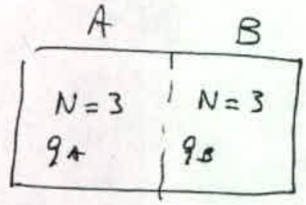
Physics 140A **Midterm 1 Solutions** Fall 2010

Problem 1:

$$\Omega(N, q) = \frac{(q + N-1)!}{q!\,(N-1)!}$$



We have 3 oscillators in each system $\Rightarrow N=3$. $q_A + q_B = q = 4$

q_A	q_B	Ω_A	Ω_B	$\Omega = \Omega_A \Omega_B$
0	4	1	15	15
1	3	3	10	30
2	2	6	6	36
3	1	10	3	30
4	0	15	1	15
				126

$$\Omega(3,2) = \frac{4!}{2!\,2!} = \frac{4\cdot 3}{2} = 6$$

$$\Omega(3,3) = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2} = 10$$

$$\Omega(3,4) = \frac{6!}{4!\,2!} = \frac{6\cdot 5}{2} = 15$$

(b) Total multiplicity from table above is 126.

For the combined system, $N=6$, $q=4 \Rightarrow$

$$\Omega(N=6, q=4) = \frac{9!}{4!\,5!} = \frac{\overset{3}{\cancel{9}}\cdot\cancel{8}\cdot 7\cdot 6}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}} = 3\times 42 = 126$$

$\Rightarrow$ it agrees, as it should.

(c) Find the 'temperature'

$$\frac{1}{T} = \frac{\partial \ln \Omega}{\partial q} = \ln \Omega(q+1) - \ln \Omega(q) = \ln \frac{\Omega(q+1)}{\Omega(q)}$$

So:

$$\left.\frac{1}{T}\right)_{q_A=1} = \ln \frac{\Omega_A(2)}{\Omega_A(1)} = \ln \frac{6}{3} = 0.693 \Rightarrow T(q_A=1) = 1.44$$

$$\left.\frac{1}{T}\right)_{q_A=2} = \ln \frac{\Omega_A(3)}{\Omega_A(2)} = \ln \frac{10}{6} = 0.511 \Rightarrow T(q_A=2) = 1.96$$

$$\left.\frac{1}{T}\right)_{q_A=3} = \ln \frac{\Omega_A(4)}{\Omega_A(3)} = \ln \frac{15}{10} = 0.405 \Rightarrow T(q_A=3) = 2.47$$

(d) The initial temperatures if $q_A = 1$, $q_B = 3$, are:

$T_A = 1.44$, $T_B = 2.47$

So B is at higher temperature. So heat should flow from B to A.

At a later time, the most likely macrostate is the one where

$q_A = q_B = 2$, where both temperatures are equal, $T = 1.96$

Problem 2

For ideal gas, $\Omega_N = C_N V^N U^{3N/2} \Longrightarrow$

$$S = k \ln \Omega = Nk \ln V + \frac{3}{2} N \ln U + \text{const}.$$

If the gas expands with constant temperature, U doesn't change.

Hence change in entropy is

$$\Delta S = Nk \ln V_{final} - Nk \ln V_{initial} = Nk \ln \frac{V_{final}}{V_{initial}}$$

$$= Nk \ln 3$$

So $\Delta S = 10^{22} \times 1.381 \times 10^{-23} \cdot \ln 3 \ \frac{J}{K} = 0.1381 \ln 3 \ \frac{J}{K} \Rightarrow$

$$\boxed{\Delta S = 0.152 \ J/K}$$

(b) For free expansion, no heat is absorbed $\Rightarrow$

$$\boxed{Q = 0}$$

For quasistatic isothermal expansion it does work and absorbs heat

$$\Delta U = Q + W = 0$$

We can find the heat absorbed from: $\boxed{Q = T\Delta S = 15.2 J}$

Or from: $Q = -W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{NkT}{V} dV = Nk \ln \frac{V_f}{V_i} \cdot T =$

$$= Nk \ln 3 \cdot T = 15.2 J$$

(c) In free expansion, environment doesn't change entropy

$$\Rightarrow \boxed{\Delta S_{universe} = 0.152 J/K}$$

In quasistatic expansion, $\Delta S_{environment} = -Q/T = -\Delta S_{gas}$

$$\Rightarrow \boxed{\Delta S_{universe} = 0}$$

Problem 3

$$\Omega = \frac{N!}{N_\uparrow ! \, N_\downarrow !}$$

Initially, $N_\uparrow = 30, N_\downarrow = 70 \Rightarrow$

$$\Omega_{initial} = \frac{N!}{30! \, 70!}$$

Then, one spin flips from $\uparrow$ to $\downarrow$, so $N_\uparrow = 29$, $N_\downarrow = 71$, and

$$\Omega_{final} = \frac{N!}{29! \, 71!}$$

The change in multiplicity is:

$$\frac{\Omega_{final}}{\Omega_{initial}} = \frac{30! \, 70!}{29! \, 71!} = \frac{30}{71} = 0.422$$

multiplicity went down.

The energy increased, since $\downarrow$ spins have higher energy than $\uparrow$ spins.

The entropy is $S = k \ln \Omega \Rightarrow$ the entropy decreased

(b) The system increased its energy and decreased its entropy.

Temperature is defined by

$$\frac{1}{T} = \frac{\partial S}{\partial U}$$

since $\Delta U > 0$ and $\Delta S < 0 \Rightarrow$ the temperature is negative.