Einstein, 1905-1906: A photon carries energy $E=hf$ and momentum $p=E/c=hf/c$
Therefore, the wavelength and frequency of an electromagnetic wave are

$\lambda=c/f$

Louis de Broglie, 1923: all forms of matter have wave as well as particle properties. The wavelength and frequency of a **matter wave** associated with any moving object are

$$\lambda = \frac{h}{p}, \qquad f = \frac{E}{h}$$

where h is Plank's constant, p is momentum and E is energy of the object

Interference leads to selection of certain waves. For example a guitar string of length L supports only standing waves that have nodes at each end, i.e. with $2L=n\lambda$. Other wavelengths rapidly vanish by destructive interference

Only certain electron radii are allowed for the electrons in the atom

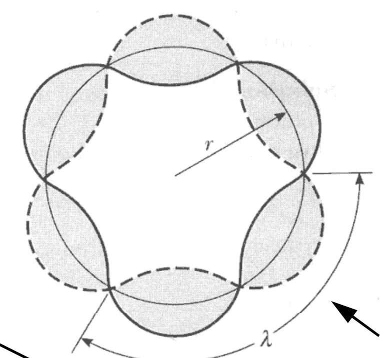
Apply this reasoning to the electrons in the atoms:
The allowed Bohr orbits arise because the electron matter waves interfere constructively when an integral number of wavelengths fits into the circumference of a circular orbit

$$r_n = \frac{n^2\hbar^2}{m_e k e^2}$$

$$\lambda = \frac{h}{p} = \frac{h}{m_e v}$$

$$m_e v \lambda = h$$

$$m_e v \frac{2\pi r}{n} = h$$



$n=3$

$n\lambda = 2\pi r$

1st Bohr's postulate: only those electron orbits occur for which the angular momentum of the electron is $nh/2\pi$, where n is an integer and h is Planck's constant.

$$m_e v r = n\frac{h}{2\pi} = n\hbar$$

Why don't we see the wave properties of macroscopic objects?

For a baseball of mass 140 g traveling at a speed of 60 mi/h=27 m/s

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(0.14\ \text{kg})\cdot(27\ \text{m/s})} = 1.7\times10^{-34}\ \text{m}$$

It is too small even compared to nucleus whose size is ~10^{-14} m
This is why wave properties of macroscopic objects are not revealed
and they appear as particle like

An object will reveal its wave properties if it exhibits interference or diffraction, which require scattering objects or apertures with a size comparable to wavelength

To observe wave properties of matter: study microscopic particles with small $m \rightarrow \lambda$ is large
study interference from small objects with a size ~ λ

EXPERIMENT #6
Electron Diffraction

GOALS

Physics

Determine the de Broglie wavelength for electrons, by diffracting them from parallel planes of atoms in a carbon film.

Techniques

Control the wavelength of the electron by varying its kinetic energy $KE = eV_a$ from an accelerating voltage.

References

Serway, Moses, Moyer §5.2

Louis de Broglie 1923: a particle with momentum p possess a wavelength $\lambda = h/p$

Clinton J. Davisson and Lester H. Germer, 1927: direct experimental proof
George P. Thomson by diffraction experiments

When an electron is accelerated through a potential difference V, it gains a kinetic energy

$$\frac{1}{2}mv^2 = eV \qquad v = \sqrt{\frac{2eV}{m}} \qquad mv = \sqrt{2meV}$$

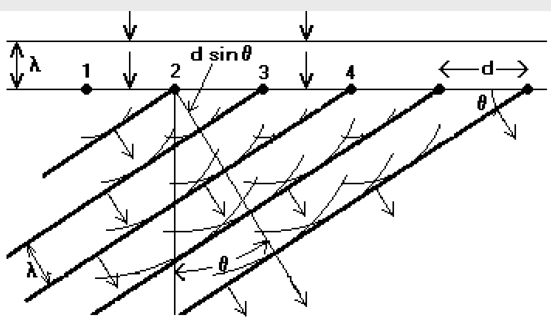
$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = (1.23\ \text{nm})\left(\frac{V}{1\ \text{Volt}}\right)^{-1/2}$$

Accelerating electrons in a voltage readily produces a beam of electrons with a sub-nanometer wavelength

For V=50 V $\quad \lambda = \frac{h}{\sqrt{2meV}} = (1.23\ \text{nm})(50)^{-1/2} = 0.17\ \text{nm} = 1.7\ \text{Å}$

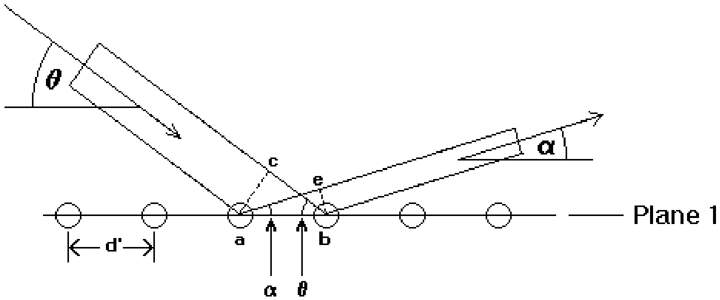
Wave properties of electrons can be revealed by diffraction. Diffraction of the electron beam form a grating requires the spacing between the rulings of the order the sub-nanometer electron wavelength $\quad d\sin\theta = n\lambda$

atomic lattices have "natural" spacings of Ångstroms 1Å=0.1 nm

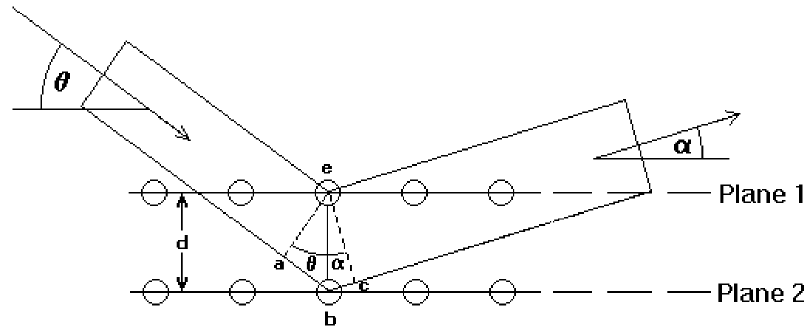


the spacing between the rulings in regular gratings are of the order of microns (10^3 nm)

Conditions for constructive interference: Constructive interference will occur for the rays scattered from atoms if the difference in path length is a whole number of wavelengths



Scattering of waves from a plane of atoms



Scattering of waves from successive planes of atoms

1. Condition for constructive interference for the rays scattered from <u>neighboring atoms</u> separated by a distance *d'*

$$\overline{ae} - \overline{cb} = d'\cos\alpha - d'\cos\theta = m\lambda$$

2. Condition for constructive interference for the rays scattered from <u>successive planes</u> separated by a distance *d*

$$\overline{ab} + \overline{bc} = d\sin\theta + d\sin\alpha = n\lambda$$

These conditions can be satisfied simultaneously if $\theta = \alpha$. In that case $m = 0$ satisfies the first condition for any *d'*, and $n\lambda = 2d\sin\theta$ satisfies the second condition.

$$\theta = \alpha$$

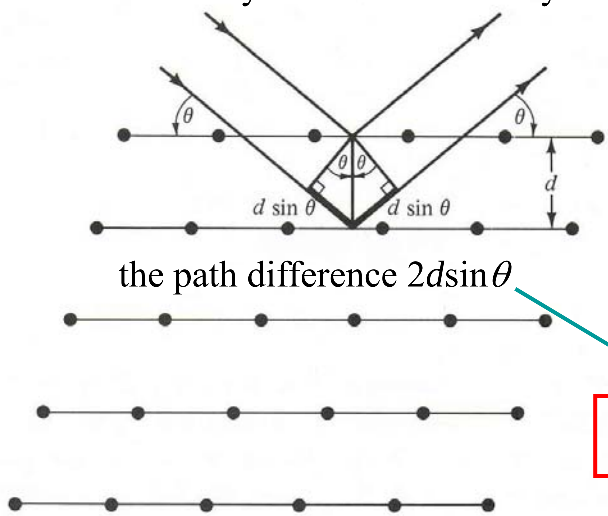
$$n\lambda = 2d\sin\theta$$

X-ray Bragg diffraction:

intense peaks of scattered radiation are observed for certain wavelengths and directions

↑ Bragg peaks

incident ray reflected ray



the path difference $2d\sin\theta$

specular reflection by a plane implies constructive interference of rays scattered by individual ions within the plane

William Henry Bragg and William Lawrence Bragg, 1912:

the conditions for a sharp peak in the intensity of scattered wave

1 – the X-ray should be specularly reflected by the ions in one plane

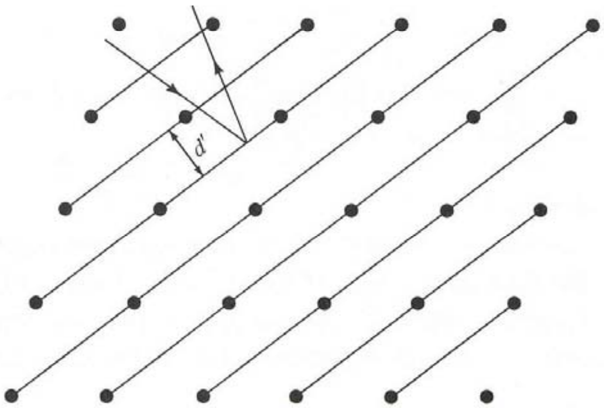
2 – the reflected waves from successive planes should interfere constructively

Bragg condition $n\lambda = 2d\sin\theta$

n – order of the corresponding reflection

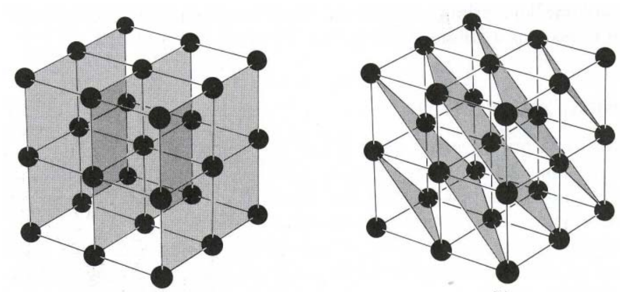
θ – Bragg angle

2θ – the angle by which the incident beam is deflected

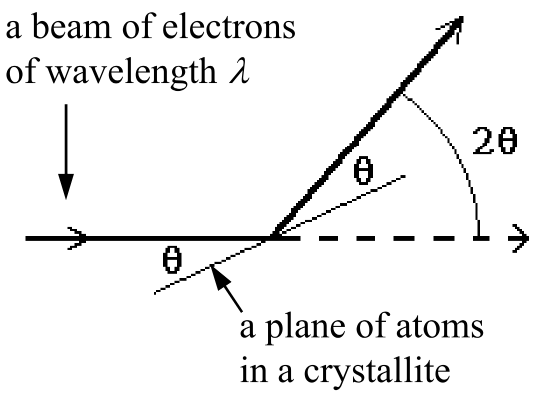


the same lattice,
the same incident ray
but different direction
and λ of the reflected ray

any family of planes
produces reflections

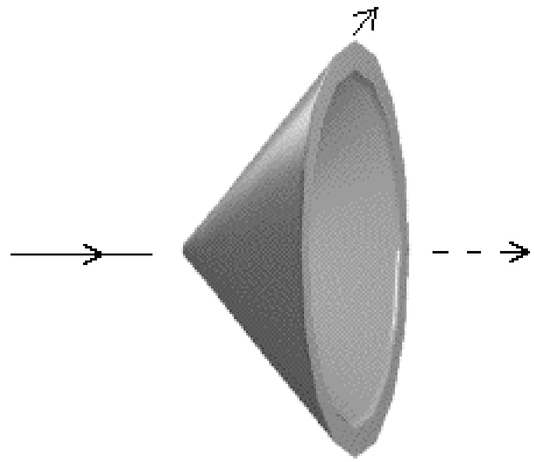


Most materials are polycrystalline. They are composed of a large number of small single crystals (crystallites) that are randomly oriented. Your electron diffraction sample is a polycrystalline film, thin enough so that the diffracted electrons can be transmitted through the film.



$$n\lambda = 2d\sin\theta$$

the beam will be diffracted by the angle $\gamma = 2\theta$



many randomly oriented crystallites in a polycrystalline film scatter the electron beam into a cone when the Bragg condition is fulfilled by planes of atoms disposed symmetrically about the incident beam

For n=1 the Bragg condition becomes $\lambda = 2d\sin\theta \approx 2d\theta = d\gamma$

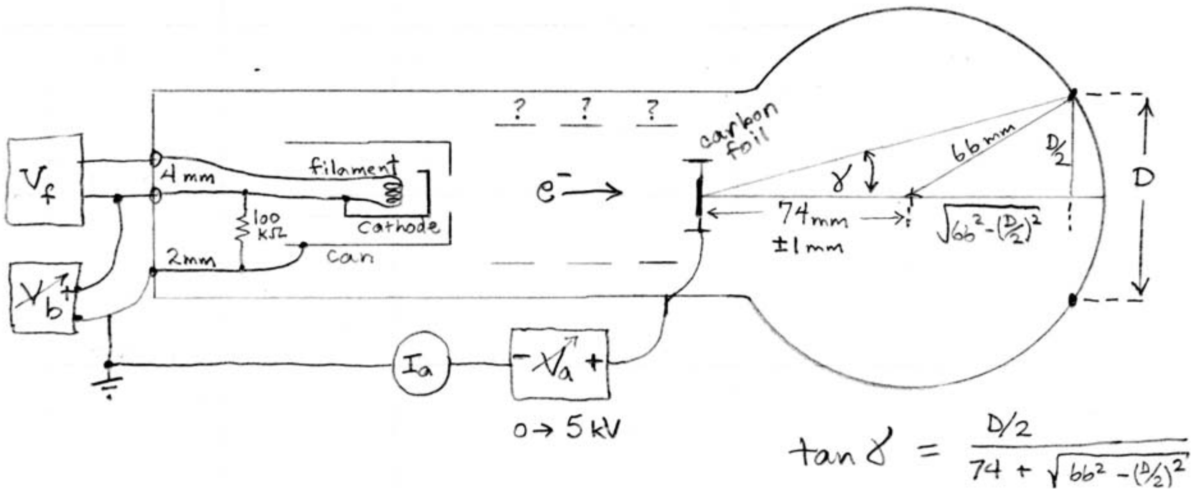
THE EXPERIMENT

Equipment

1. Electron diffraction tube with carbon thin film target.
2. High and low voltage power supplies.
3. Calipers for measuring diffraction ring diameters.

CAUTION

1. The 5kV power source can give you a very nasty shock. Verify that your circuit is correctly wired before turning on power. Have your instructor or TA check the circuit.
2. Check that the anode current monitoring meter is on the grounded side of the circuit as shown in the diagram below.
3. Never permit the anode current to exceed 0.2 mA; otherwise the target may be damaged.



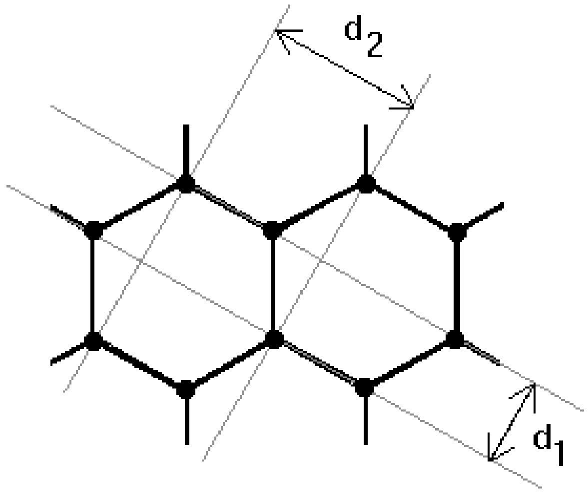
The electron diffraction tube

The carbon film is mounted in the anode.
The anode voltage $+V_a$ accelerates the electrons and then they hit the carbon film.
The variable anode voltage V_a is provided by the 5kV dc supply.

The electrons are emitted from a heated oxide-coated cathode.
The heater voltage, V_f, is supplied by the 6 Volt output.

The external bias V_{bias} for the can surrounding the cathode serves to focus the electron beam.

The beam current I_a varies with both anode and bias voltages. Be sure to keep the beam current below 0.2 mA. The energetic electron beam deposits its power $P = I_a \cdot V_a \leq 0.2\ \text{mA} \cdot 5\ \text{kV} = 1\ \text{Watt}$ as heat in the carbon target. If it's glowing dull red, it's too hot. To prevent surprising increases in current, you should stabilize the filament heater current for about a minute before turning on the anode voltage.



The atoms in a carbon crystal are located on the corners of hexagons. The two sets of planes produce the diffraction rings

the spacings are
$d_1 = 0.123$ nm
$d_2 = 0.213$ nm

● As you turn up the anode voltage you will see two rings on the screen. Each ring corresponds to one of the carbon d spacings (d_1 or d_2).

● Calculate de Broglie wavelength of the electrons $\lambda = \frac{h}{p} = (1.23 \text{ nm}) \cdot (V_a - V_b)^{-1/2}$ as a function of V_a

● Measure the ring diameter D on the screen with calipers and calculate γ from D. See the geometric construction on the previous viewgraph.

● For each ring, plot $(V_a - V_b)^{-1/2}$ as a function of γ for a number of values of V_a. Determine d_1 and d_2 from the slopes of these curves using the diffraction condition derived for the polycrystalline carbon film $\lambda = (1.23 \text{ nm}) \cdot (V_a - V_b)^{-1/2} = d\,\gamma$

● Using error analysis, compare your values to the d spacings expected for carbon.

$$\lambda = \frac{h}{p}$$

light

$$p = E/c$$

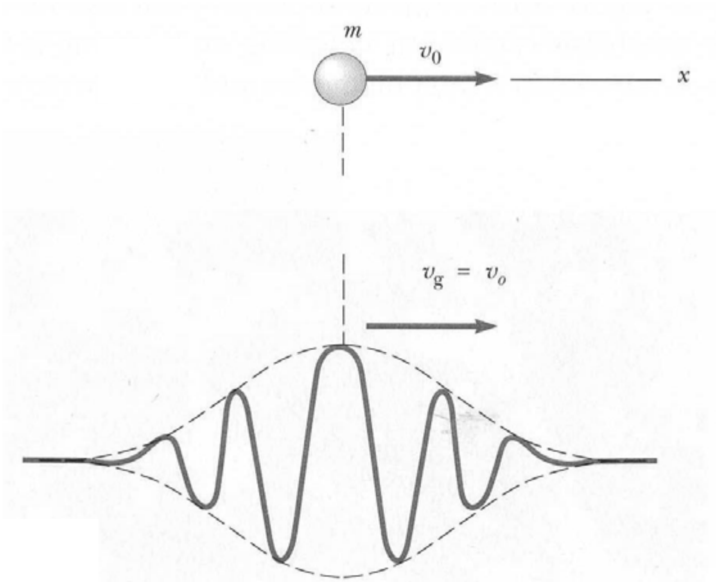
$$\lambda = \frac{h}{p} = \frac{hc}{E}$$

matter (nonrelativistic)

$$p = \sqrt{2mE} \longleftarrow E = \frac{p^2}{2m}$$

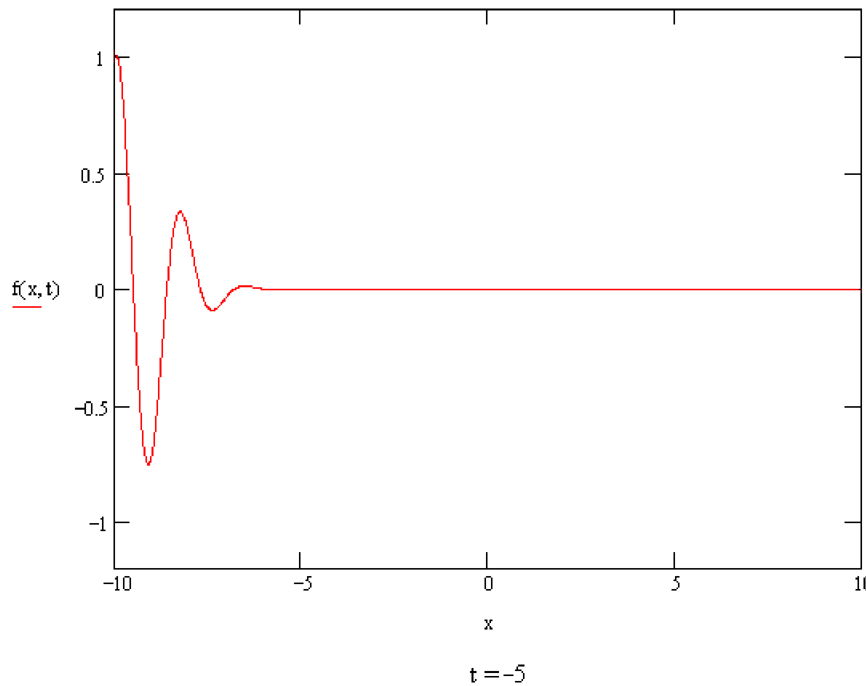
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

	Light	**Matter**
Wave property	**Light interference** Revealed in experiments on light diffraction and interference (e.g. Michelson interferometer)	**Matter interference** Revealed in Davisson-Germer experiments on electron diffraction
Particle (corpuscular) property	**Light carries momentum** Revealed in the Compton effect	**Matter carriers momentum** Revealed e.g. in electron scattering experiments



A particle is represented by a <u>wave group or wave packets</u> of limited spatial extent,
which is a superposition of many matter waves with a spread of wavelengths centered on $\lambda_0 = h/mv$

The wave group moves with a speed v_g – <u>the group speed</u>, which corresponds to the classical particle speed



sinusoidal wave propagating with a <u>phase speed</u> v_p

$$y = A\cos\left(\frac{2\pi x}{\lambda} - 2\pi ft\right)$$

$$v_p = \lambda f$$

$$\omega = 2\pi f \qquad k = 2\pi/\lambda$$

angular frequency (ω)

wave number (k)

$$y = A\cos(kx - \omega t)$$ ← compact form

$$v_p = \frac{\omega}{k}$$

superposition of two waves

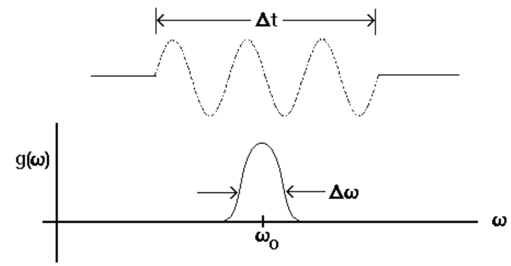
$$y = A_1 \cos(k_1 x - \omega_1 t) + A_2 \cos(k_2 x - \omega_2 t)$$

superposition of many waves

$$V(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} g(\omega) e^{i\omega t} d\omega$$

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} V(t) e^{-i\omega t} dt$$

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$



$$v_p = \frac{\omega}{k}$$ phase velocity

$$v_g = \left.\frac{d\omega}{dk}\right|_{k_0}$$ group velocity

$$\omega = k v_p \quad \rightarrow \quad v_g = \left.\frac{d\omega}{dk}\right|_{k_0} = \left.v_p\right|_{k_0} + k \left.\frac{dv_p}{dk}\right|_{k_0}$$

connection between group velocity and phase velocity

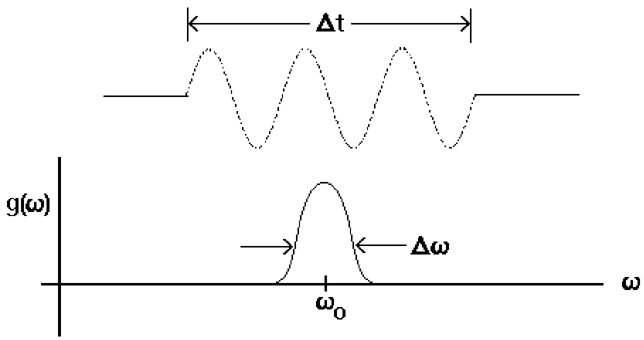
The Heisenberg uncertainty principle

Heisenberg, 1927:

It is impossible to determine simultaneously with unlimited precision the position and momentum of a particle.

If a measurement of position is made with precision Δx and a simultaneous measurement of momentum in the x direction is made with precision Δp_x, then

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$



for a <u>wave train</u> of length $\Delta\chi = c\Delta\tau$

$$\Delta\omega\Delta\tau = 2\pi$$

$$\frac{\Delta\lambda\Delta\chi}{\lambda_0^2} = 1$$

for a particular wave packet

$$p = \frac{h}{\lambda} \quad \rightarrow \quad |\Delta p| = h\left|\frac{\Delta\lambda}{\lambda^2}\right| \quad \rightarrow \quad \Delta p\Delta\chi = h$$