PHYSICS 110A : CLASSICAL MECHANICS
MIDTERM EXAM #2

A mechanical system consists of a ring of radius a and mass M, and a point particle of mass m configured as shown in the sketch below. The ring is affixed to a massless rigid rod of length ℓ which is free to swing in a plane (the plane of the ring). The point mass m moves along the inner surface of the ring. The apparatus moves under gravity.

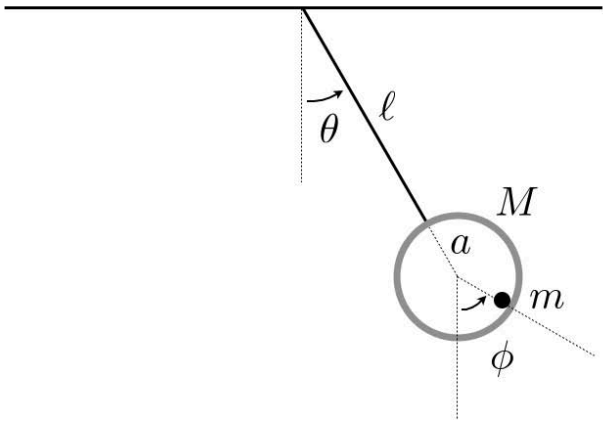

Figure 1: A point mass m slides frictionlessly inside a ring of radius a and mass M which is affixed to a rigid rod of length ℓ. The apparatus moves under the influence of gravity.

(a) Choose as generalized coordinates the angles θ and ϕ shown in the figure. Express the Cartesian coordinates (x, y) of the point mass in terms of the angles θ and ϕ and the lengths ℓ and a. Note that the center of the ring lies a distance $(\ell + a)$ from the fulcrum.
[20 points]

(b) Find the Lagrangian $L(\theta, \phi, \dot{\theta}, \dot{\phi}, t)$. You may find it convenient to abbreviate $\ell + a \equiv b$.
[20 points]

(c) Find p_θ, p_ϕ, F_θ, and F_ϕ.
[20 points]

(d) Write down the equations of motion in terms of the generalized coordinates and their first and second time derivatives.
[20 points]

(e) What, if anything is conserved? Express all conserved quantities in terms of the generalized coordinates and velocities.
[20 points]

(f) Introduce another generalized coordinate, r, defined to be the instantaneous distance from the mass m to the center of the ring. Then impose the constraint $r = a$. Find the force of constraint Q_r.
[20 quatloos extra credit]