Significant Figures

~~$$g = 9.82 \pm 0.02385 \text{ m/s}^2$$~~

$$g = 9.82 \pm 0.02 \text{ m/s}^2$$

Experimental uncertainties should be rounded to one significant figure (to two significant if the leading digit in the uncertainty is a 1)

~~$$g = 9.82 \pm 0.01437 \text{ m/s}^2$$~~

$$g = 9.82 \pm 0.014 \text{ m/s}^2$$

The last significant figure in any answer should be of the same order of magnitude (in the same decimal position) as the uncertainty

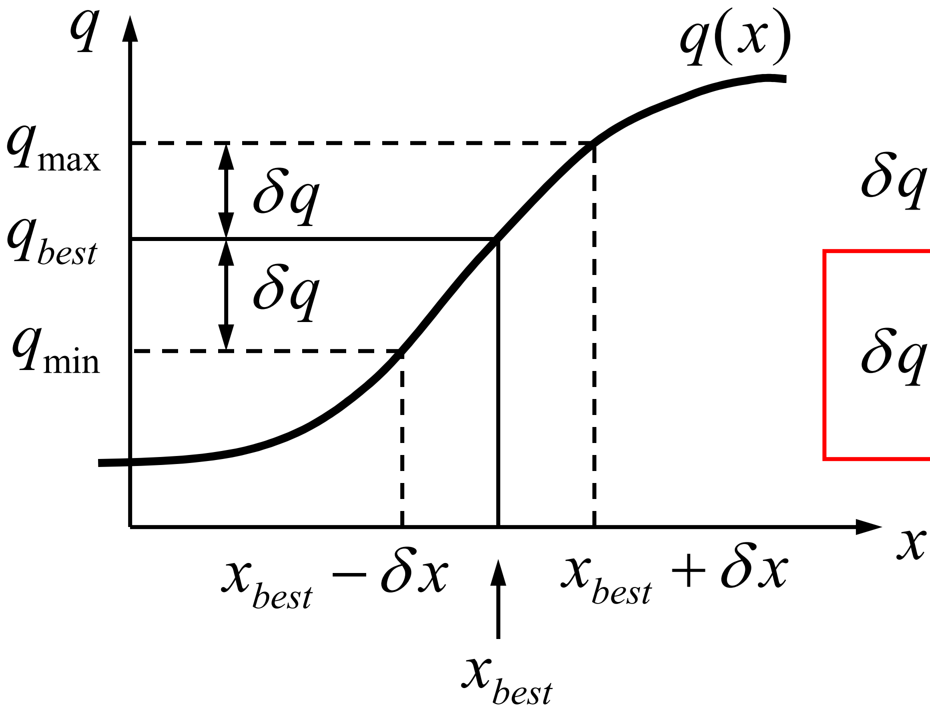
~~$$g = 9.82378 \pm 0.02 \text{ m/s}^2$$~~

$$g = 9.82 \pm 0.02 \text{ m/s}^2$$

$g = 9.82378 \pm 0.02385 \text{ m/s}^2 \rightarrow g = 9.82378 \pm 0.02 \text{ m/s}^2 \rightarrow$ **$g = 9.82 \pm 0.02 \text{ m/s}^2$**

$v = 6051.78 \pm 32 \text{ m/s} \rightarrow v = 6051.78 \pm 30 \text{ m/s} \rightarrow$ **$v = 6050 \pm 30 \text{ m/s}$**

Arbitrary Functions of One Variable



$$\delta q = q(x_{best} + \delta x) - q(x_{best}) = \frac{dq}{dx}\delta x$$

$$\delta q = \left|\frac{dq}{dx}\right|\delta x$$

Example

Find side a of a square with area $S = 25 \pm 2$ cm^2.

$$a = \sqrt{S} = \sqrt{25} = 5 \text{ cm}$$

$$\delta a = \left|\frac{da}{dS}\right|\delta S = \frac{1}{2\sqrt{S}}\delta S = \frac{1}{2\cdot\sqrt{25}}2 = 0.2 \text{ cm}$$

$$a = 5.0 \pm 0.2 \text{ cm}$$

General Formula for Error Propagation

$$q = q(x, y, z)$$

$$q_{best} = q(x_{best}, y_{best}, z_{best})$$

partial derivatives of q with respect to x, y, and z

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$$

for independent random errors δx, δy, and δz

$$\sqrt{a^2 + b^2} = a \oplus b$$

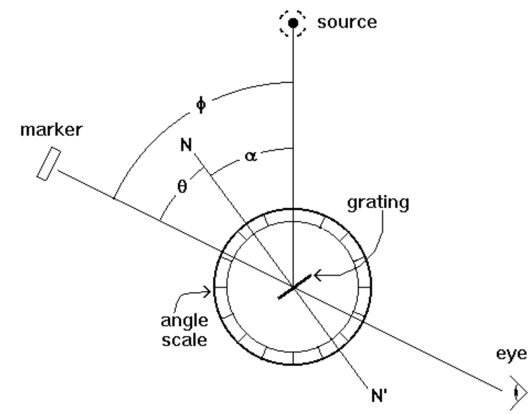
shorthand notation for quadratic sum

quadratic sum = addition in quadrature

for independent random errors $\delta x \leftrightarrow \sigma_x$

σ = (sigma) = Standard Deviation

Error analysis: Calculating $\delta\lambda$



$$d\left(\sin\alpha+\sin\theta\right)=n\lambda$$ ← Grating Equation

$$\lambda=\frac{d}{n}\left(\sin\alpha+\sin\theta\right)$$

$$\delta\lambda=\sqrt{\left(\frac{\partial\lambda}{\partial\alpha}\delta\alpha\right)^2+\left(\frac{\partial\lambda}{\partial\theta}\delta\theta\right)^2}=\frac{d}{n}\sqrt{\left(\cos\alpha\cdot\delta\alpha\right)^2+\left(\cos\theta\cdot\delta\theta\right)^2}$$

always use radians when calculating the errors on trig functions

estimate: $\delta\lambda\approx d\sqrt{\left(\delta\alpha\right)^2+\left(\delta\theta\right)^2}$

for a 0.5 degree error in α and θ ($\delta\alpha=\delta\theta=0.5^0=\frac{2\pi\text{ rad}}{360^0}\cdot 0.5^0=0.009\text{ rad}$)

$$\delta\lambda=1016\sqrt{\left(0.009\right)^2+\left(0.009\right)^2}\text{ nm}=13\text{ nm}$$

make error analysis using α, θ, $\delta\alpha$, $\delta\theta$ of your experiment

The mean

$x_1, x_2, \ldots, x_N$ N measurements of the quantity x

$x_{best} = \overline{x}$ the best estimate for $x \rightarrow$ the average or mean

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{\sum x_i}{N}$$

The standard deviation

$d_i = x_i - \overline{x}$ deviation of x_i from x

$$\sigma_x = \sqrt{\frac{1}{N-1}\sum\left(x_i - \overline{x}\right)^2}$$

standard deviation of x

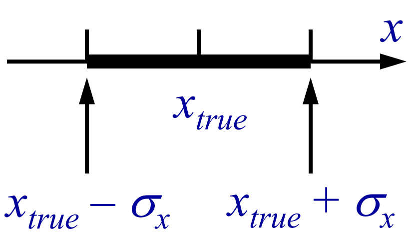
average uncertainty of the measurements $x_1, \ldots, x_N$

RMS (route mean square) deviation

uncertainty in any one measurement of $x \rightarrow \delta x = \sigma_x$

↓

68% of measurements will fall in the range $x_{true} \pm \sigma_x$



The standard deviation of the mean

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

uncertainty in $\bar{x}$
is the standard deviation of the mean

based on the N measured values $x_1, \ldots, x_N$ we can state our final answer for the value of x:

$$(\text{value of } x) = x_{best} \pm \delta x_{best}$$

$$x_{best} = \bar{x} \qquad \delta x = \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$

$$(\text{value of } x) = \bar{x} \pm \sigma_{\bar{x}}$$

Exp. 1: 4 measurements for each spectral line $\quad \lambda_1, \lambda_2, \lambda_3, \lambda_4 \qquad N = 4$

$$\bar{\lambda} = \frac{\sum \lambda_i}{N} \qquad \delta\bar{\lambda} = \frac{\delta\lambda}{\sqrt{N}} = \frac{1}{\sqrt{N}}\sqrt{\frac{1}{N-1}\sum\left(\lambda_i - \bar{\lambda}\right)^2}$$

EXPERIMENT #2
Coherence of Light and the Interferometer

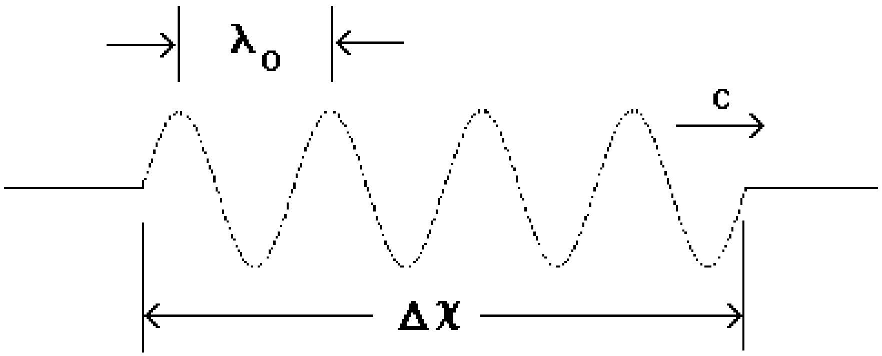
GOALS
Physics
Measure the coherence length of light using an interferometer.
Establish that “filtering” increases the coherence length.

References
Serway, Moses, Moyer §1.3

coherence

The degree of coherence of a source of light
is the degree to which that light consists of long, unbroken trains of sinusoidal waves.

Suppose we have a train of sine waves with wavelength λ_0
having some total length in space $\Delta\chi$, and propagating at velocity c.



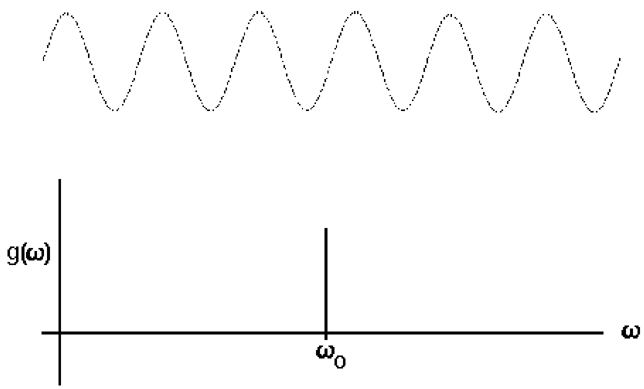
$$E(t) = E_o \sin \omega_o t \quad \text{for } 0 < t < \Delta\tau$$
$$= 0 \quad \text{for } t < 0 \text{ or } t > \Delta\tau$$

$$\Delta\tau = \Delta\chi / c$$

What frequencies are present in this wave packet?
The wave packet turns on and turns off, i.e., is not continuously oscillating.
Therefore, it cannot be presented by a single frequency $\omega_0 = 2\pi c/\lambda_0$.

Fourier analysis: Any arbitrary function can be represented as a sum of sine or cosine functions of different frequencies and different strengths

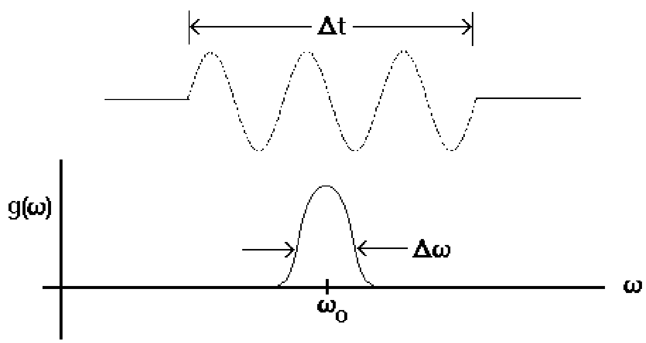
$g(\omega)$ represents the strength of various frequencies in the original function



For an infinitely long train of waves, i.e.

$$E(t) = E_0 \sin \omega_0 t, \quad \text{for} \ -\infty < t < \infty$$

$g(\omega)$ is a "delta function" $g(\omega) = \delta(\omega - \omega_0)$, i.e., a single infinitely narrow peak at $\omega = \omega_0$, indicating that there is only one frequency

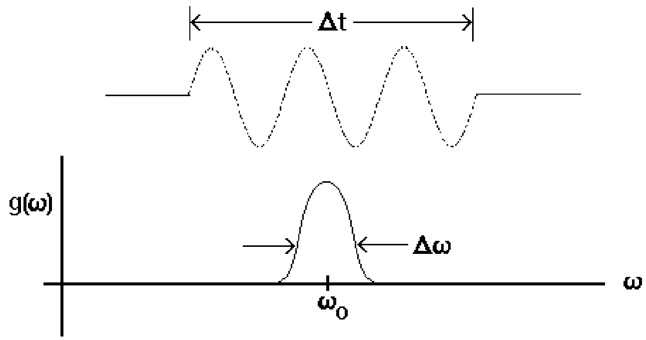


For a wave train of length $\Delta\chi = c\Delta\tau$ a band of frequencies of width $\Delta\omega$ appears in $g(\omega)$, centered at ω_0

A general result: a wave train of frequency ω_0 truncated to a duration $\Delta\tau$ has its frequency spectrum spread over a range $\Delta\omega$, such that $\boxed{\Delta\omega\,\Delta\tau = 2\pi}$

$$\omega_o = \frac{2\pi c}{\lambda_0} \quad \rightarrow \quad \Delta\omega = \frac{2\pi c}{\lambda_0^2}\Delta\lambda$$

$$2\pi = \Delta\omega\Delta\tau = \frac{2\pi c}{\lambda_0^2}\Delta\lambda\frac{\Delta\chi}{c} \quad \rightarrow \quad \frac{\Delta\lambda\Delta\chi}{\lambda_0^2} = 1$$

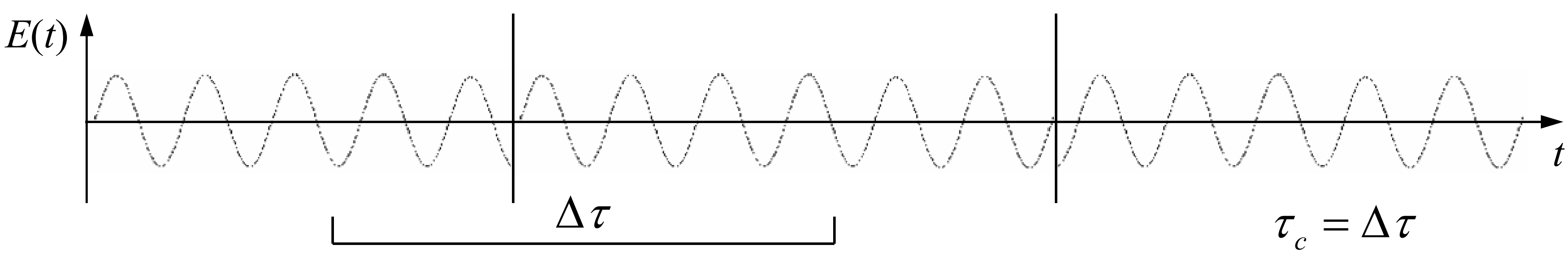


for a wave train of length $\Delta\chi = c\Delta\tau$

$$\Delta\omega\Delta\tau = 2\pi$$

$$\frac{\Delta\lambda\Delta\chi}{\lambda_0^2} = 1$$

the spread of wavelengths $\Delta\lambda$ is called the linewidth
the spread in frequencies $\Delta\omega$ is called the bandwidth
the packet length $\Delta\chi$ is called the coherence length
the time associated with the coherence length $\Delta\tau = \Delta\chi/c$ is called the coherence time



The electric field amplitude of the wave train radiated by an atom. The vertical lines represent collisions separated by periods of free flight with the mean duration $\Delta\tau$.