PHYSICS 110A : CLASSICAL MECHANICS
MIDTERM EXAM #1

[1] A particle of mass m moves in the one-dimensional potential

$$U(x) = k\left(x^2 - a^2\right) e^{-x/a} \ . \tag{1}$$

(a) What are the dimensions of the constants k and a?

(b) Sketch $U(x)$. Identify the location(s) of any local minima and/or maxima, and be sure that your sketch shows the proper behavior as $x \to \pm\infty$.

(c) Sketch a representative set of phase curves. Identify any and all fixed points, find their energies, and assess their stability. Find the energy of every separatrix.

(d) Find the frequency of small oscillations about the minimum of $U(x)$.

[2] Consider the electrical circuit depicted below. The inductance is $L = 1\,\mathrm{mH}$ and the capacitances are $C_1 = 100\,\mu\mathrm{F}$ and $C_2 = 150\,\mu\mathrm{F}$. The system is forced by a time-dependent voltage source $V(t) = V_0\,\cos(\Omega t)$, where $V_0 = 8\,\mathrm{mV}$ and $\Omega = 10^3\,\mathrm{s}^{-1}$. The charge Q_1 on the upper plate of capacitor C_1 is found to lead the voltage source $V(t)$ (*i.e.* the difference in potential between the upper and lower termini of the source in the figure) by a phase angle $\delta = \frac{\pi}{4}$. Recall the relevant MKS units: $1\,\Omega = 1\,\mathrm{V}\cdot\mathrm{s}\,/\,\mathrm{C}$, $1\,\mathrm{F} = 1\,\mathrm{C}\,/\,\mathrm{V}$, and $1\,\mathrm{H} = 1\,\mathrm{V}\cdot\mathrm{s}^2/\,\mathrm{C}$.

(a) The voltage drops across the two capacitors are the same. Use this fact to express Q_1 in terms of the total charge $Q = Q_1 + Q_2$. Do the same for Q_2.

(b) Write town the equation of motion for $Q(t)$.

(c) What is the value of the resistance R?

(d) Find the current $I(t)$ flowing through the resistor. Your expression should involve no unknown quantities other than the time variable t.

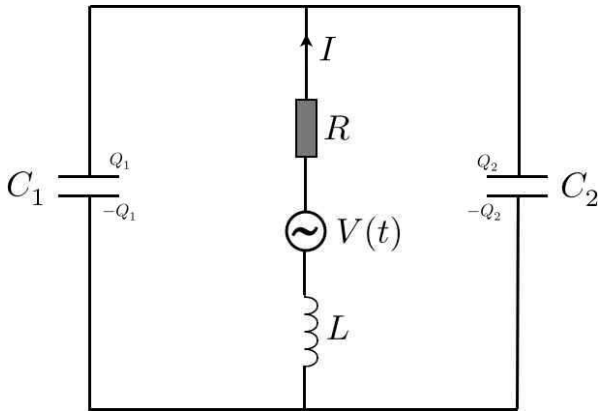

Figure 1: The circuit for problem 2.