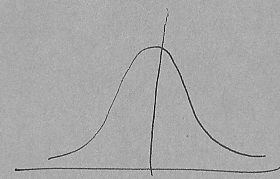
Spreading Gaussian Wavepacket.

$$f(k) = e^{-\frac{\alpha k^2}{2}}$$



$$\omega(k) = \omega(0) + \frac{\partial \omega}{\partial k} k + \frac{1}{2}\frac{\partial^2 \omega}{\partial k^2} k^2$$

$$\psi(x,t) = \int dk\, e^{-\alpha k^2/2}\, e^{ikx}\, e^{-it\left(\omega_0 + \frac{\partial \omega}{\partial k} k + \frac{1}{2}\frac{\partial^2 \omega}{\partial k^2} k^2\right)}$$

$$e^{i\omega t_0}\cdot \psi(x,t) = \int dk\, e^{-\frac{\alpha k^2}{2} + ik\left(x - \frac{\partial \omega}{\partial k} t\right) - \frac{i}{2}\frac{\partial^2 \omega}{\partial k^2} k^2 t}$$

$$= \int dk' \sqrt{\frac{2}{\alpha}}\, e^{-k'^2 + i\sqrt{\frac{2}{\alpha}} k'\left(x - \frac{\partial \omega}{\partial k} t\right) - i\frac{\partial^2 \omega}{\partial k^2}\frac{k'^2}{\alpha} t}$$

$$\left(\begin{array}{l} k' = \sqrt{\frac{\alpha}{2}}\, k \\ dk' = \sqrt{\frac{\alpha}{2}}\, dk \end{array} \right)$$

$$= \sqrt{\frac{2}{\alpha}} \int dk'\, e^{-k'^2\left(1 - i\frac{\partial^2 \omega}{\partial k^2}\frac{t}{\alpha}\right) + i\sqrt{\frac{2}{\alpha}} k'(x - v_g t)}$$

$$\left(k'' = k'\sqrt{1 - i\frac{\partial^2 \omega}{\partial k^2}\frac{t}{\alpha}} \right)$$

$$= \sqrt{\frac{2}{\alpha}} \int_{\frac{-\infty}{\sqrt{1 - i\frac{\partial^2 \omega}{\partial k^2}\frac{t}{\alpha}}}}^{\frac{\infty}{\sqrt{1 - i\frac{\partial^2 \omega}{\partial k^2}\frac{t}{\alpha}}}} dk''\, e^{-\left(k''^2 - i\sqrt{\frac{2}{\alpha}}\frac{k''(x - v_g t)}{\sqrt{1 - i\frac{\partial^2 \omega}{\partial k^2}\frac{t}{\alpha}}}\right)} \left(\frac{1}{\sqrt{1 - i\#}}\right)$$

$$= \sqrt{\frac{2}{\alpha}} \int_{-\infty'}^{\infty'} dk''\, e^{-\left(k'' - i\frac{1}{\sqrt{2\alpha}}\frac{x - v_g t}{\sqrt{1 - i\#}}\right)^2} e^{\frac{2}{\alpha}\frac{(x - v_g t)^2}{1 - i\#}} \cdot \left(\frac{1}{\sqrt{1 - i\#}}\right)$$

$$= \sqrt{\frac{2}{\alpha}}\, e^{\frac{2}{\alpha}\frac{(x - v_g t)^2}{1 - i\#}} \left(\int_{-\infty''}^{\infty''} dk'''\, e^{-k'''^2}\right) \frac{1}{\sqrt{1 - i\#}}$$

$$= \sqrt{\frac{2\pi}{\alpha}} \cdot e^{\frac{2}{\alpha}\frac{(x - v_g t)^2}{1 - i\#}} \cdot \frac{1}{\sqrt{1 - i\#}}$$

$$\boxed{|\psi(x,t)|^2 = \frac{2\pi}{\alpha}\frac{1}{\sqrt{1 + \#^2}}\, e^{\frac{4(x - v_g t)^2}{\alpha(1 + \#^2)}}}$$

$$\boxed{\# = \frac{\beta}{\alpha} t} \qquad \beta = \left.\frac{\partial^2 \omega}{\partial k^2}\right|_{k=0}$$

At $t = 0$,

$$\sigma_x = \frac{\sqrt{\alpha}}{2}$$

$$\sigma_k = \frac{1}{\sqrt{\alpha}}$$

$$\sigma_x \sigma_k = \frac{1}{2}$$ ☑

4-43. $\lambda = \left[R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right]^{-1}$ $\quad \Delta\lambda = \frac{d\lambda}{d\mu}\Delta\mu = \left(-R^{-2}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}\frac{dR}{d\mu}\Delta\mu$

Because $R \propto \mu$, $\quad dR/d\mu = R/\mu$. $\quad \Delta\lambda \approx \left(-R^{-2}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}(R/\mu)\Delta\mu = -\lambda\left(\Delta\mu/\mu\right)$

$$\mu_H = \frac{m_e m_p}{m_e + m_p} \qquad \mu_D = \frac{m_e m_d}{m_e + m_d}$$

$$\frac{\Delta\mu}{\mu} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_d/\left(m_e + m_d\right)}{m_e m_p/\left(m_e + m_p\right)} - 1 = \frac{m_d/\left(m_e + m_d\right)}{m_p/\left(m_e + m_p\right)} - 1 = \frac{m_e\left(m_d - m_p\right)}{m_p\left(m_e + m_d\right)}$$

If we approximate $m_d = 2m_p$ and $m_e \ll m_d$, then $\frac{\Delta\mu}{\mu} \approx \frac{m_e}{2m_p}$ and

$$\Delta\lambda = -\lambda\left(\Delta\mu/\mu\right) = -\left(656.3nm\right)\frac{0.511MeV}{2\left(938.28MeV\right)} = -0.179nm$$

4-44. For maximum recoil energy for the Hg atoms, the collision is 'head-on'.

(a)

	Before collision	After collision
kinetic energy	$E_k = \frac{1}{2}mv_{ei}^2$	$E_k' = \frac{1}{2}mv_{ef}^2$ $E_{Hg} = \frac{1}{2}Mv_{Hg}^2$
momentum	$p_{ei} = mv_{ei}$	$p_{ef} = mv_{ef}$ $p_{Hg} = Mv_{Hg}$

Conservation of momentum requires:

$$mv_{ei} = -mv_{ef} + Mv_{Hg} \rightarrow v_{Hg} = \frac{m}{M}\left(v_{ei} + v_{ef}\right)$$

Therefore, the maximum Hg recoil kinetic energy is given by:

$$\frac{1}{2}Mv_{Hg}^2 = \frac{1}{2}M\left(\frac{m}{M}\right)^2\left(v_{ei} + v_{ef}\right)^2 = \frac{m^2}{2M}\left(v_{ei}^2 + 2v_{ei}v_{ef} + v_{ef}^2\right)$$

$$\approx \frac{m^2}{2M}\left(4v_{ei}^2\right) \text{ since } m \ll M,\ v_{ei} \approx v_{ef}$$

$$\approx \frac{4m}{M}\left(\frac{1}{2}mv_{ei}^2\right) = \frac{4m}{M}E_k$$

(b) Since the collision is elastic, kinetic energy is conserved, so the maximum kinetic energy gained by the Hg atom equals the maximum kinetic energy lost by the electron. If $E_k = 2.5eV$, then the maximum lost is equal to:

$$4\frac{m}{M}(2.5eV) = 4\frac{9.11\times10^{-31}kg\,(2.5eV)}{(201u)(1.66\times10^{-27}kg/u)} = 2.7\times10^{-5}eV$$