Physics 2a, Nov 15, lecture 23

⋆Reading: chapters 9 and 10.

• Recall circular motion: $\vec{r}(t) = r(\hat{i}\cos\theta(t) + \hat{j}\sin\theta(t))$. From now on, measure angle θ in radians. Then distance on circle is $s = r\theta$.

• Angular velocity: $\omega = \frac{d\theta}{dt}$. If an object is rotating around some axis, given by a unit vector $\hat{n}$, then can define the angular velocity vector $\vec{\omega} = \omega\hat{n}$, with sign determined by the right hand rule. E.g. for above circular motion in the x, y plane, we have $\vec{\omega} = \frac{d\theta}{dt}\hat{z}$.

• Angular acceleration: $\vec{\alpha} = \frac{d}{dt}\vec{\omega}$. For example, if $\vec{\alpha} = \alpha_z\hat{z}$ is a constant, then $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_z t^2$, and $\omega^2 - \omega_0^2 = 2\alpha_z(\theta - \theta_0)$.

• Obvious similarity with what we've seen before:

$$x \to \theta$$
$$v \to \omega$$
$$a \to \alpha.$$

• Outline of remainder of this week: extend this analogy with

$m \to I$	moment of intertia
$\vec{p} \to \vec{L}$	angular momentum
$\vec{F} \to \vec{\tau}$	torque.

• Definitions:

$$I = \sum_i m_i r_i^2 = \int r^2 dm$$

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i = \vec{r}_{cm} \times M\vec{v}_{cm} + I\vec{\omega}.$$

$$\vec{\tau} = \sum_i \vec{r}_i \times \vec{F}_i = \frac{d\vec{L}}{dt}.$$

• If no torque, $\vec{\tau} = 0$, angular momentum is conserved, $\vec{L} =$ constant. Newton's 3rd law $\to \vec{\tau}_{total} = \vec{\tau}_{external}$, which vanishes for a closed system. So closed systems have conserved angular momentum. At a fundamental level, angular momentum is always conserved, though it can flow in and out of a system. Conservation of angular momentum is a deep principle, like conservation of energy and conservation of momentum. (They are related to symmetries: energy to time translations, momentum to space translations, and angular momentum to rotational invariance).

• Torque does work, $W = \int \vec{\tau} \cdot d\vec{\theta}$. Rotation of rigid bodies, kinetic energy

$$K = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2} M v_{cm}^2 + \tfrac{1}{2} I\omega^2.$$

• Compute some examples of moments of inertia: stick through middle, stick through end, solid cylinder, hollow cylinder, solid sphere, etc.