Physics 2a, Nov 22, lecture 26

$\star$Reading: sections 11.1, 11.2, 11.3. (The later sections of Ch 11 won't be covered here. You'll see them in Physics 2C.)

- Conditions for equilibrium: $\vec{F}_{tot} = 0$, $\vec{\tau}_{tot} = 0$.
- Center of gravity. Recall $\vec{r}_{cm} = \sum_i m_i \vec{r}_i / M$, where $M = \sum_i m_i$.

For a rigid body that's near earth and not too big, all masses m_i feel $\vec{F}_i^{grav} = m_i \vec{g}$ and thus torque $\tau = \sum_i m_i \vec{r}_i \times \vec{g} = \vec{r}_{cm} \times M\vec{g}$. This would vanish if we take the origin at $\vec{r}_{cm} = \vec{0}$, so the center of mass is the center of gravity: a body is in equilibrium if it's supported there.

- Examples, cars on curved road and tipping risk, Romeo and Juliet, pumping iron.

Romeo and Juliet. Romeo has weight $M_R g$, ladder has weight $M_L g$ and length L. Romeo climbs distance d along ladder. Ladder makes angle θ with respect to the ground. Find minimum coefficient of friction between ladder and ground, taking zero coefficient of friction between ladder and wall. $\vec{F}_{tot} = 0$ gives $f_1 = n_2$ and $n_1 = (M_R + M_L)g$. So need $\mu = f_1/n_1 = n_2/(M_R + M_L)g$. Now solve for n_2 using $\tau = 0$ condition: $\tau = -(M_R d + \frac{1}{2} M_L L) g \cos\theta + n_2 L \sin\theta$, using that the center of mass of the ladder is at $L/2$. So $n_2 = (M_R(d/L) + \frac{1}{2} M_L) g \cot\theta$ and thus need $\mu = (M_R(d/L) + \frac{1}{2} M_L)(M_R + M_L)^{-1} \cot\theta$. Check: for small θ, need more friction, which makes sense if you think about the ladder wanting to slide-out if it isn't vertical enough. Also, need more friction as d/L increases, which also makes sense – as Romeo climbs up the ladder, the sliding-out risk increases.