

**Formulas:**

First law  $\Delta U = Q + W$  ;  $C_V = \left(\frac{\partial U}{\partial T}\right)_V$  ;  $H = U + PV$  ;  $C_P = \left(\frac{\partial H}{\partial T}\right)_P$

$k = 1.381 \times 10^{-23} \text{ J/K}$  ,  $N_A = 6.02 \times 10^{23}$  ,  $R = 8.31 \text{ J/(mol}^\circ\text{K)}$

Ideal gas :  $PV = NkT$   $U = N \frac{f}{2} kT$   $W = -P\Delta V$  ,  $W = -\int PdV$  ;  $C_P = C_V + Nk$

adiabatic process:  $PV^\gamma = \text{const.}$  ,  $\gamma = (f + 2)/f$

$S = k \ln \Omega$  ;  $\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V$  ;  $dS = \frac{Q}{T} = \frac{C_V dT}{T}$  (constant volume)

Ideal gas (monoatomic) :  $\Omega_N = C_N V^N U^{3N/2} / N!$

Stirling's approximation :  $n! = n^n e^{-n} \sqrt{2\pi n}$

Einstein solid :  $\Omega(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!} \sim \left(\frac{eq}{N}\right)^N$  for  $q \gg N$

Two - state system :  $\Omega = \frac{N!}{N_\uparrow! N_\downarrow!}$

Paramagnetism :  $M = N\mu \tanh(\mu B/kT)$  ,  $U = -MB$  ,  $M = \mu(N_\uparrow - N_\downarrow)$

**Justify all your answers to all problems**

**Problem 1**

Consider two systems A and B of three Einstein oscillators each, that can exchange energy between themselves but not with the rest of the universe. Assume they have a total of four energy units to share between them.

(a) Calculate the multiplicity of each of the possible macrostates of the composite system. A macrostate is defined by the values of  $q_A$  and  $q_B$ , with  $q_A$  the number of energy units in system A and same for B.

Hint: For e.g. system A only, the multiplicity for  $q_A=3$  is 10 as you can verify from the formula for the Einstein solid given above.

(b) Add the multiplicities for the composite system found in (a) and check whether or not the answer agrees with the multiplicity of a system of 6 Einstein oscillators with 4 energy units. If it doesn't agree, explain why.

(c) Define a dimensionless 'temperature' by the usual formula omitting Boltzmann constant, assuming the energy unit for the oscillator ( $hf$ ) is 1, and discretizing the derivative. I.e., for e.g. system A with  $q_A$  energy units, the discrete derivative of a given function  $f(q_A)$  would be

$$\frac{\partial f(q_A)}{\partial q_A} \equiv \frac{f(q_A + 1) - f(q_A)}{(q_A + 1) - q_A}$$

Find the numerical value for the 'temperature' of system A for:

(i)  $q_A=1$ , (ii)  $q_A=2$ , (iii)  $q_A=3$ .

(d) Explain what happens if the combined A-B system is initially in a microstate corresponding to the macrostate  $q_A=1$ ,  $q_B=3$ . What are the initial temperatures, in which direction will heat flow, what is the most likely macrostate the system will be found at a later time and what are the 'temperatures' of A and B at that time?

**Problem 2**

An ideal gas of  $10^{22}$  molecules expands from an initial volume  $2\text{m}^3$  to a final volume  $6\text{m}^3$ . Its temperature is  $T=100^\circ\text{K}$  throughout the process.

- (a) Find the change in its entropy. Give your answer in units J/K.
- (b) What is the heat absorbed by the gas from the environment for the cases where the expansion was (i) free expansion and (ii) quasistatic expansion? Give your answer in J.
- (c) What is the change in the entropy of the universe (gas + environment) for the processes (i) and (ii) in (b)? Give your answers in J/K.

**Problem 3**

A system of 100 spin  $\frac{1}{2}$  particles has 30 particles with magnetic moment pointing parallel to an external magnetic field  $B$  and 70 particles with magnetic moment antiparallel to the field. The energy of a magnetic moment is  $-\mu B$  or  $+\mu B$  when it is parallel or antiparallel to the field respectively.

- (a) Suppose the system is briefly allowed to exchange energy with another system, and it flips one spin from parallel to the field to antiparallel to the field. (i) Did the system gain or lose energy? (ii) Did the multiplicity of the corresponding macrostate of this system increase or decrease, and by which factor? Did this system's entropy increase or decrease in this process?
- (b) Is the temperature of this system positive or negative? Justify using the answers from (a).

**Justify all your answers to all problems**