PHYSICS 110A : CLASSICAL MECHANICS
MIDTERM EXAM #1

[**1**] A particle of mass m moves in the one-dimensional potential

$$U(x) = \frac{U_0}{a^4}\left(x^2 - a^2\right)^2 \ . \tag{1}$$

(a) Sketch $U(x)$. Identify the location(s) of any local minima and/or maxima, and be sure that your sketch shows the proper behavior as $x \to \pm\infty$.
`[15 points]`

(b) Sketch a representative set of phase curves. Be sure to sketch any separatrices which exist, and identify their energies. Also sketch all the phase curves for motions with total energy $E = \frac{1}{2}\,U_0$. Do the same for $E = 2\,U_0$.
`[15 points]`

(c) The phase space dynamics are written as $\dot{\boldsymbol{\varphi}} = \boldsymbol{V}(\boldsymbol{\varphi})$, where $\boldsymbol{\varphi} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$. Find the upper and lower components of the vector field $\boldsymbol{V}$.
`[10 points]`

(d) Derive and expression for the period T of the motion when the system exhibits small oscillations about a potential minimum.
`[10 points]`

[**2**] An R-L-C circuit is shown in fig. 1. The resistive element is a light bulb. The inductance is $L = 400\,\mu$H; the capacitance is $C = 1\,\mu$F; the resistance is $R = 32\,\Omega$. The voltage $V(t)$ oscillates sinusoidally, with $V(t) = V_0 \cos(\omega t)$, where $V_0 = 4\,$V. In this problem, you may neglect all transients; we are interested in the late time, steady state operation of this circuit. Recall the relevant MKS units:

$$1\,\Omega = 1\,\mathrm{V \cdot s / C} \quad , \quad 1\,\mathrm{F} = 1\,\mathrm{C / V} \quad , \quad 1\,\mathrm{H} = 1\,\mathrm{V \cdot s^2 / C} \; .$$

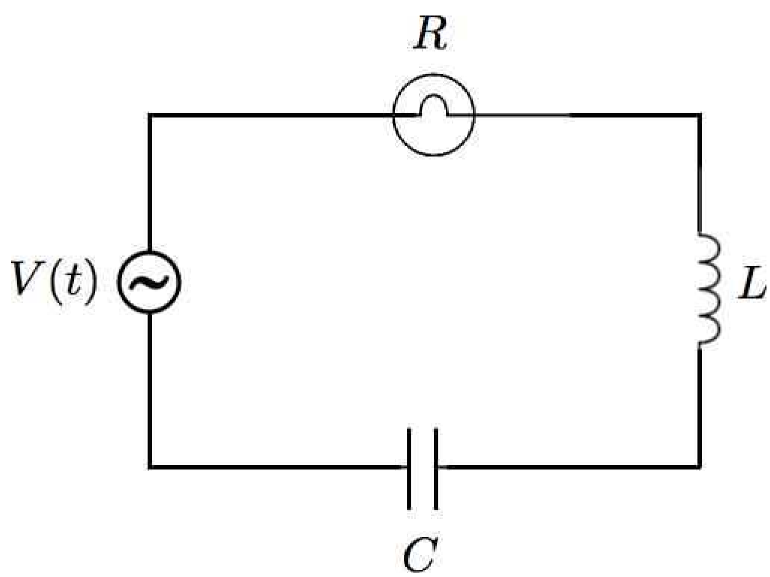

Figure 1: An R-L-C circuit in which the resistive element is a light bulb.

(a) Is this circuit underdamped or overdamped?
`[10 points]`

(b) Suppose the bulb will only emit light when the average power dissipated by the bulb is greater than a threshold $P_{\mathrm{th}} = \frac{2}{9}\,W$. For fixed $V_0 = 4\,$V, find the frequency range for ω over which the bulb emits light. Recall that the instantaneous power dissipated by a resistor is $P_R(t) = I^2(t) R$. (Average this over a cycle to get the average power dissipated.)
`[20 points]`

(c) Compare the expressions for the instantaneous power dissipated by the voltage source, $P_V(t)$, and the power dissipated by the resistor $P_R(t) = I^2(t) R$. If $P_V(t) \neq P_R(t)$, where does the extra power go or come from? What can you say about the averages of P_V and $P_R(t)$ over a cycle? Explain your answer.
`[20 points]`