Physics 2a, Nov 10, lecture 22

⋆Reading: chapter 8.

• More on elastic collisions, in 1d. $m_1v_1+m_2v_2 = m_1v_1'+m_2v_2'$ and $\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2$. Example: $v_1 = v$, $v_2 = 0$, $v_1' = \frac{m_1-m_2}{m_1+m_2}v$, $v_2' = \frac{2m_1}{m_1+m_2}v$, consider cases $m_1 \ll m_2$, and $m_1 \gg m_2$ and $m_1 = m_2$. Relative velocity changes sign before and after collision.

• Gravitational slingshot. Spacecraft has $v_1 = 10.4 \times 10^3$ and Saturn has $v_2 = -9.6 \times 10^3$. So $v_1 - v_2 = -(v_1' - v_2')$ and $v_2 = v_2'$. So $v_1' = 2v_2 - v_1 = -29.6 \times 10^3$. Craft's speed is almost tripled and kinetic energy is increased by factor of 8.

• Center of mass. For a system of objects with masses m_i and locations $\vec{r}_i$, the center of mass is a kind of average position: $\vec{r}_{cm} = \sum m_i\vec{r}_i/\sum m_i$, so $M\frac{d}{dt}\vec{r}_{cm} = \vec{P}$. Any total external force acts on the center of mass, and internal forces cancel out.

• Rocket equations: $p(t) = mv$, and $p(t+dt) = (m+dm)(v+dv) + (-dm)(v - v_{ex})$, where $-dm > 0$ is the ejected mass and $-v_{ex}$ is its velocity relative to the rocket. So $dp = (m+dm)(v+dv)+(-dm)(v-v_{ex})-mv = mdv+v_{ex}dm$ and thus $F_{ext} = m\frac{dv}{dt}+v_{ex}\frac{dm}{dt}$. If $F_{ext} = 0$ (e.g. in space) then we can integrate to get $v - v_0 = v_{ex}\ln(m_0/m)$. Or if $F_{ext} = -mg$, integrate to get $v - v_0 = -gt + v_{ex}\ln(m_0/m)$.