identical to asymptotic velocity in flow without finite grid

(1) Low Reynolds number regime

For $Re < 40$ the flow is steady and vortex formation is not detected in the wake

(a) Calculate the drag force and the drag coefficient C_D for $Re = 0.5, 1, 5, 10, 20, 40$

Although the absolute normalization is not yet known, it is only an additive constant on the Tritton log plot

(b) Plot your drag results on the Tritton plot after you adjusted the constant.

(c) Can you determine the absolute normalization?

(2) Critical Reynolds number

(a) Try to determine the Reynolds number where the flow becomes unsteady.
It is somewhere below $Re = 100$.
First, show that the flow is unsteady at $Re = 100$

(b) Present some graphical evidence

(3) <u>Drag in unsteady flow</u>

(a) Calculate the drag for $Re = 60, 80, 100, 120$ and plot it on the Tritton plot

(b) Illustrate the vortex shedding mechanism using some graphics