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Formulas:

Relativistic energy - momentum relation $E = \sqrt{m^2c^4 + p^2c^2}$; $c = 3 \times 10^8 m/s$

Electron rest mass : $m_e = 0.511\ MeV/c^2$;Proton : $m_p = 938.26\ MeV/c^2$;Neutron : $m_n = 939.55\ MeV/c^2$

Planck's law : $u(\lambda) = n(\lambda)\bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda}\frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67 \times 10^{-8} W/m^2K^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda)d\lambda$

Wien's displacement law : $\lambda_m T = \frac{hc}{4.96k_B}$

Photons : $E = pc$; $E = hf$; $p = h/\lambda$; $f = c/\lambda$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Constants : $hc = 12{,}400\ eV\,A$; $\hbar c = 1{,}973\ eV\,A$; $k_B = 1/11{,}600\ \ eV/K$; $ke^2 = 14.4 eVA$

Electrostatics : $F = \frac{kq_1q_2}{r^2}$ (force) ; $U = q_0V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum : $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097 \times 10^7\ m^{-1} = \frac{1}{911.3A}$

Bohr atom : $E_n = -\frac{ke^2Z}{2r_n} = -\frac{Z^2E_0}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{mk^2e^4}{2\hbar^2} = 13.6eV$; $E_n = E_{kin} + E_{pot}, E_{kin} = -E_{pot}/2 = -E_n$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529A$; $L = mvr = n\hbar$ angular momentum

X - ray spectra : $f^{1/2} = A_n(Z - b)$; K : $b = 1$, L : $b = 7.4$

de Broglie : $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Wave function $\Psi(x,t) = |\Psi(x,t)|e^{i\theta(x,t)}$; $P(x,t)\,dx = |\Psi(x,t)|^2\,dx$ = probability

Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well : $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2n^2}{2mL^2}$; $x_{op} = x$, $p_{op} = \frac{\hbar}{i}\frac{\partial}{\partial x}$; $\langle A \rangle = \int_{-\infty}^{\infty} dx\psi^* A_{op}\psi$

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Eigenvalues and eigenfunctions: $A_{op}\ \Psi = a\ \Psi$ (a is a constant) ; uncertainty: $\Delta A = \sqrt{<A^2> - <A>^2}$

Harmonic oscillator: $\Psi_n(x) = C_n H_n(x) e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Step potential: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - V)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T \sim e^{-2\alpha\Delta x}$; $T \sim e^{-2\int_a^b \alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[V(x) - E]}{\hbar^2}}$

3D square well: $\Psi(x,y,z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{n,\ell,m}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$; $Y_{\ell m}(\theta,\phi) = f_{lm}(\theta)e^{im\phi}$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$; $L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$; $L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$; $L_z = m\hbar$

Radial probability density: $P(r) = r^2 |R_{n,\ell}(r)|^2$; Energy: $E_n = -13.6eV\frac{Z^2}{n^2}$

Ground state of hydrogen and hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$

Orbital magnetic moment: $\vec{\mu} = \frac{-e}{2m_e}\vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} eV/T$

Spin 1/2: $s = \frac{1}{2}$, $|S| = \sqrt{s(s+1)}\hbar$; $S_z = m_s\hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e}g\vec{S}$

Total angular momentum: $\vec{J} = \vec{L} + \vec{S}$; $|J| = \sqrt{j(j+1)}\hbar$; $|l - s| \le j \le l + s$; $-j \le m_j \le j$

Orbital+spin mag moment: $\vec{\mu} = \frac{-e}{2m}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu}\cdot\vec{B}$

Two particles : $\Psi(x_1,x_2) = +/- \Psi(x_2,x_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $Z \to Z_{eff}$, $1 < Z_{eff} < Z$

Orbital ordering:
1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d ~ 5f

$I_n(\lambda) = \int_0^\infty dx\, x^n e^{-\lambda x^2}$; $I_0 = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$; $I_1 = \frac{1}{2\lambda}$; $I_{n+2} = -\frac{\partial}{\partial\lambda}I_n$

$f_{MB}(E) = Ce^{-E/kT}$; $f_{BE}(E) = \frac{1}{e^\alpha e^{E/kT} - 1}$; $f_{FD}(E) = \frac{1}{e^\alpha e^{E/kT} + 1}$; $n(E) = g(E)f(E)$

$F(v_x,v_y,v_z) = f(v_x)f(v_y)f(v_z)$; $f(v_x) = (\frac{m}{2\pi kT})^{1/2}e^{-mv_x^2/2kT}$ (kinetic theory)

Rotation: $E_R = \frac{L^2}{2I}$, $I = \mu R^2$, vibration: $E_v = \hbar\omega(n+\frac{1}{2})$, $\omega = \sqrt{K/\mu}$, $\mu = m_1 m_2/(m_1 + m_2)$

$g(E) = [2\pi(2m)^{3/2}V/h^3]E^{1/2}$ (translation, per spin) ; Equipartition: $<E> = k_B T/2$ per degree of freedom

Lasers: $B_{12} = B_{21}$, $A_{21}/(B_{21}u(f)) = e^{hf/kT} - 1$; $u(f) = (8\pi hf^3/c^3)/(e^{hf/kT} - 1)$

Justify all your answers to all problems

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Problem 1 (10 pts)
1mol of Ne gas (monatomic) occupies a volume $8cm^3$. The atomic weight of Ne is 20, and its total spin is 0, so it's a boson.
(a) Give the average kinetic energy (in eV) of a Ne atom in this gas at temperature 300°K.
(b) Estimate the de Broglie wavelength (in A) for a Ne atom in this gas at 300°K.
(c) Explain why you can use the Maxwell-Boltzmann distribution for this gas at temperature 300K.
(d) Estimate at what temperature (in °K) quantum effects will become sufficiently important that you can no longer use the Maxwell Boltzmann distribution for this gas (assuming the system remains a gas rather than becoming liquid or solid).

Problem 2 (10 pts)
Consider a system of N distinguishable particles that has only two possible energy states. Each particle can be in an energy state of energy $E_1=0$ or in energy states of energy $E_2=\varepsilon=4.4$ eV. The statistical weight (degeneracy) is $g_1=1$ for the state of energy E_1 and $g_2=9$ for the states of energy E_2. No other states are available for these particles. Use the Boltzmann distribution since the particles are distinguishable.
(a) At what temperature are there on average an equal number of particles with energy E_1 as there are with energy E_2? Give your answer in °K.
(b) What is the average energy per particle at the temperature found in (a)? Give your answer in eV.
(c) What is the average energy per particle as T--> oo? (very high temperature). Give your answer in eV.
(d) For one mol of this system, <u>estimate</u> the heat capacity C_V at temperature 300°K. Your estimated answer should not differ from the exact answer by more than 0.1R, with R the gas constant ($R=N_Ak$, $N_A=6.02x10^{23}$).

Problem 3 (10 pts)
The heat capacity of 1 mol of a diatomic gas has values:
$C_V=1.5R$ at temperature T=5K.
$C_V=2.4R$ at temperature T=10K.
$C_V=2.8R$ at temperature T=100K.
The distance between the two atoms in the molecule is 1.5A. The two atoms are identical.
(a) Estimate the atomic weight of the atoms in this molecule. Justify your answer.
(b) Estimate the vibration frequency of this molecule, in s^{-1}.
(Use $\hbar = 6.58 \times 10^{-16} eV\, s$ or $h = 4.136 \times 10^{-15} eV\, s$)
(c) Estimate the value of its heat capacity at T=2K, assuming the system remains a gas at T=2K.

Justify all your answers to all problems