Physics 110A: Problem Set #5

`Reading: MT chapters 6 and 7; lecture notes pp. 186-270.`

[1] Two particles of masses m_1 and m_2 attract each other according to the logarithmic potential $U(r) = U_0 \ln(r/a)$.

(a) Write down and sketch the effective potential $U_{\text{eff}}(r)$.

(b) Find the radius r_0 and period τ_0 of a circular orbit.

(c) For small deviations about a circular orbit, write $r(t) = r_0 + \eta(t)$. Derive the equation of motion for the deviation $\eta(t)$ and solve this equation assuming η is small. (What do we mean by "small"?)

(d) What is the *geometric* equation of the perturbation $\eta(\phi)$? Is the perturbed orbit closed? Why or why not?

[2] Assuming the earth's orbit to be circular, what would happen if the sun were to suddenly lose a fraction $(1 - \alpha)$ of its mass? You should assume that the mass simply vanishes (perhaps in an isotropic neutrino burst), so that $M' = \alpha M_\odot$. Describe the resulting orbit of the earth.

[3] Find the central force law which results in a spiral orbit of shape $r(\phi) = C\phi^2$, where C is a constant.

[4] An orbiting space station always remains vertically above an observer on the earth's surface. Where on earth must the observer be located? What is the radius of the space station's orbit?

[5] A particle moves in an elliptical orbit in an inverse square central force field. If the ratio of its maximum angular velocity to its minimum angular velocity is λ, show that the orbit has eccentricity

$$\varepsilon = \frac{\sqrt{\lambda} - 1}{\sqrt{\lambda} + 1} \; .$$

[6] A spacecraft is designed to dispose of nuclear waste. Two possible missions are envisaged: *(i)* carry the waste to the sun, and *(ii)* carry the waste out of the solar system. Assume that no planetary flybys are permitted. If the thrusts occur only in the orbital plane, which mission requires the least energy? Explain your answer.

[7] A particle moves in a central force $\vec{f}(\vec{r}) = \hat{\boldsymbol{r}} f(r)$.

(a) Using the equation for the shape of the orbit,

$$\frac{d^2 s}{d\phi^2} + s = -\frac{\mu}{\ell^2 s^2} \, f(s^{-1}) \; ,$$

where $s = 1/r$, derive the force law under which the shape of the orbit is $r(\phi) = a/\cos(\phi - \phi_0)$. Explain why your answer makes excellent sense.

(b) What force law will result in the shape $r(\phi) = 2b/\phi^2$? Is the force attractive or repulsive? Sketch the orbit over the interval $\phi \in (0, \infty)$.