PHYSICS 110A : CLASSICAL MECHANICS
MIDTERM EXAM #2

[**1**] A point mass m slides frictionlessly, under the influence of gravity, along a massive ring of radius a and mass M. The ring is affixed by horizontal springs to two fixed vertical surfaces, as depicted in fig. 1. All motion is within the plane of the figure.

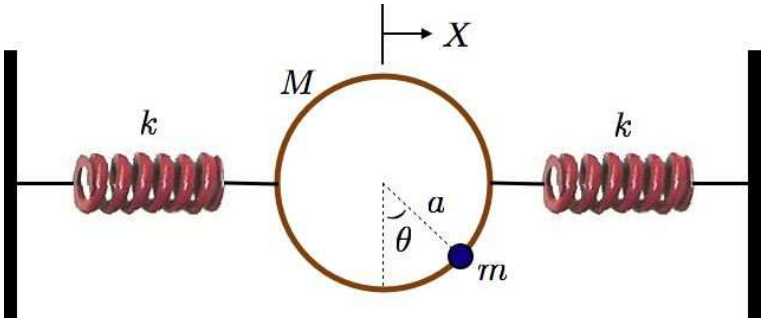

Figure 1: A point mass m slides frictionlessly along a massive ring of radius a and mass M, which is affixed by horizontal springs to two fixed vertical surfaces.

(a) Choose as generalized coordinates the horizontal displacement X of the center of the ring with respect to equilibrium, and the angle θ a radius to the mass m makes with respect to the vertical (see fig. 1). You may assume that at $X = 0$ the springs are both unstretched. Find the Lagrangian $L(X, \theta, \dot{X}, \dot{\theta}, t)$.
[15 points]

(b) Find the generalized momenta p_X and p_θ, and the generalized forces F_X and F_θ
[10 points]

(c) Derive the equations of motion.
[15 points]

(d) Find expressions for all conserved quantities.
[10 points]

[2] A point particle of mass m moves in three dimensions in a helical potential

$$U(\rho, \phi, z) = U_0 \, \rho \, \cos\left(\phi - \frac{2\pi z}{b}\right) .$$

We call b the pitch of the helix.

(a) Write down the Lagrangian, choosing (ρ, ϕ, z) as generalized coordinates.
`[10 points]`

(b) Find the equations of motion.
`[20 points]`

(c) Show that there exists a continuous one-parameter family of coordinate transformations which leaves L invariant. Find the associated conserved quantity, Λ. Is anything else conserved?
`[20 points]`