

Formulas:

Relativistic energy - momentum relation $E = \sqrt{m^2 c^4 + p^2 c^2}$; $c = 3 \times 10^8 \text{ m/s}$

Electron rest mass : $m_e = 0.511 \text{ MeV}/c^2$; Proton : $m_p = 938.26 \text{ MeV}/c^2$; Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Planck's law : $u(\lambda) = n(\lambda) \bar{E}(\lambda)$; $n(\lambda) = \frac{8\pi}{\lambda^4}$; $\bar{E}(\lambda) = \frac{hc}{\lambda} \frac{1}{e^{hc/\lambda k_B T} - 1}$

Energy in a mode/oscillator : $E_f = nhf$; probability $P(E) \propto e^{-E/k_B T}$

Stefan's law : $R = \sigma T^4$; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$; $R = cU/4$, $U = \int_0^\infty u(\lambda) d\lambda$

Wien's displacement law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Photons : $E = pc$; $E = hf$; $p = h/\lambda$; $f = c/\lambda$

Photoelectric effect : $eV_0 = (\frac{1}{2}mv^2)_{\max} = hf - \phi$, $\phi \equiv$ work function

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$

Rutherford scattering: $b = \frac{kq_\alpha Q}{m_\alpha v^2} \cot(\theta/2)$; $\Delta N \propto \frac{1}{\sin^4(\theta/2)}$

Constants : $hc = 12,400 \text{ eV \AA}$; $k_B = 1/11,600 \text{ eV/K}$; $ke^2 = 14.4 \text{ eV \AA}$

Electrostatics : $F = \frac{kq_1 q_2}{r^2}$ (force) ; $U = q_0 V$ (potential energy) ; $V = \frac{kq}{r}$ (potential)

Hydrogen spectrum: $\frac{1}{\lambda} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ \AA}}$

Bohr atom: $E_n = -\frac{ke^2 Z}{2r_n} = -\frac{Z^2 E_0}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{mk^2 e^4}{2\hbar^2} = 13.6 \text{ eV}$; $E_n = E_{kin} + E_{pot}$, $E_{kin} = -E_{pot}/2 = -E_n$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{mke^2} = 0.529 \text{ \AA}$; $L = mvr = n\hbar$ angular momentum

Justify all your answers to all problems

Problem 1 (10 pts)

In a different universe, the energy of electromagnetic radiation is quantized according to the law

$$E = nIf^2$$

instead of Planck's law $E = nhf$. f is the frequency of the radiation, n is an integer, and I is the equivalent of Planck's constant h in that universe. The Boltzmann distribution, counting of modes, etc., are the same in that universe as in ours.

(a) The equivalent of Stefan's law in that universe is

$$P = CT^\alpha$$

where C and α are constants. Find α .

(b) The equivalent of Wien's displacement law in that universe is

$$\lambda_m T^\beta = D$$

where D and β are constants. Find β .

(c) What are the dimensions of the constant I , and what might it represent physically?

Problem 2 (10 pts)

In a Compton scattering experiment, incident photons have wavelength 1\AA and scattered electrons have kinetic energy 493 eV .

(a) Find the wavelength of the scattered photons, in \AA .

(b) Find the angle θ at which the photons are scattered, in degrees.

Hint: use energy conservation.

Problem 3 (10 pts)

(a) The emission line spectrum emitted by a hydrogen-like ion extends down to wavelengths smaller than 15 \AA . Clearly, this is not hydrogen ($Z=1$). What is the minimum Z that this ion has to have?

(b) A hydrogen-like ion has the electron orbiting at speed faster than $0.1c$. What is the minimum Z that this ion has to have? (Hint: use angular momentum).

(c) A hydrogen-like ion has the electron in an orbit of radius 0.0529\AA . Give two possible values for the Z of this ion.

Justify all your answers to all problems