EXPERIMENT #4
Charge to Mass ratio of the Electron

GOALS
Physics
Measure the charge-to-mass ratio e/m for electrons.

Errors
You should calculate your best estimate for $e/m = \bar{x}$ and your "standard error" $\sigma_{\bar{x}}$ as

$$\sigma_{\bar{x}} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{N(N-1)}}$$

from your N measurements x_i of e/m.

Question: Which causes a larger systematic error in your e/m measurement:
(a) a 5% error in V, the acceleration voltage; or
(b) a 2% error in z_0, the separation between the coils of the Helmholtz magnet?

References
Serway, Moses, Moyer §4.2

Joseph John Thompson, 1897: The cathode rays are negatively charged material particles electrons with $e/m \sim 10^{11}$ Coulomb/kg

Prior experiments on the electrolysis of hydrogen ions had given $q/m \sim 10^8$ for hydrogen. Thomson had discovered a particle with a mass ~ 1000 times smaller than the smallest atoms.

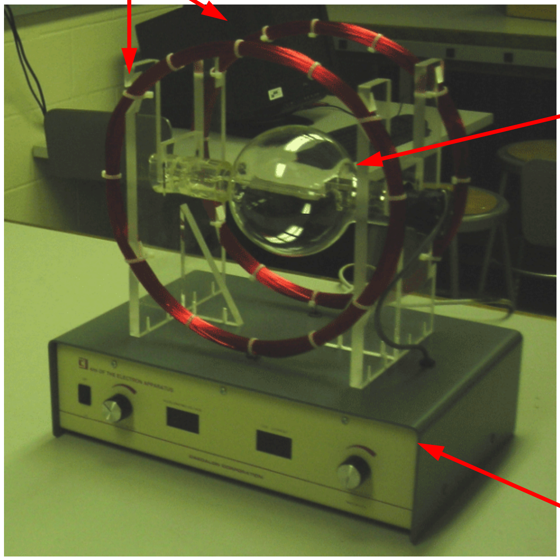
The atoms is not an indivisible entity

e/m was independent of the discharge gas and cathode metal. The particles emitted when electrical discharges were passed through different gases were found to be the same as those observed in photoelectric effect.

Based on these observations, Thomson concluded that these particles must be a universal constituent of all matter

The experiment

Helmholtz coils

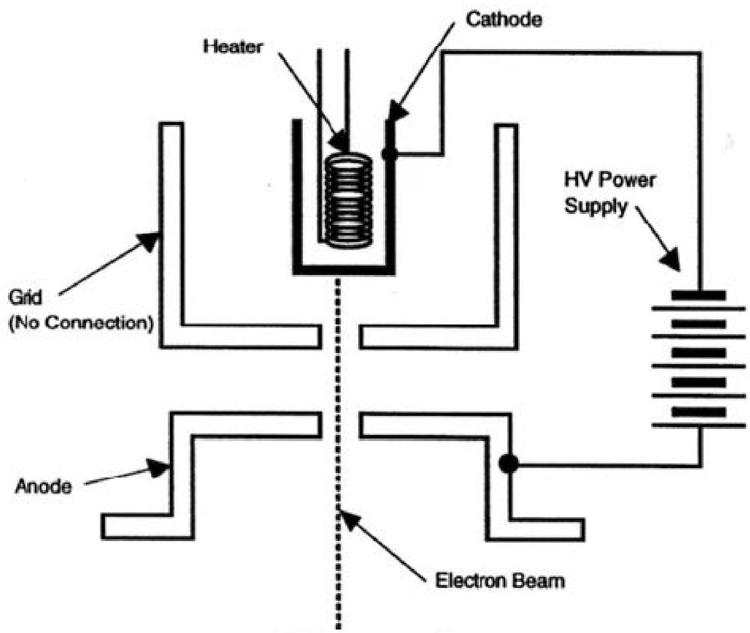


e/*m* tube contains

- an electron gun
  to produce a beam of electrons,
- a low pressure helium vapor atmosphere
  to render the beam visible,
- a glass scale
  to permit measurements of the trajectory of the electrons

a power supply which controls
accelerating potential and the coil current

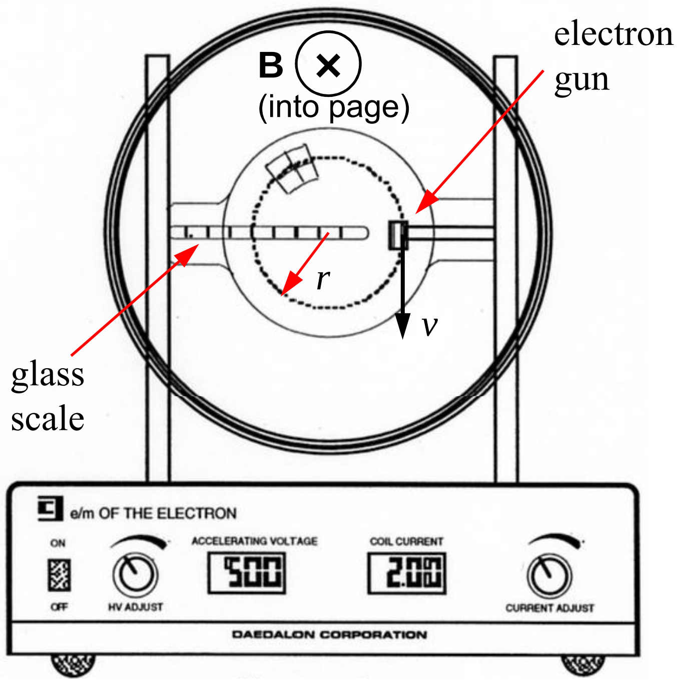
The electron gun



The cathode filament is a tungsten wire with a low "work function" energy $W \sim 2.6$ eV.
The filament is heated up to ~2000 Kelvin, giving $kT \sim 0.2$ eV, by the current flowing through it.
A small fraction $f \sim \exp\left(-W/k_B T\right) \sim 2\times10^{-6}$ of the "surface" electrons escape the metal; this is called "thermionic emission."
These electrons are accelerated toward the anode by the voltage V_a applied between the filament and the anode.
Some electrons, pass out through a narrow slot in the anode forming a flat ribbon beam of electrons which we use for the experiment.

kinetic energy of the electrons $\dfrac{mv^2}{2} = eV_a$

The tube has a small amount of helium vapor added to its otherwise good vacuum.
A few of the beam electrons strike helium atoms and excite them (Expt. 5); then the excited helium atoms emit visible radiation and we can thus, "see" where the electron beam is.



An electron moving in magnetic field experiences a force perpendicular to its velocity – the Lorentz force. It results to centripetal acceleration and curved motion.

$$evB = m\frac{v^2}{r}$$

$$\frac{e}{m}Br = v$$

$$\frac{m}{2}v^2 = eV_a$$

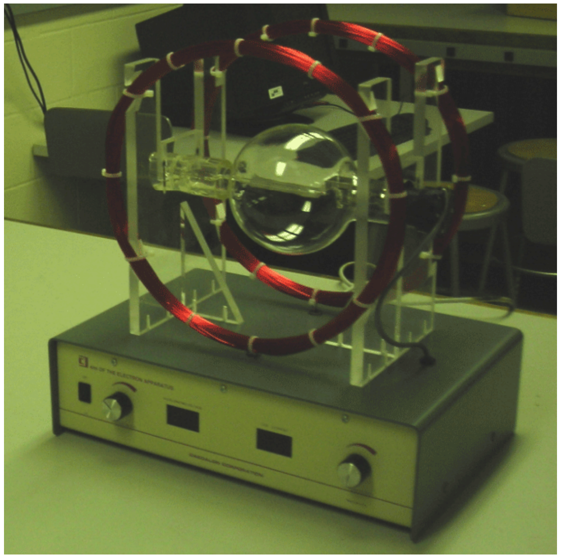
The electron velocity is determined by the voltage V_a applied to the gun

$$\frac{m}{2}\left(\frac{e}{m}Br\right)^2 = eV_a$$

$$\boxed{\frac{e}{m} = \frac{2V_a}{B^2r^2}}$$

Helmholtz coils

are coils, which are positioned on a common axis and separated by a distance z_0 equal to their radius a. They are connected in series with the same polarity and have the same current I.



Helmholtz coils give a homogeneous magnetic field near the center

$$B = \frac{\mu_0 N I a^2}{(a^2 + (z_0/2)^2)^{3/2}} = \frac{8\mu_o N I}{\sqrt{125}a}$$

for $z_0 = a$

Here,

N=130 is the number of turns per coil,

I is the current,

a~0.15m is the coil radius, measure a

and $\mu_0 = 4\pi \times 10^{-7}$ Weber/Ampere-meter (permeability of empty space).

B is in Webers/m^2, called Tesla; 1 Tesla = 10^4 Gauss.

Basic lab measurements

● Set the gun anode power supply to V_a=200 Volts and measure the beam diameter for 8 coil current settings.

● Repeat the measurements at V_a=300 Volts and V_a=400 Volts.

● Make a separate computation of e/m for each combination of V_a and B. Compute the average, and estimate the statistical error. Estimate your systematic errors from a, V_a and any other effects you can think of.

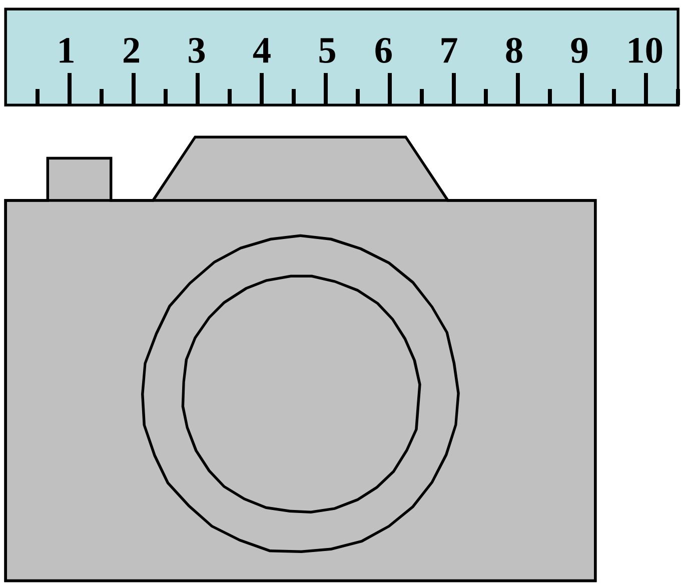
The accepted value of e/m is 1.76×10^{11} Coulomb/kg.

Optional: Verify if the Earth's magnetic field of 6×10^{-5} T (0.6 Gauss) is negligible in this experiment by rotating the apparatus.

Random and Systematic Errors

$$\delta x = \sqrt{\left(\delta x_{ran}\right)^2 + \left(\delta x_{sys}\right)^2}$$

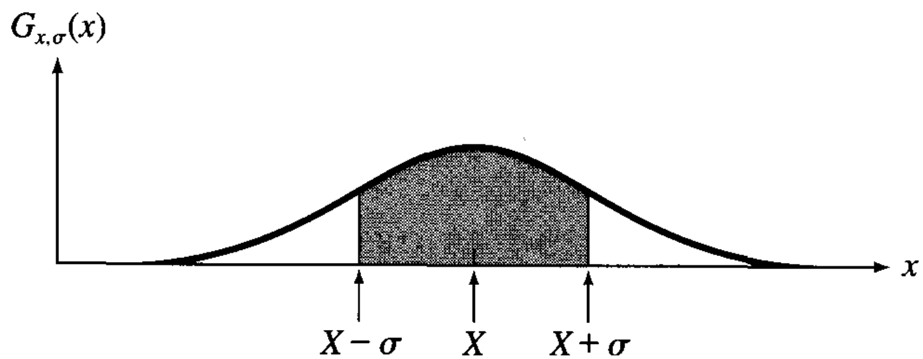
random component systematic component



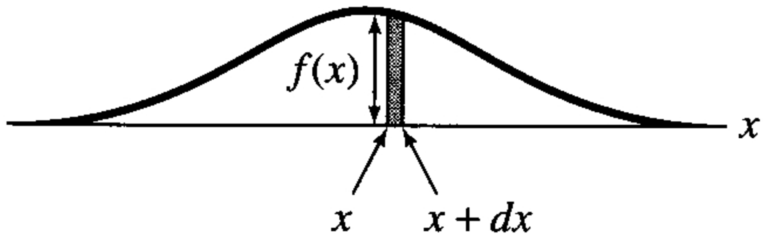
The Gauss, or Normal Distribution

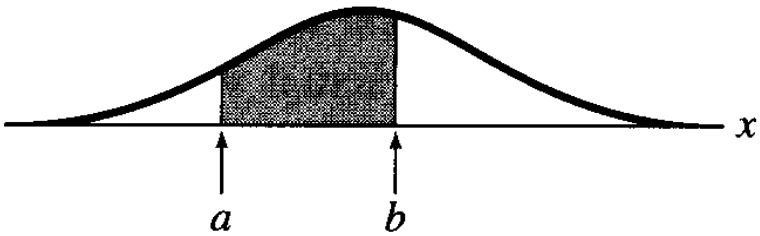
$$\boxed{G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}}$$

standard deviation σ_x = width parameter of the Gauss function σ
the mean value of x = true value X



Distributions





$f(x)\,dx$ = fraction of measurements that fall between x and $x+dx$
= probability that any measurement will give an answer between x and $x+dx$

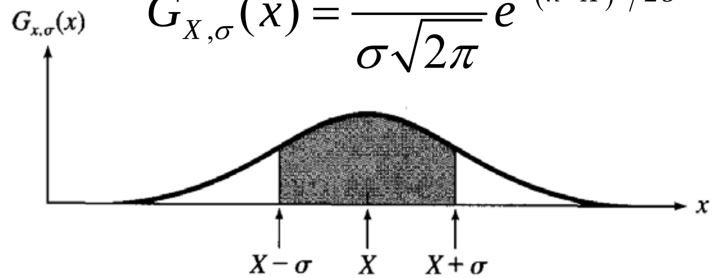
$\int_a^b f(x)dx$ = fraction of measurements that fall between $x=a$ and $x=b$
= probability that any measurement will give an answer between $x=a$ and $x=b$

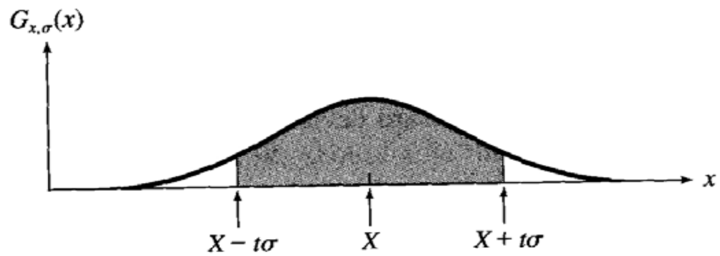
$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

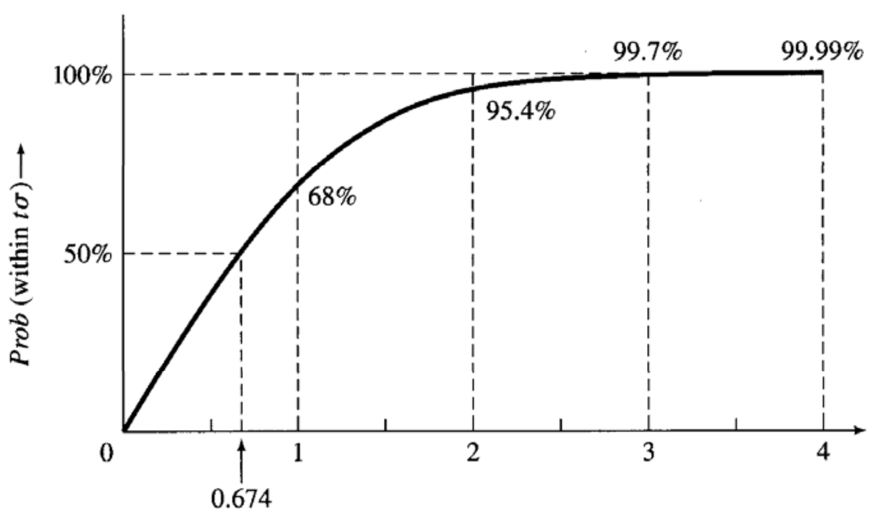
normalization condition

The standard Deviation as 68% Confidence Limit

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}$$







$$\text{Prob}(\text{within } \sigma) = \int_{X-\sigma}^{X+\sigma} G_{X,\sigma}(x)dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{X-\sigma}^{X+\sigma} e^{-(x-X)^2/2\sigma^2} dx$$

$$(x - X)/\sigma = z$$

$$\text{Prob}(\text{within } \sigma) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-z^2/2} dz$$

$$\text{Prob}(\text{within } t\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-t}^{t} e^{-z^2/2} dz$$

← *erf(t)* – error function

the probability that a measurement will fall within one standard deviation of the true answer is 68 %

$x = x_{best} \pm \delta x \qquad \delta x = \sigma$

Example:

A student measures a quantity x many times and calculates the mean as $\overline{x} = 10$ and the standard deviation as $\sigma_x = 1$. What fraction of his readings would you expect to find between 11 and 12?

$$G_{X,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}$$

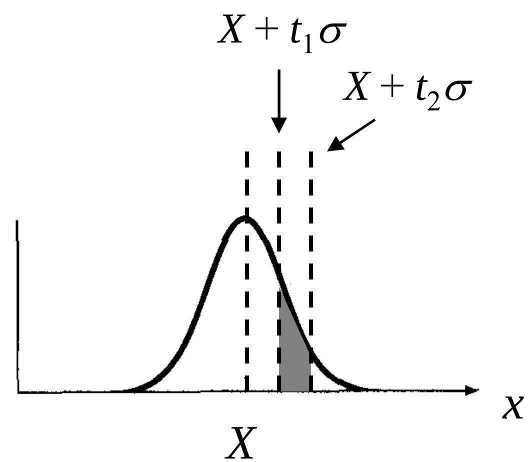
$$t = \frac{x - X}{\sigma}$$

Table B

$$\text{Prob}(X \le x \le X + t\sigma) = \int_X^{X+t\sigma} G_{X,\sigma}(x)dx = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-z^2/2} dz$$

The probability of a measurement to be between $X + t_1\sigma$ and $X + t_2\sigma$

$$\text{Prob}(X + t_1\sigma \le x \le X + t_2\sigma) = \frac{1}{\sqrt{2\pi}} \int_0^{t_2} e^{-z^2/2} dz - \frac{1}{\sqrt{2\pi}} \int_0^{t_1} e^{-z^2/2} dz$$



$$t_1 = \frac{x_1 - X}{\sigma} = \frac{11 - 10}{1} = 1$$

$$t_2 = \frac{x_2 - X}{\sigma} = \frac{12 - 10}{1} = 2$$

$$\text{Prob}(X + \sigma \le x \le X + 2\sigma) = \frac{1}{\sqrt{2\pi}} \int_0^{2} e^{-z^2/2} dz - \frac{1}{\sqrt{2\pi}} \int_0^{1} e^{-z^2/2} dz = 48\% - 34\% = \underline{14\%}$$

Table B

Acceptability of a Measured Answer

(value of x) = $x_{best} \pm \sigma$

x_{exp} - expected value of x, e.g. based on some theory

x_{best} differs from x_{exp} by t standard deviations $\longrightarrow$ $t = \dfrac{|x_{best} - x_{\exp}|}{\sigma}$

Table A.

$$\text{Prob(outside } t\sigma) = 1 - \text{Prob(within } t\sigma)$$

$\downarrow$

< 5 % - significant discrepancy, $t > 1.96$

< 1 % - highly significant discrepancy, $t > 2.58$

$\uparrow$

boundary for unreasonable improbability

the result is beyond the boundary for unreasonable improbability

$\longrightarrow$ the result is unacceptable

Example Problem

Two students measure the radius of a planet and get final answers R_A=25,000±3,000 km and R_B=19,000±2,500 km.
(a) Assuming all errors are independent and random, what is the discrepancy and what is its uncertainty?
(b) Assuming all quantities are normally distributed as expected, what would be the probability that the two measurements would disagree by more than this?
Do you consider the discrepancy in the measurements significant (at the 5% level)?

(a) $$R_A - R_B = 25,000 - 19,000 = \underline{6,000km}$$

$$\sigma_{R_A - R_B} = \sqrt{{\sigma_{R_A}}^2 + {\sigma_{R_B}}^2} = \sqrt{3,000^2 + 2,500^2} = 3,905km \to \underline{4,000km}$$

$$R_A - R_B = 6,000 \pm 4,000km$$

(b) $$t = \frac{6,000}{4,000} = 1.5$$

Table A: Probability to be within 1.5σ is 86.64 % ≈ 87 %. Therefore, the probability that the two measurements would disagree by more than this is 100 – 87 = 13 %.
The discrepancy in the measurements is not significant (at the 5% level).