PHYSICS 110A : CLASSICAL MECHANICS
MIDTERM EXAM #2

[1] Two blocks connected by a spring of spring constant k are free to slide frictionlessly along a horizontal surface, as shown in Fig. 1. The unstretched length of the spring is a.

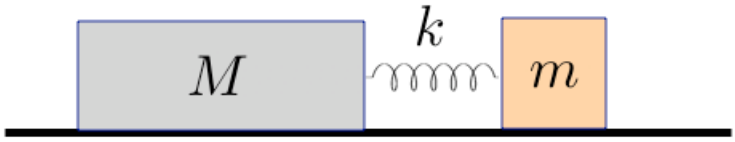

Figure 1: Two masses connected by a spring sliding horizontally along a frictionless surface.

(a) Identify a set of generalized coordinates and write the Lagrangian.
`[15 points]`

(b) Find the equations of motion.
`[15 points]`

(c) Find all conserved quantities.
`[10 points]`

(d) Find a complete solution to the equations of motion. As there are two degrees of freedom, your solution should involve 4 constants of integration. You need not match initial conditions, and you need not choose the quantities in part (c) to be among the constants.
`[10 points]`

[2] A uniformly dense ladder of mass m and length 2ℓ leans against a block of mass M, as shown in Fig. 2. Choose as generalized coordinates the horizontal position X of the right end of the block, the angle θ the ladder makes with respect to the floor, and the coordinates (x, y) of the ladder's center-of-mass. These four generalized coordinates are not all independent, but instead are related by a certain set of constraints.

Recall that the kinetic energy of the ladder can be written as a sum $T_{\rm CM} + T_{\rm rot}$, where $T_{\rm CM} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$ is the kinetic energy of the center-of-mass motion, and $T_{\rm rot} = \frac{1}{2}I\dot{\theta}^2$, where I is the moment of inertial. For a uniformly dense ladder of length 2ℓ, $I = \frac{1}{3}m\ell^2$.

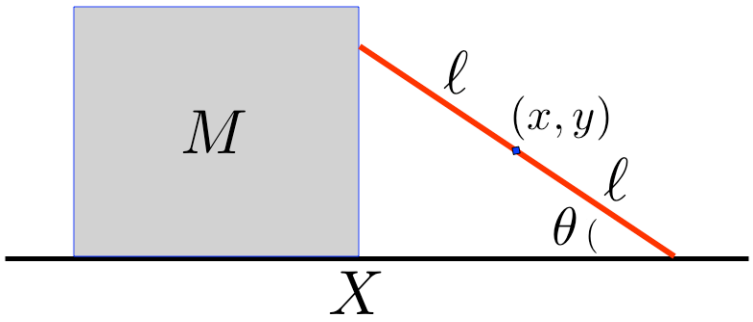

Figure 2: A ladder of length 2ℓ leaning against a massive block. All surfaces are frictionless..

(a) Write down the Lagrangian for this system in terms of the coordinates X, θ, x, y, and their time derivatives.
`[10 points]`

(b) Write down all the equations of constraint.
`[10 points]`

(c) Write down all the equations of motion.
`[10 points]`

(d) Find all conserved quantities.
`[10 points]`

(e) What is the condition that the ladder detaches from the block? You do not have to solve for the angle of detachment! Express the detachment condition in terms of any quantities you find convenient.
`[10 points]`