

★Reading: sections 12.1, 12.2, 12.3.

• Last time: Gravity! Newton's result (1687): any two masses attract each other with a force of magnitude

$$F_{gravity} = \frac{G_N m_1 m_2}{r^2}.$$

Henry Cavendish directly measured G in 1798 (!)

$$G = 6.67 \times 10^{-11} m^3/kg s^2.$$

This gave a way to determine $M_e = 5.97 \times 10^{24} kg$.

On the surface of a planet of mass M and radius R , get $F_{gravity} = GMm/R^2 = ma$, so the downward gravitational acceleration felt by a mass m near the surface is $g = G_N M/R^2$. In particular, this applies for the earth.

Example: planet X has $M_X = 2M_e$ and $R_X = 2R_e$. How much does a 150 lb person weigh on planet X? A: the weight is $w = m(GM_X/R_X^2) = m(G2M_e/4R_e^2) = \frac{1}{2}mg_e$, where $g_e = GM_e/R_e^2$ is the acceleration on earth. So the 150 lb person weighs 75lbs on planet X. Of course their mass is unchanged, regardless of the environment.

Find the gravitational acceleration of a mass m , due to the earth, as a function of the distance r from the center of the earth, $g(r)$, with $g(r = R_e) \equiv g_e = 9.8m/s^2$. The result is $\sim 1/r^2$ above the earth. Someone asked the interesting question about what would happen if you dug a hole inside the earth. The result is that only the mass inside the sphere of radius r contributes, that outside cancels out. If we approximate the mass density of the earth as uniform (this is not true, since the center of the earth is a very dense core), then $M(r) \sim r^3$, proportional to the volume of a sphere of radius r . So $F(r) \sim M(r)/r^2 \sim r$: the force grows linearly with r for r inside the earth. The result is thus:

$$g(r) = \begin{cases} g_e \left(\frac{R_e}{r}\right)^2 & \text{for } r \geq R_e \\ g_e \frac{r}{R_e} & \text{for } r \leq R_e. \end{cases}$$

If there is a little hollow cavity in the center of the earth, objects inside would float around, weightless, like they're floating in empty space – the gravity from the earth canceling out.

• Superposition of forces, $\vec{F}_{tot} = \sum_i \vec{F}_i$. For example, the total gravitational force on an apple is a superposition of forces coming from the earth, sun, moon, distant stars etc. Satellite between earth and moon example, at radial equilibrium at a distance x from

center of earth and $d - x$ from center of moon. Draw force arrows, show this is unstable equilibrium.

- Note how weak gravity usually is. If we take two 1000kg masses and separate them by say 1mm , the force of gravity between them is $F_g = G(10^3)^4 \approx 66.7\text{N}$. If the distance is instead 1cm the force would be $F_g \approx 0.66\text{N}$. Actually, it's totally unrealistic to have two 1000kg masses separated by a mm or a cm, since this would have to be the separation between their centers of mass, and then those huge masses would have to be smaller than a mm, requiring unrealistically large densities. (Actually, the density deep inside a neutron star can be up to nearly $10^{18}\text{kg}/\text{m}^3$, so a cubic mm of that would have mass 10^9kg . There the force of gravity is enormous, balanced by quantum effects: the Pauli exclusion principle. If that isn't enough to stop the gravitational collapse, it results instead in a black hole.)

- Electric forces also go like $1/r^2$, $F_e = kq_1q_2/r^2$, but there the force can be attractive or repulsive and, since opposites attract, but objects tend to be electrically neutral and then F_e is weaker. But gravity is always attractive, so its effect always adds and accumulates at larger and larger length scales. So gravity is the only important force for the motion of planets, stars, and what the Universe is doing right now.

- $\vec{F}_{gravity} = -\vec{\nabla}U_{gravity}(r)$, with the gravitational potential energy

$$U_{gravity}(r) = -G_N \frac{m_1 m_2}{r}.$$

So the work done against gravity is conservative and

$$W = \int_{r_i}^{r_f} \vec{F} \cdot \vec{dr} = U_{grav}(r_i) - U_{grav}(r_f).$$

For r near the surface of the earth we can write $r = R_e + y$ and then $U_{grav}(r) \approx U_0 + mgy$, where U_0 is a irrelevant constant, reproducing what we saw before.

For a point mass outside a spherical shell, the potential is the same as if the entire mass of the shell is concentrated at its center. For a point mass inside such a shell, the potential is a constant. (See text ch 12.6 for details.)

- Application, escape velocity: $\frac{1}{2}mv_e^2 + U_{grav}(R) = 0 \rightarrow v_e = \sqrt{2GM/R}$.
- **Next time:** Orbital motion.