

## Physics 110A: Final Exam

INSTRUCTIONS: Do problems 1, 2, and 3 and either 4 or 5.

[1] A particle of mass  $m$  moves in these one-dimensional potentials: **(a)**  $U(x) = U_0|x/a|^\beta$  ( $\beta > 0$ ), **(b)**  $U(x) = -U_0 \operatorname{sech}^2(x/a)$ , and **(c)**  $U(x) = U_0 \tan^2(x/a)$ . In all cases,  $U_0 > 0$ . For each of these cases, sketch the potential and phase curves, and find the range of energy  $E$  over which bounded motion occurs. **(d)** For the potential  $U(x) = U_0|x/a|$ , determine the period  $T(E)$  for motion at energy  $E$ .

*Reminder:*  $\operatorname{sech} u = 1/\cosh u = 2/(e^u + e^{-u})$ .

[2] Evil space aliens send a probe into our solar system to observe the earth. The probe orbits the sun with its perihelion at distance  $r_p = \frac{3}{4}a_\oplus$ , where its velocity is  $v_p = \frac{\sqrt{8}}{3}v_\oplus$ . ( $a_\oplus$  and  $v_\oplus$  are the orbital radius and velocity of the earth, respectively.) The probe's orbit is coplanar with that of the earth, and you may neglect the interaction of the probe with all bodies other than the sun.

**(a)** Compute the eccentricity of the probe's orbit.

**(b)** Compute the probe's distance from the sun at aphelion, and its velocity at aphelion.

**(c)** Write down the geometric equation for the probe's orbit.

**(d)** Let perihelion occur when the azimuthal angle is  $\phi = \pi$ . At what value of  $\phi$  does the probe cross the earth's orbit?

**(e)** Compute the period of the probe's orbit.

[3] A particle moves under the influence of the two-dimensional force field  $\vec{F} = Ay e^{-x} \hat{x} + Bx^2 \hat{y}$ .

**(a)** Compute the work done on the particle in moving along a rectangular trajectory  $(0, 0) \rightarrow (u, 0) \rightarrow (u, v) \rightarrow (0, v) \rightarrow (0, 0)$ .

**(b)** Show that the work done in part **(a)** can be computed by evaluating the two-dimensional integral of  $\vec{\nabla} \times \vec{F}$  over the area of the rectangle.

[4] A particle of mass  $m$  is constrained to move frictionlessly along a wire shaped according to the symmetric function  $z = f(x)$ . The wire is rotated about the symmetry axis ( $\hat{\mathbf{z}}$ ) at constant angular frequency  $\omega$ .

- (a) Derive the equations of motion for the particle.
- (b) Write down and identify any conserved quantities.
- (c) Suppose  $f(x) = C|x|^k/ka^{k-1}$ . At what value(s) of  $x$  is the particle in equilibrium?
- (d) Under what conditions is the equilibrium in part (c) stable? Compute the frequency of small oscillations about stable equilibria.

[5] A man of mass  $m$  is attached to a harness and connected to a wall by a spring of spring constant  $k$ , as shown in the diagram below. The man walks on a flat platform of mass  $M$  which itself slides frictionlessly along an icy surface. Relative to the center of the platform, the man executes a prescribed motion  $x = b(t)$ .

- (a) Write down the equation of motion for the position  $X$  of the center of the platform relative to the (fixed) wall. *Remark:* Your equation should involve the function  $b(t)$ . (Does your equation make sense when  $b(t) = 0$  for all  $t$ ?)
- (b) If the man paces according to  $b(t) = b \sin \Omega t$ , write down and solve the equations of motion for the position of the platform, assuming that at  $t = 0$  the spring is unstretched and the velocity of the platform is  $V_0$ .