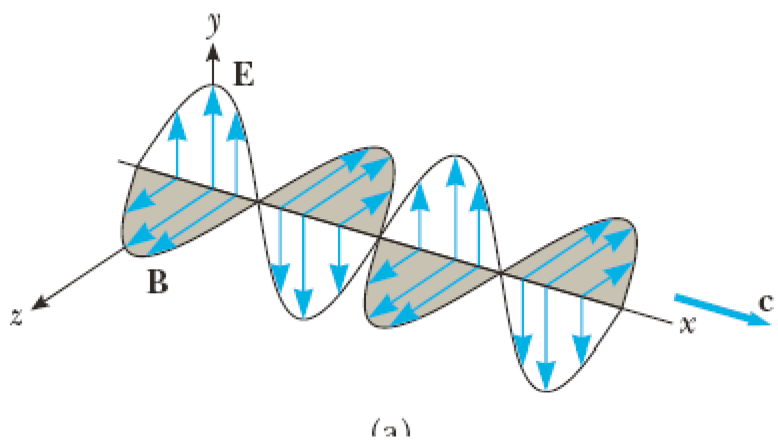
Physics 2D Lecture Slides
Week of May 4,2009
Part 2

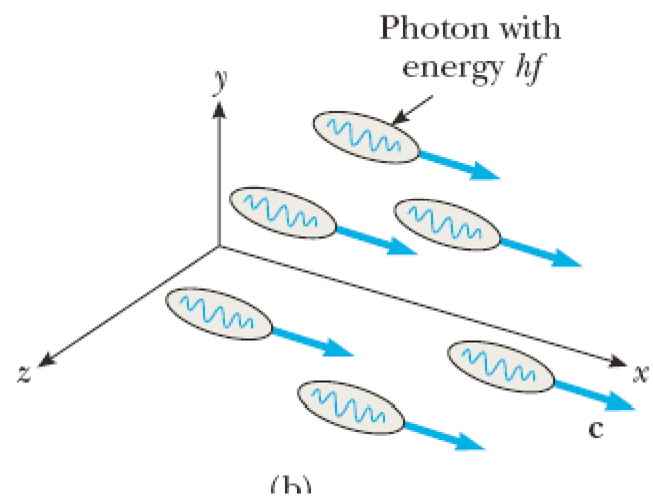
(Oleg Shpyrko)

Sunil Sinha

UCSD Physics



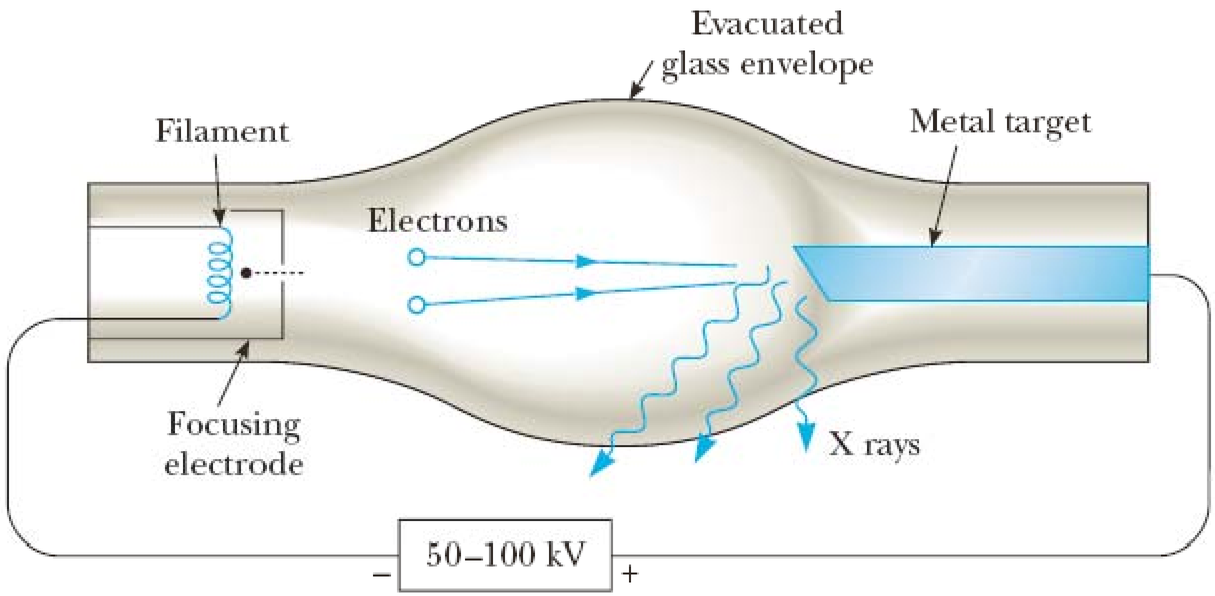
(a)

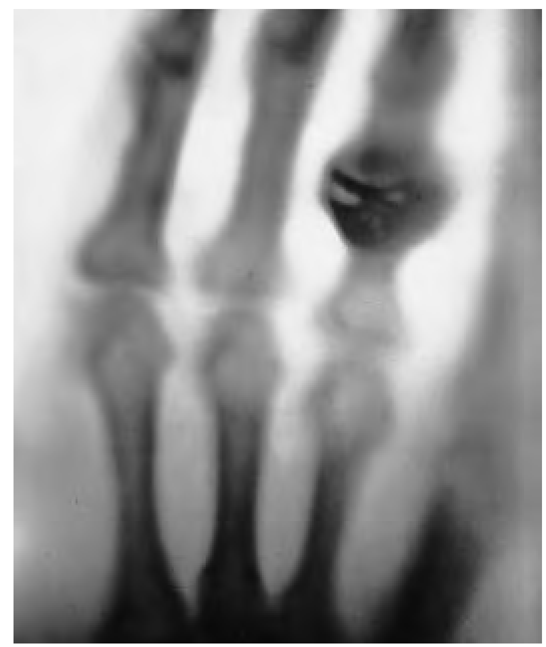


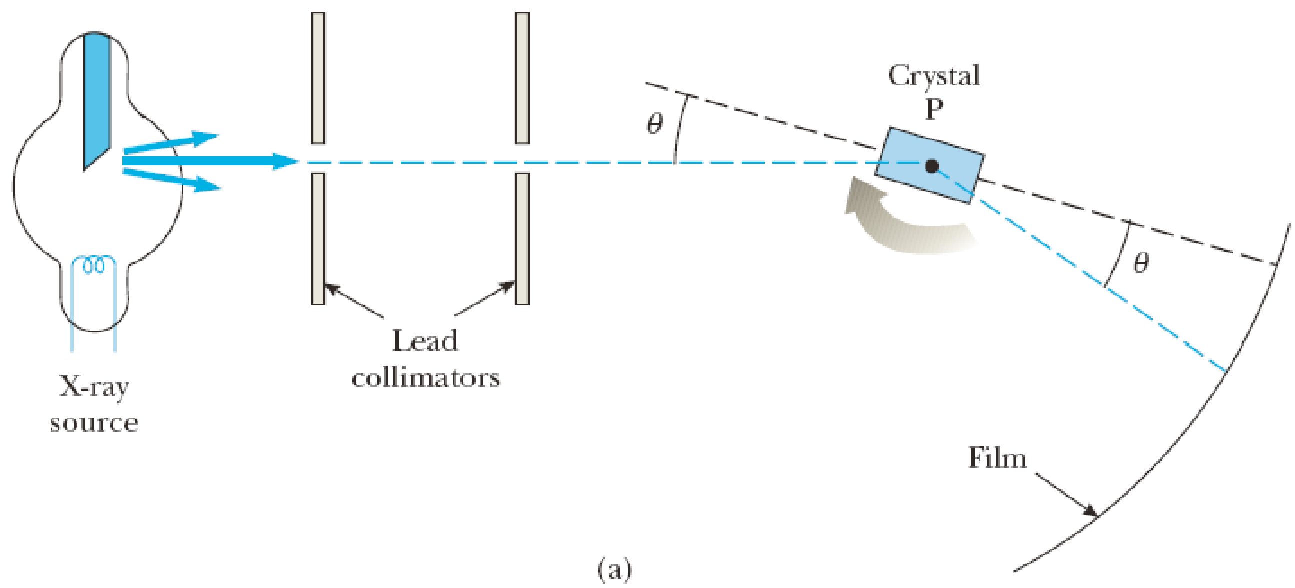
(b)

$$p = \frac{E}{c}$$

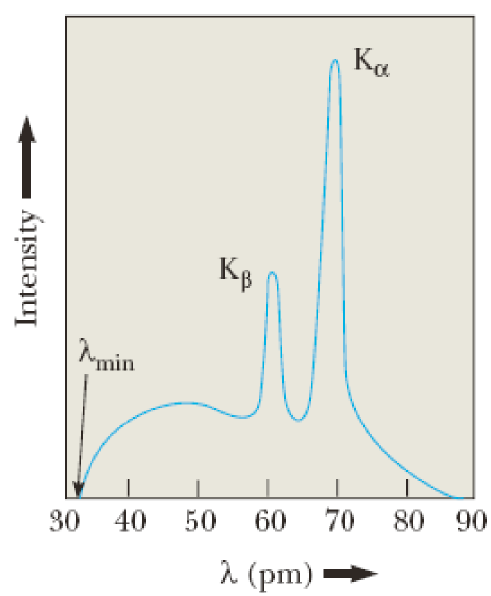
$$E^2 = (mc^2)^2 + (pc)^2$$

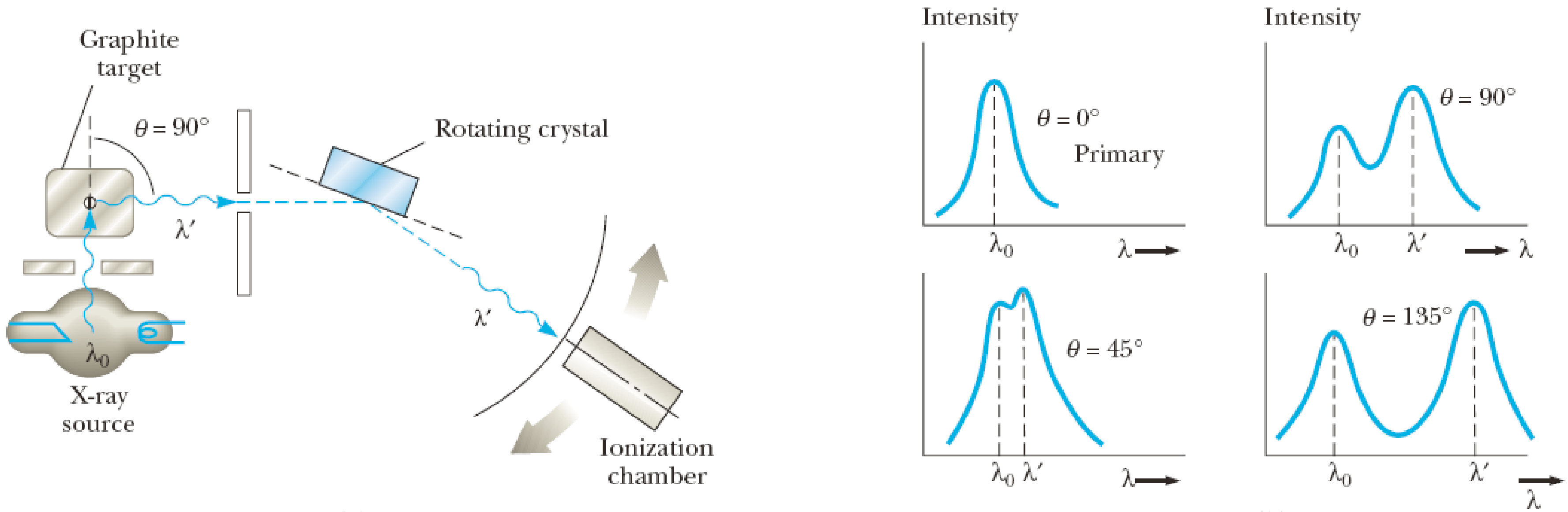

Evacuated
glass envelope
Filament
Metal target
Electrons
Focusing
electrode
X rays
50–100 kV
–
+




Crystal
P
θ
θ
X-ray
source
Lead
collimators
Film

(a)


Kα
Kβ
λmin
Intensity
30
40
50
60
70
80
90
λ (pm)


Graphite
target
θ = 90°
Rotating crystal
λ′
λ′
λ0
X-ray
source
Ionization
chamber
Intensity
θ = 0°
Primary
λ0
λ
θ = 45°
λ0 λ′
λ
Intensity
θ = 90°
λ0 λ′
λ
θ = 135°
λ0
λ′
λ

(a) (b)

de Broglie’s matter-wave theory

Louis de Broglie

Bohr's Explanation of Hydrogen like atoms

- Bohr's Semiclassical theory explained some spectroscopic data → Nobel Prize : 1922
- The "hotch-potch" of classical & quantum attributes left many (Einstein) unconvinced
  - "appeared to me to be a miracle – and appears to me to be a miracle today ...... One ought to be ashamed of the successes of the theory"
- Problems with Bohr's theory:
  - Failed to predict INTENSITY of spectral lines
  - Limited success in predicting spectra of Multi-electron atoms (He)
  - Failed to provide "time evolution " of system from some initial state
  - Overemphasized Particle nature of matter-could not explain the wave-particle duality of light
  - No general scheme applicable to non-periodic motion in subatomic systems
- Without fundamental insight …raised the question : Why was Bohr successful?

Prince Louis de Broglie & Matter Waves

- Key to Bohr atom was <u>Angular momentum quantization</u>
- Why this Quantization: $mvr = |L| = nh/2\pi$?
- Invoking symmetry in nature, Louis de Broglie conjectured:

Because photons have wave <u>and</u> particle like nature → particles may have <u>wave</u> like properties !!

Electrons have accompanying "pilot" wave (not EM) which guide particles thru spacetime

A PhD Thesis Fit For a Prince

- Matter Wave !
  - **“Pilot wave” of $\lambda = h/p = h / (\gamma mv)$**
  - **frequency $f = E/h$**

- **Consequence:**
  - **If matter has wave like properties then there would be interference (destructive & constructive)**
    - **Use analogy of standing waves on a plucked string to explain the quantization condition of Bohr orbits**

Matter Waves : How big, how small

1. Wavelength of baseball, m=140g, v=27m/s

$$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J.s}{(.14kg)(27m/s)} = 1.75\times10^{-34}\,m$$

$\Rightarrow$ $\lambda_{baseball} <<<$ size of nucleus

$\Rightarrow$ Baseball "looks" like a particle

2. Wavelength of electron K=120eV (assume NR)

$$\mathrm{K} = \frac{\mathrm{p}^2}{2\mathrm{m}} \Rightarrow p = \sqrt{2mK}$$

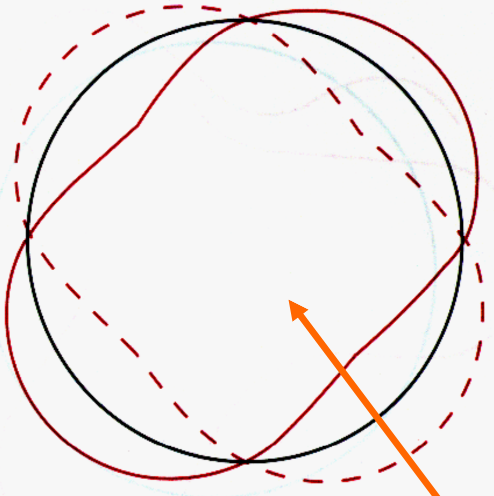
$$= \sqrt{2(9.11\times10^{-31})(120eV)(1.6\times10^{-19})}$$

$$= 5.91\times\ 10^{-24}\,Kg.m/s$$

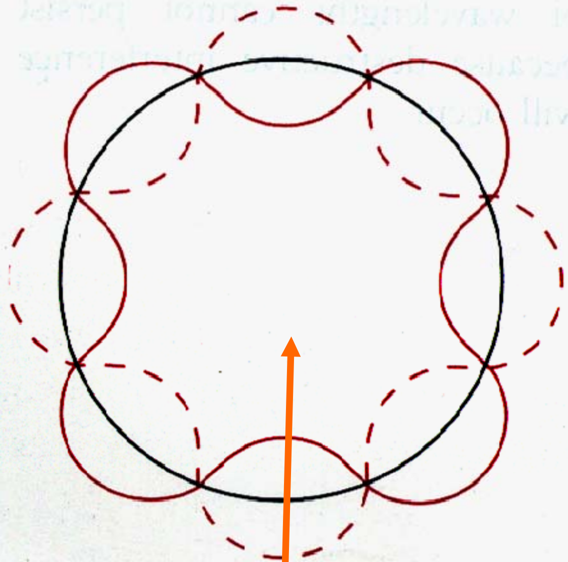
$$\lambda_e = \frac{h}{p} = \frac{6.63\times10^{-34}\,J.s}{5.91\times10^{-24}\,kg.m/s} = 1.12\times10^{-10}\,m$$

$\Rightarrow$ λ_e □ Size of atom !!

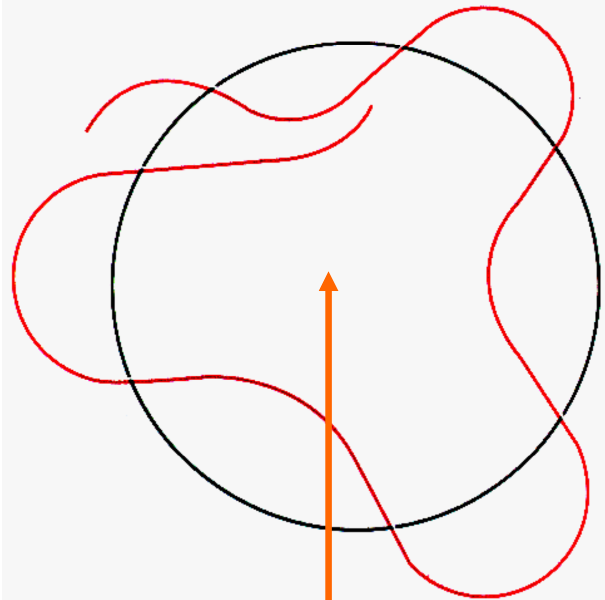
Models of Vibrations on a Loop: Model of e in atom

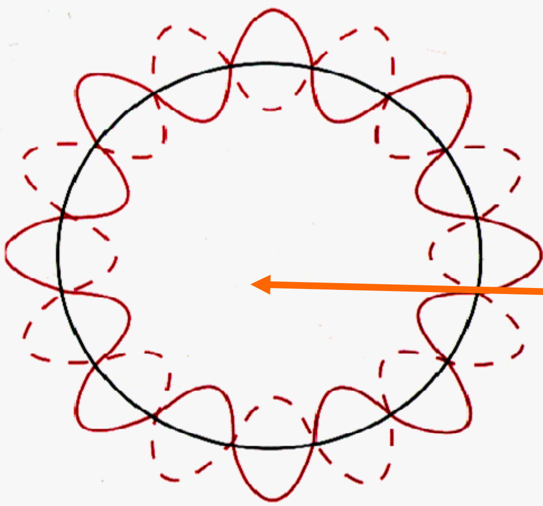


Circumference = 2 wavelengths



Circumference = 4 wavelengths





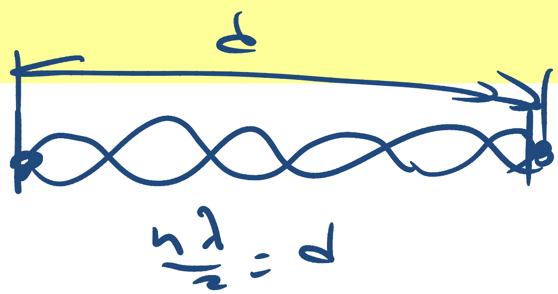
Circumference = 8 wavelengths

Modes of vibration when a integral # of λ fit into loop

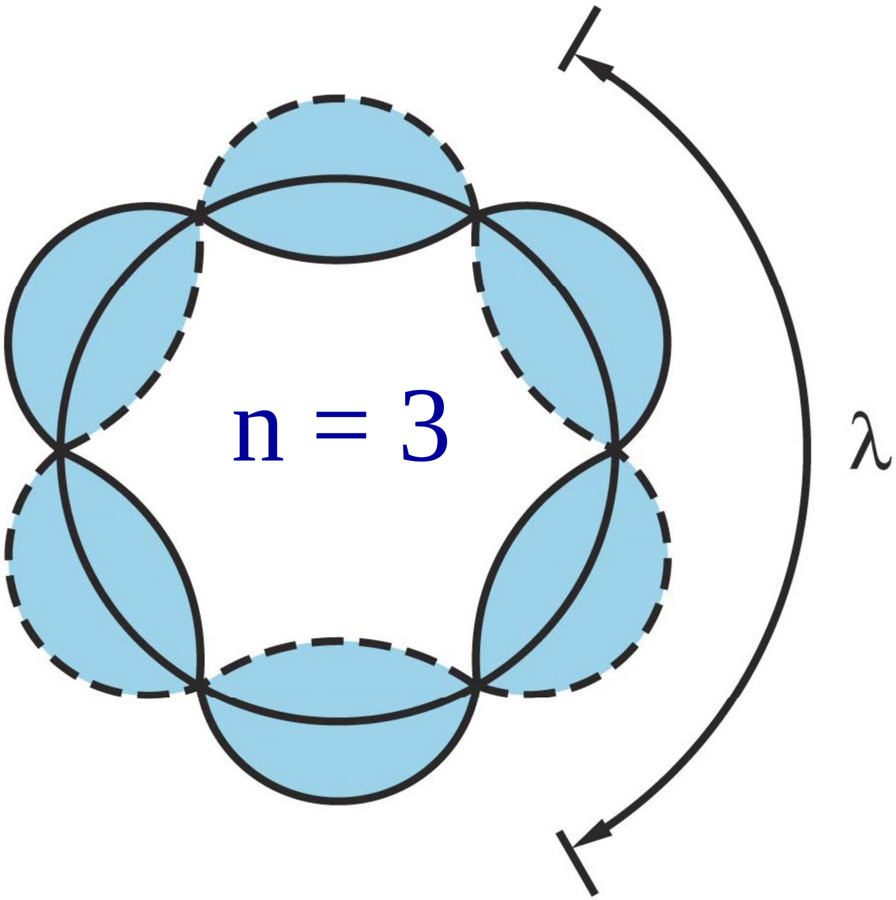
(Standing waves)

vibrations continue Indefinitely

Fractional # of waves in a loop can not persist due to destructive interference



De Broglie's Explanation of Bohr's Quantization



Standing waves in H atom:

Constructive interference when

$n\lambda = 2\pi r$

since $\lambda = \frac{h}{p} = \frac{h}{mv}$ ......(*NR*)

$$\Rightarrow \frac{nh}{mv} = 2\pi r$$

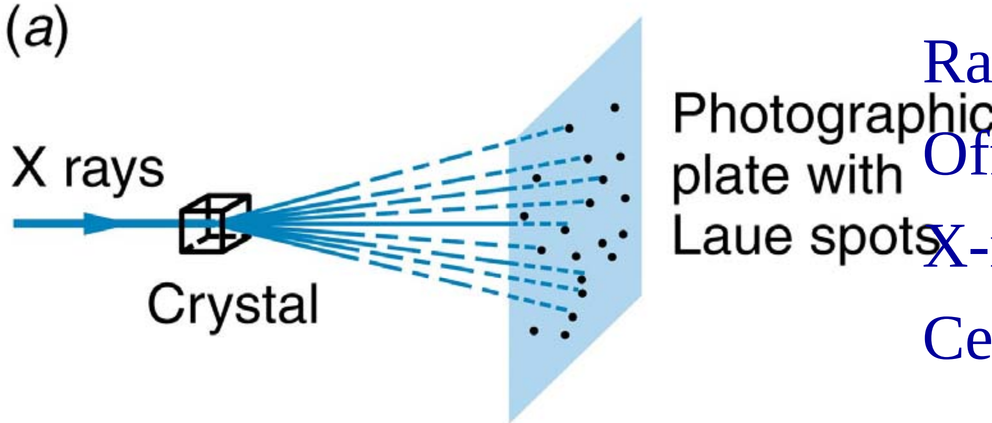
$$\Rightarrow \boxed{n\hbar = mvr}$$

Angular momentum

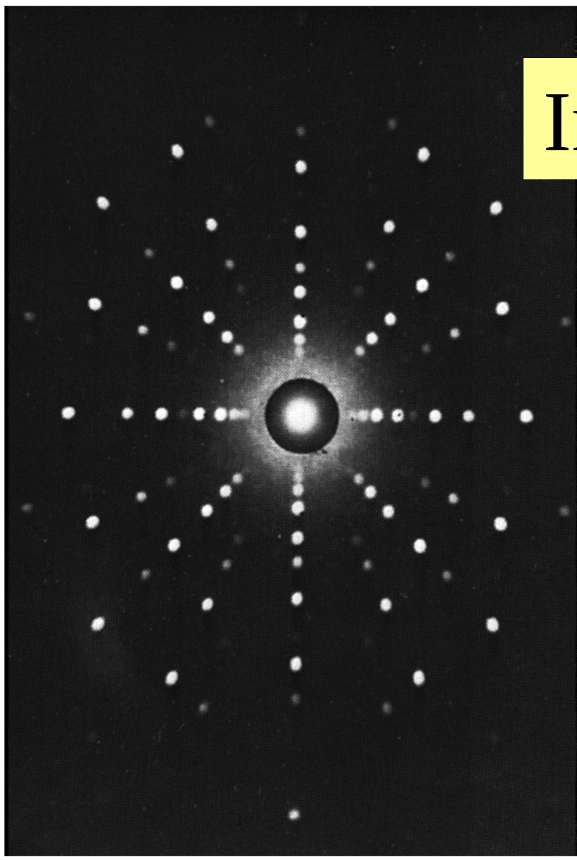
Quantization condition!

This is too intense ! Must verify such "loony tunes" with experiment

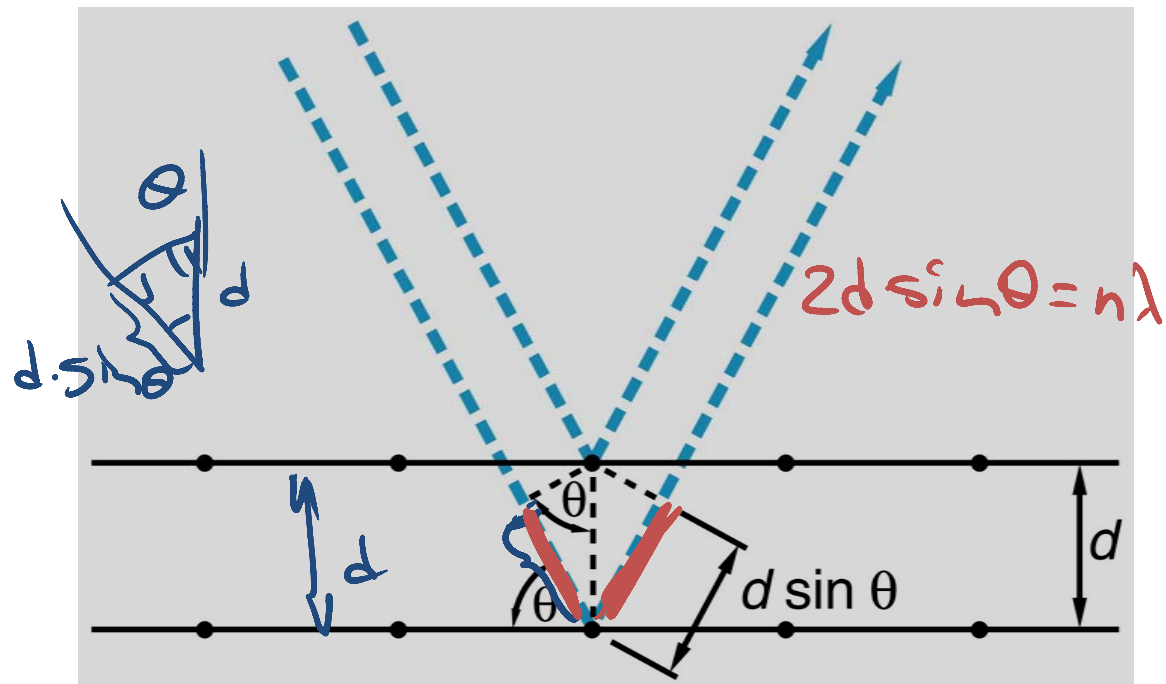
Reminder: Light as a Wave : Bragg Scattering Expt



Range of X-ray wavelengths scatter
Off a crystal sample
X-rays constructively interfere from
Certain planes producing bright spots



Interference →Path diff=2dsinϑ = nλ



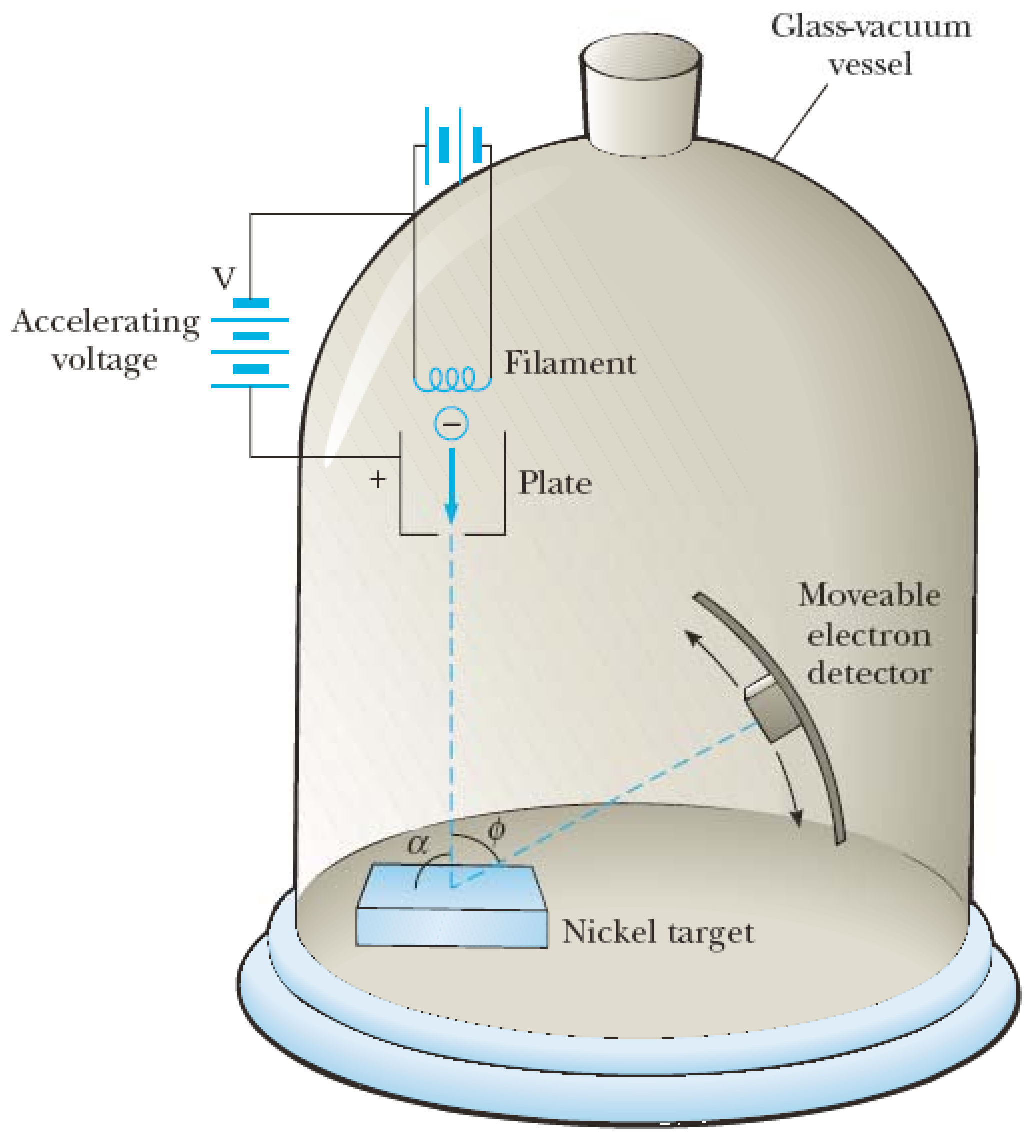
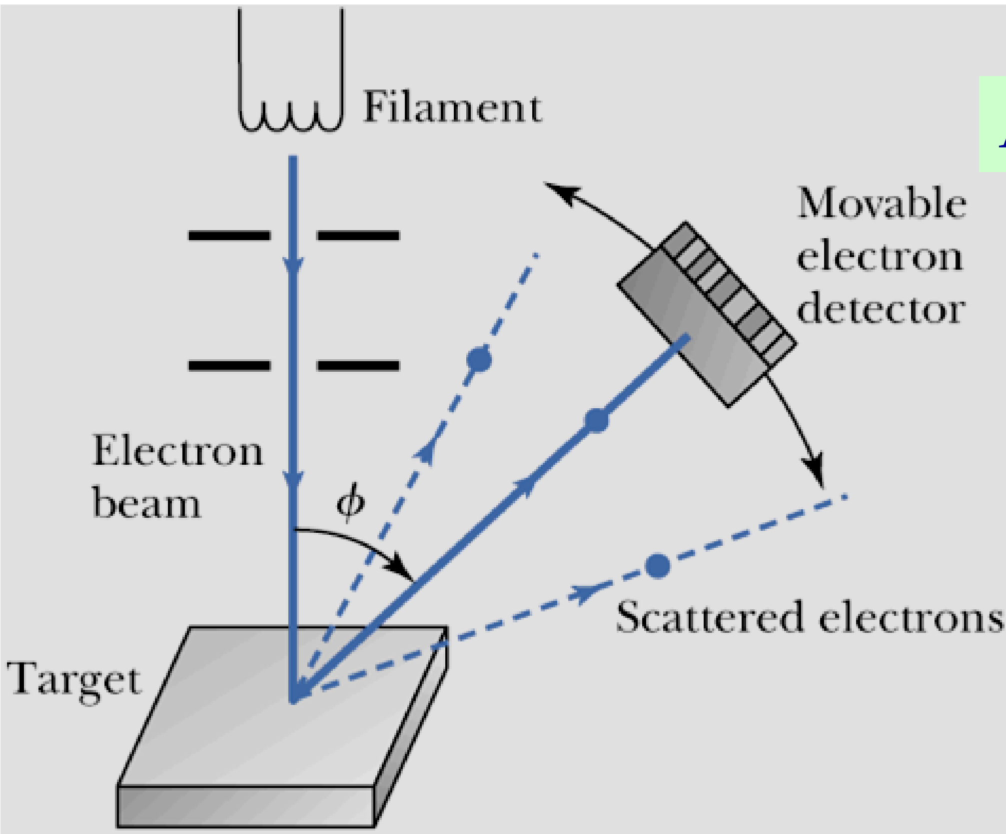


Figure 5.4 A schematic diagram of the Davisson–Germer apparatus.

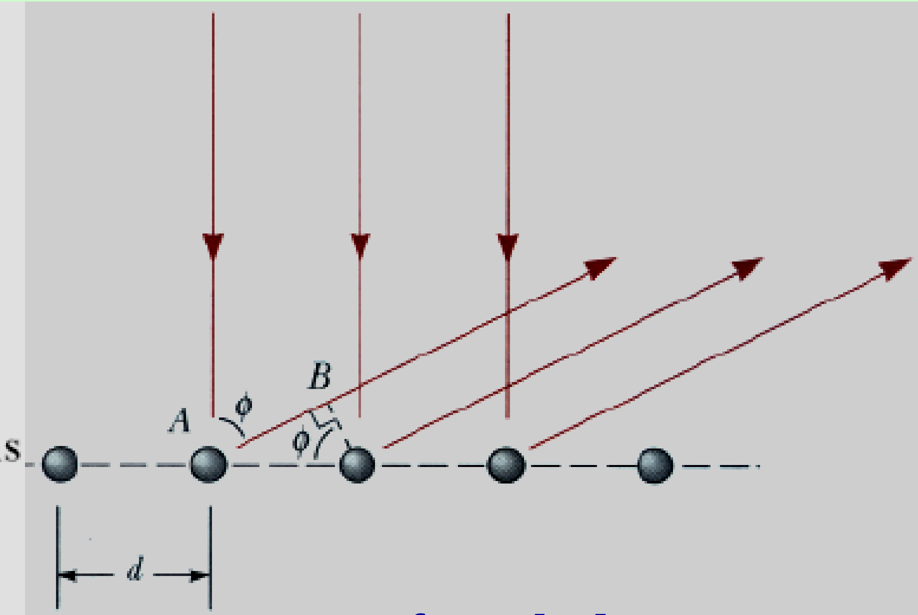
Verification of Matter Waves: Davisson & Germer Expt

If electrons have associated wave like properties → expect interference pattern when incident on a layer of atoms (reflection diffraction grating) with inter-atomic separation d such that

$$\text{path diff AB} = d\sin\varphi = n\lambda$$

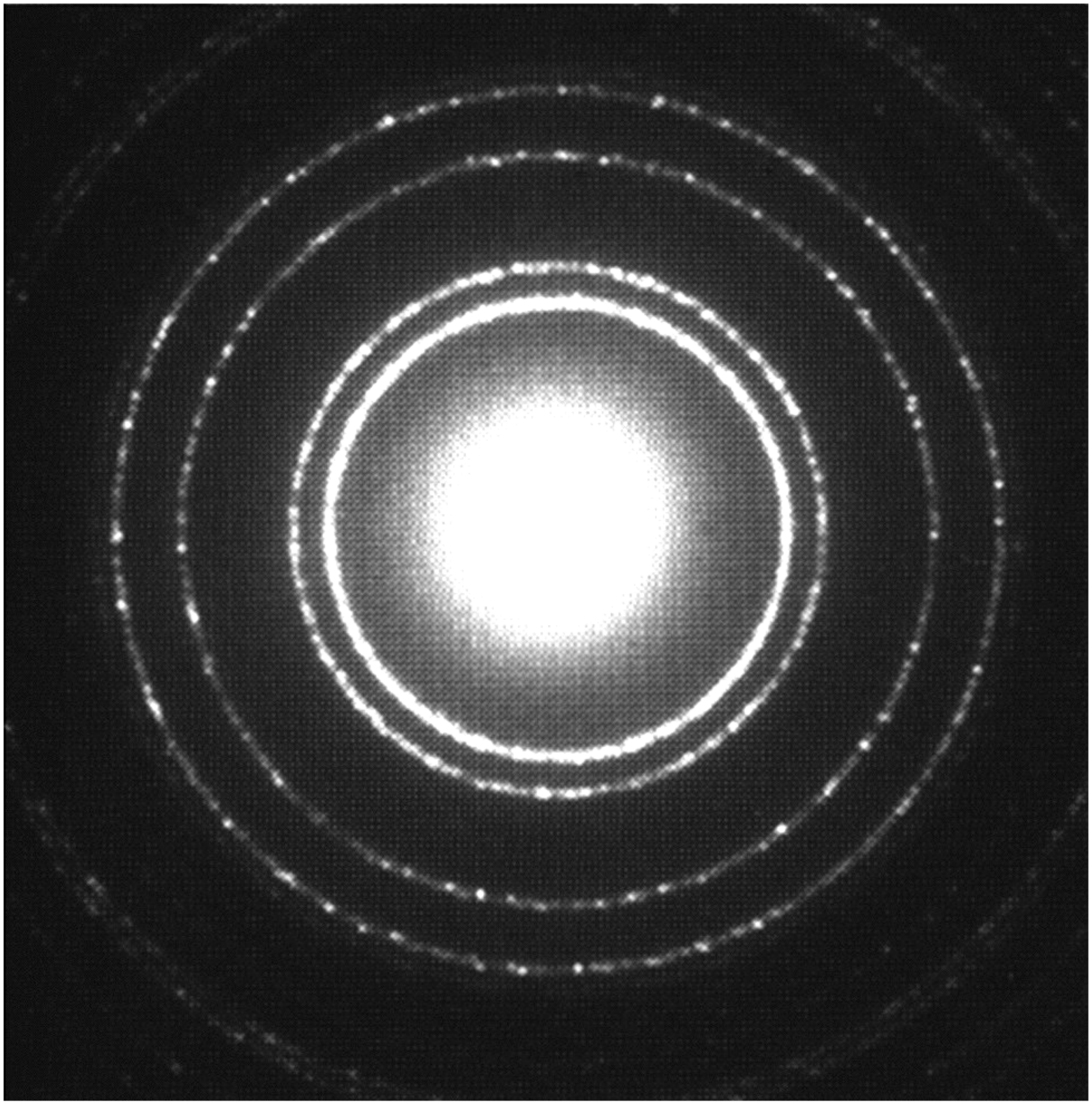


Atomic lattice as diffraction grating



Layer of Nickel atoms

Electrons Diffract in Crystal, just like X-rays

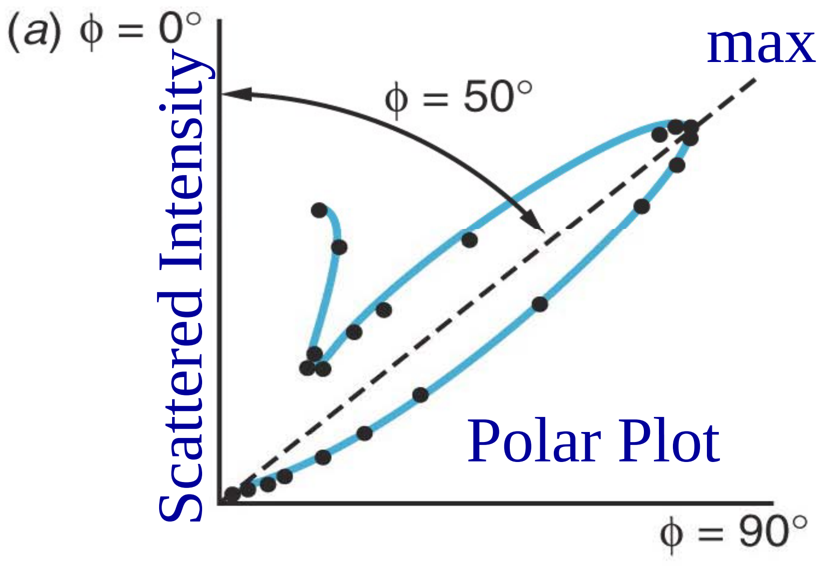


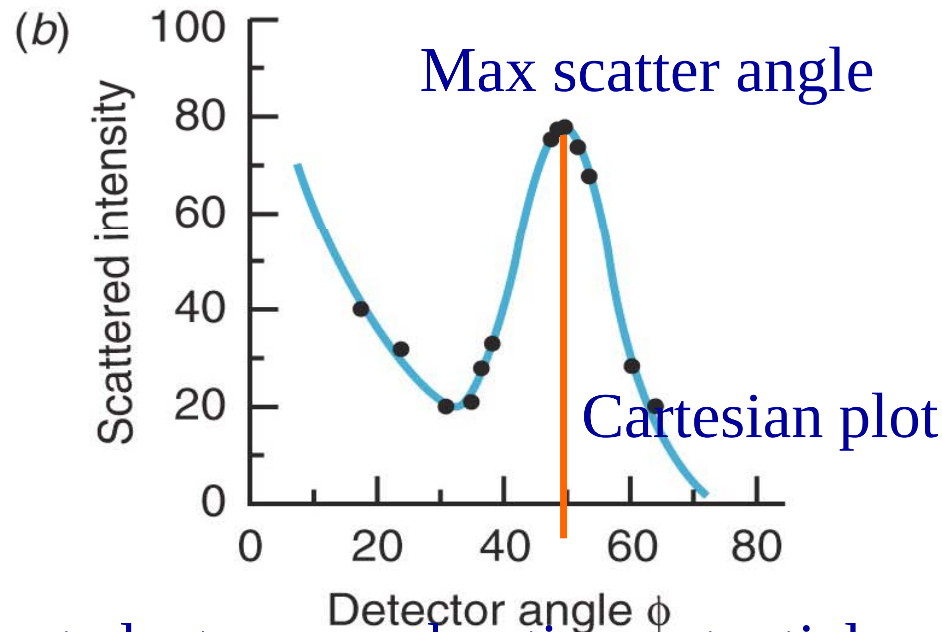
Diffraction pattern produced by 600eV electrons incident on a Al foil target

Notice the waxing and waning of scattered electron Intensity.

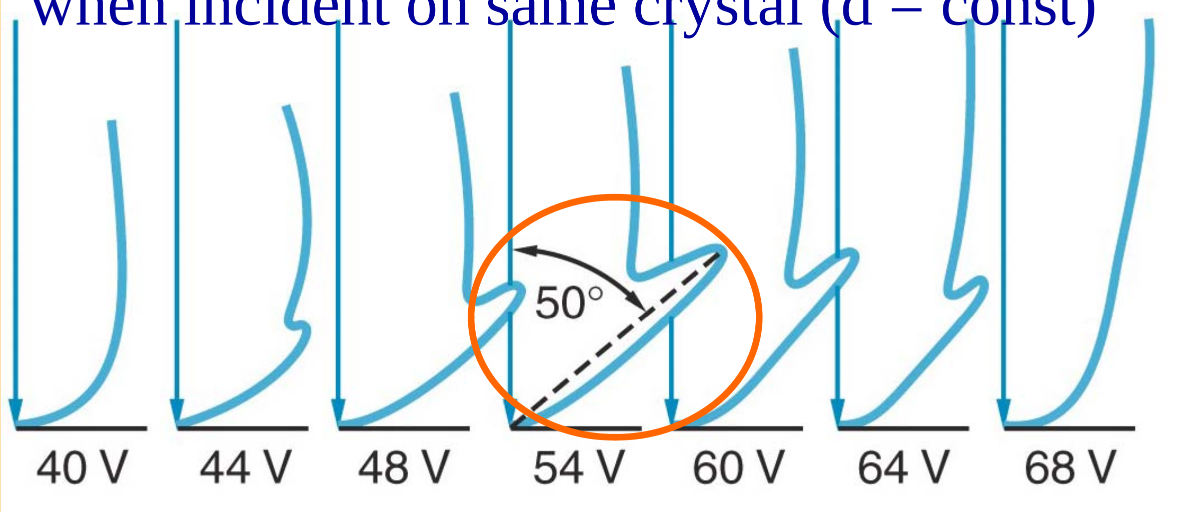
What to expect if electron had no wave like attribute

Davisson-Germer Experiment: 54 eV electron Beam





Polar graphs of DG expt with different electron accelerating potential when incident on same crystal (d = const)



Peak at $\Phi=50^o$

when $V_{acc} = 54$ V

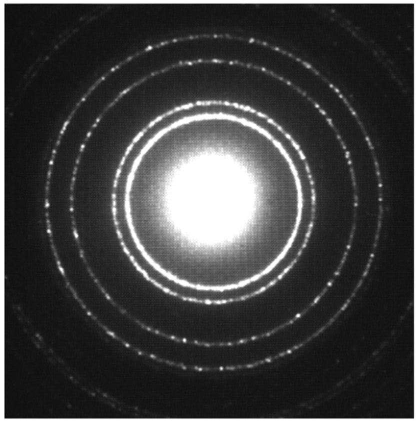
Analyzing Davisson-Germer Expt with de Broglie idea

de Broglie λ for electron accelerated thru V_{acc} =54V

- $\frac{1}{2}mv^2 = K = \frac{p^2}{2m} = eV \Rightarrow v = \sqrt{\frac{2eV}{m}}$; $p = mv = m\sqrt{\frac{2eV}{m}}$

If you believe de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \boxed{\frac{h}{\sqrt{2meV}} = \lambda^{predict}}$$



For V_{acc} = 54 Volts $\Rightarrow \lambda = 1.67 \times 10^{-10} m$ (de Broglie)

Exptal data from Davisson-Germer Observation:

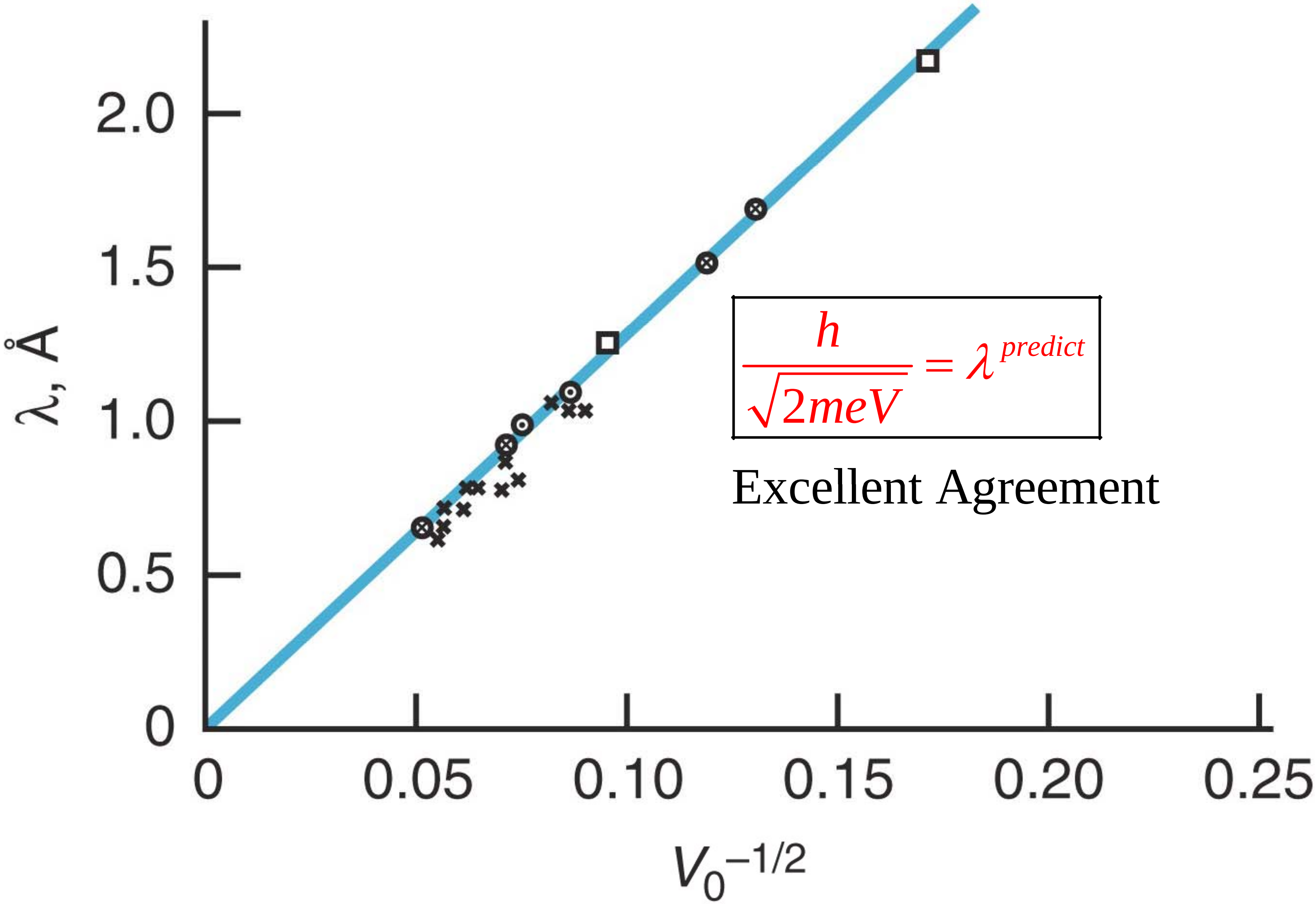
d_{nickel} =2.15 Å =$2.15 \times 10^{-10} m$ (from Bragg Scattering)

$\theta_{diff}^{max} = 50^o$ (observation from scattering intensity plot)

Diffraction Rule : d sinϕ = nλ

$\Rightarrow$ For Principal Maxima (n=1); λ^{meas} =(2.15 Å)(sin 50°) $\boxed{\begin{array}{l}\lambda^{predict} = 1.67\,\dot{A} \\ \lambda^{observ} = 1.65\,\dot{A}\end{array}}$ *Excellent* agreement

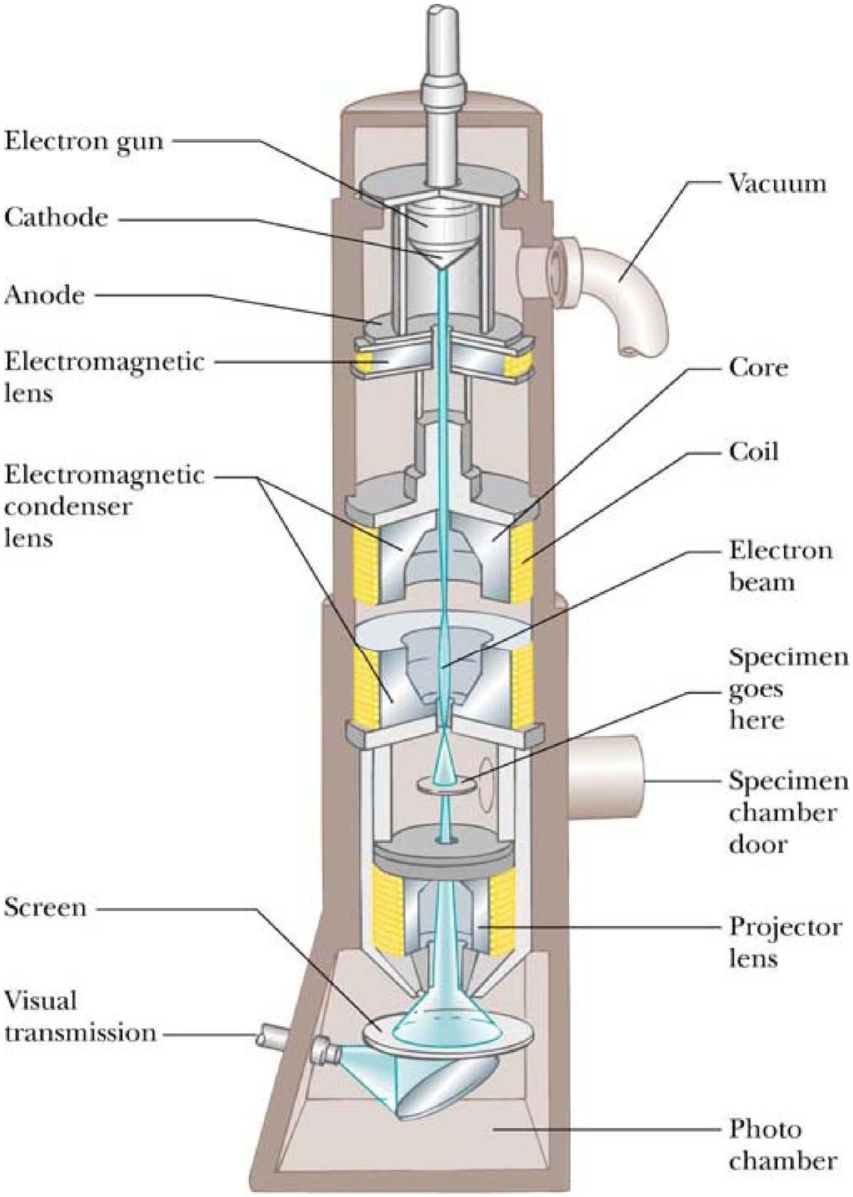
Davisson Germer Experiment: Matter Waves !

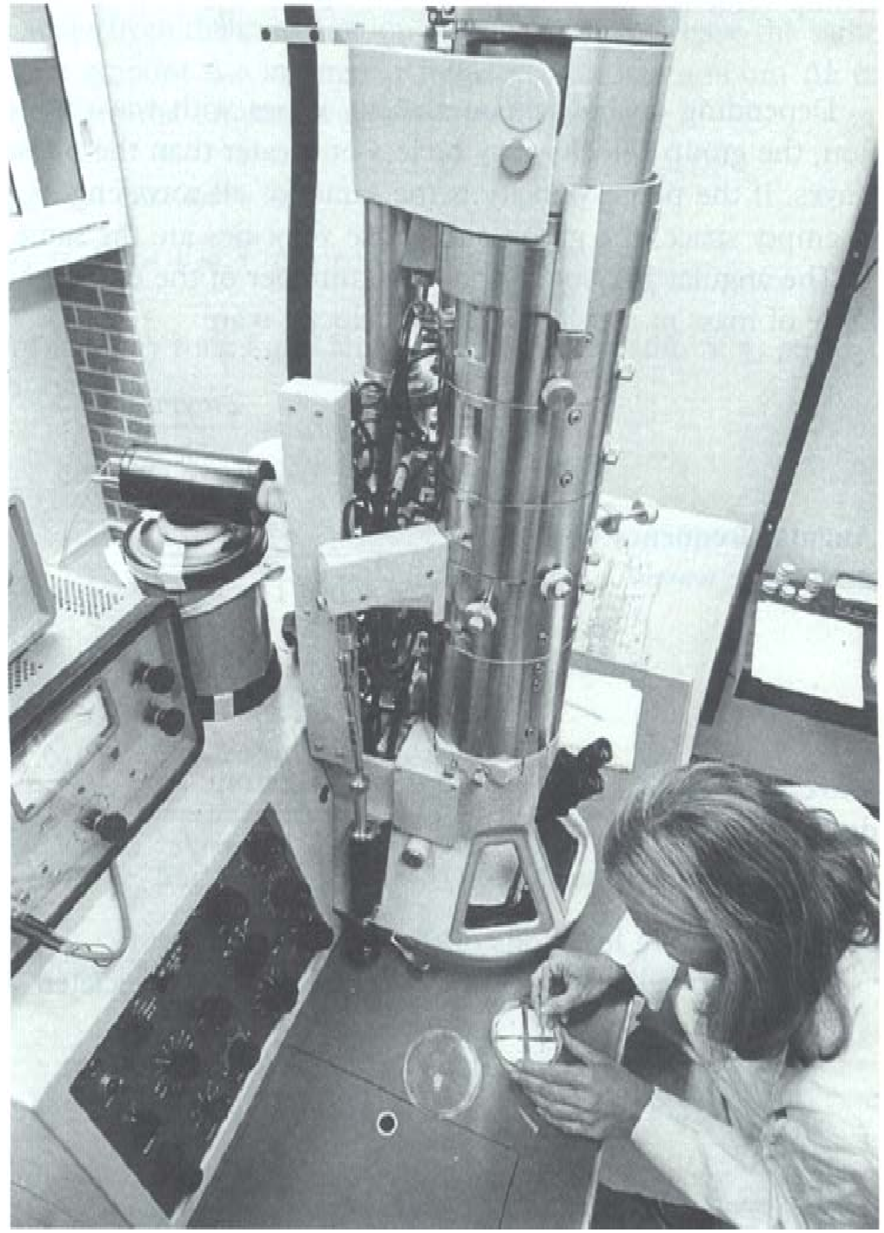


$$\frac{h}{\sqrt{2meV}} = \lambda^{predict}$$

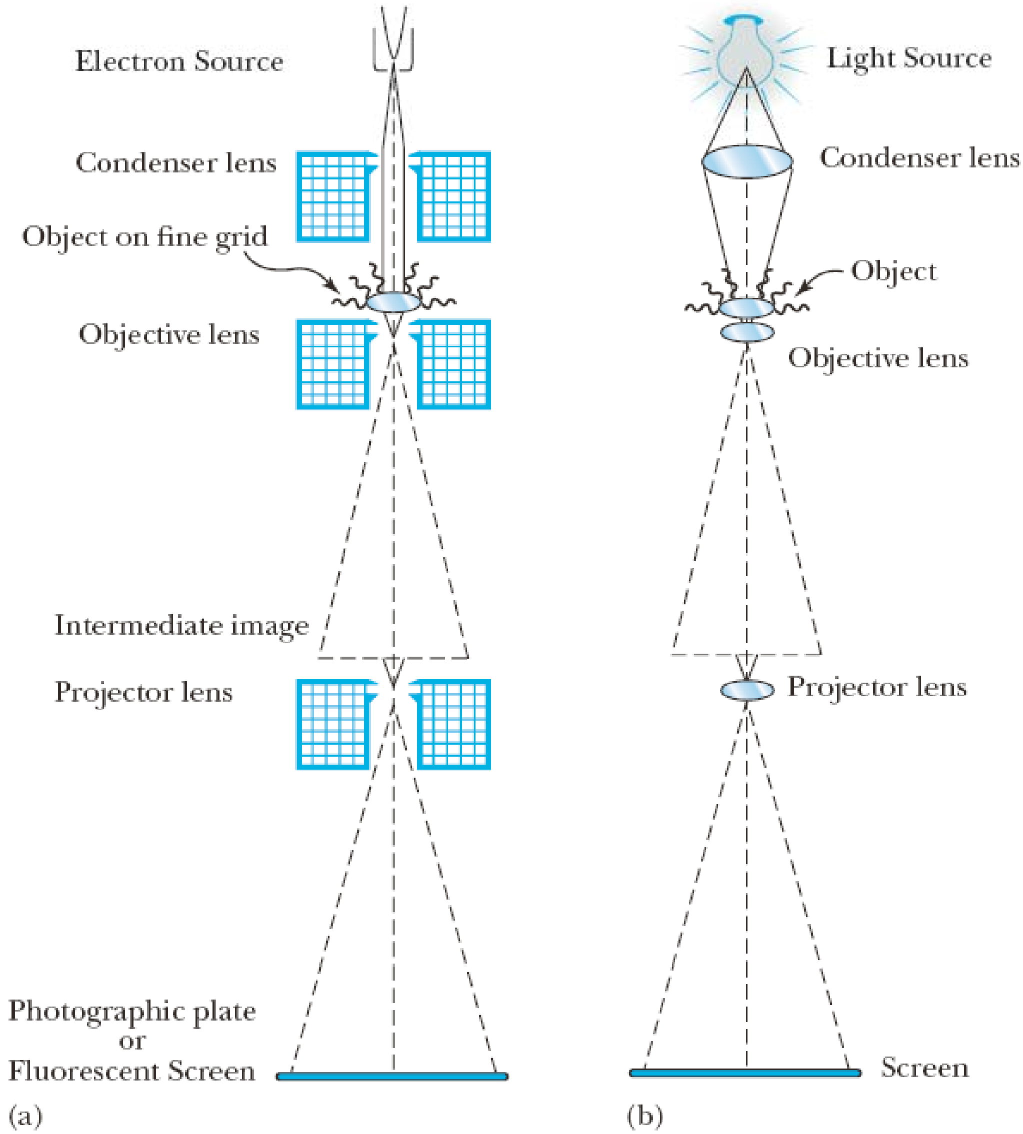
Excellent Agreement

Practical Application : Electron Microscope

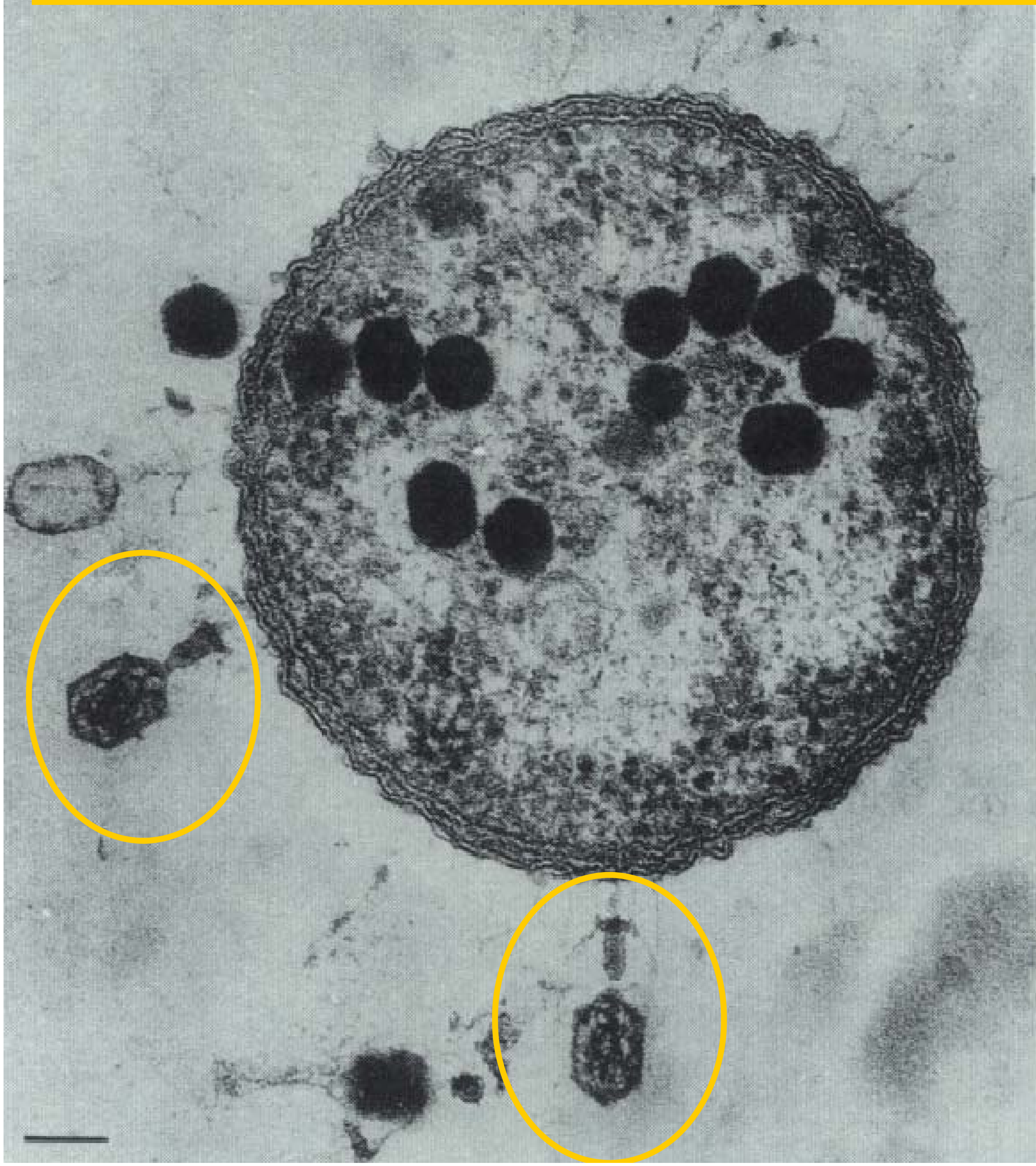




Electron Microscopy



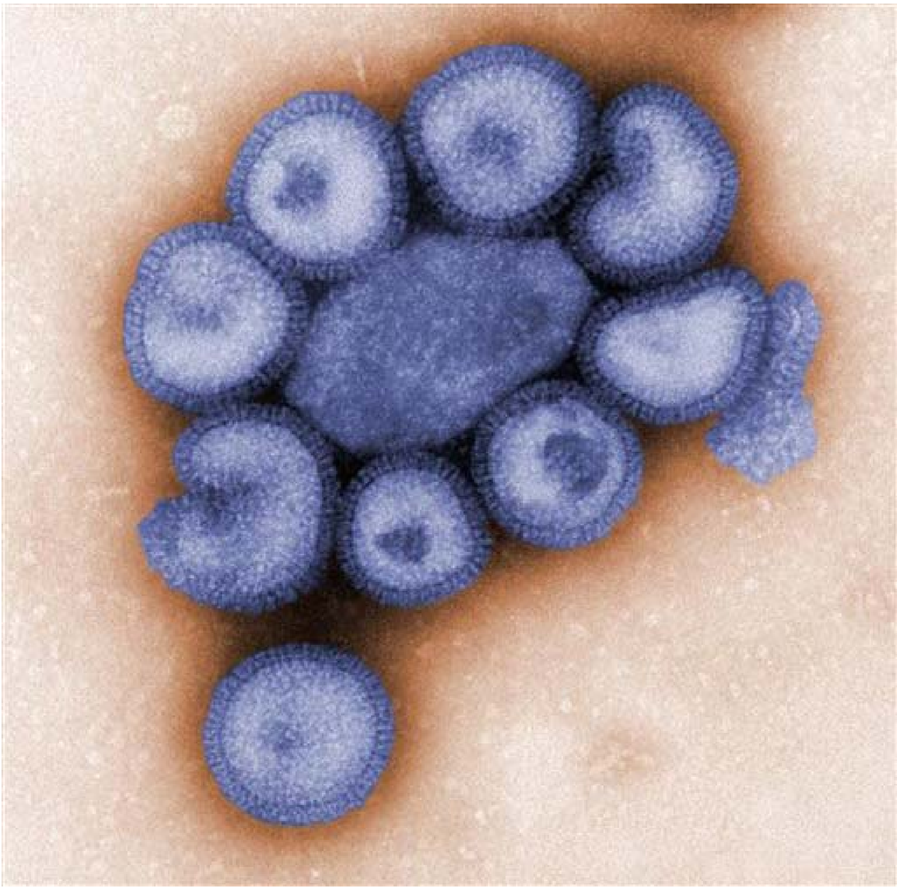
Electron Microscope : Excellent Resolving Power



Electron Micrograph
Showing Bacteriophage
Viruses in E. Coli bacterium

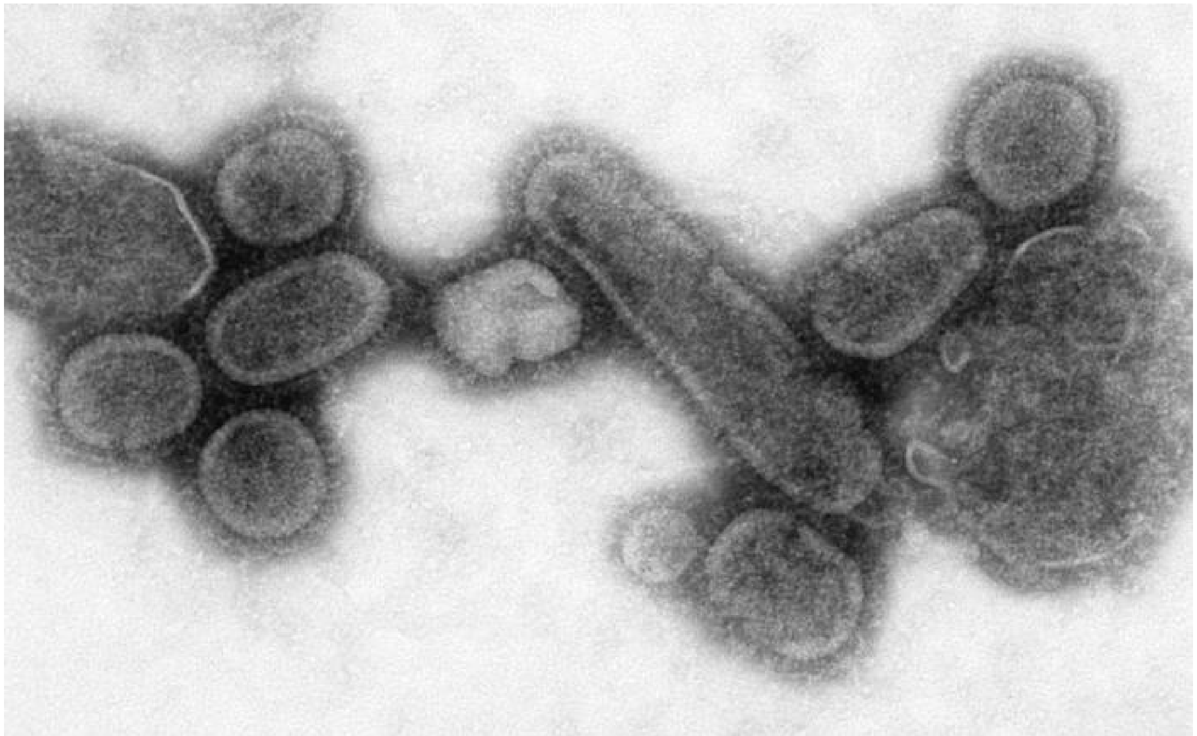
The bacterium is ≅ 1μ size

Swine Flu Virus



Electron microscope image of the H1N1 virus, April 27, 2009, at the U.S. Centers for Disease Control and Prevention's headquarters in Atlanta, Georgia (AP Photo/Center for Disease Control and Prevention, C. S. Goldsmith and A. Balish) The viruses are 80–120 nanometres in diameter.

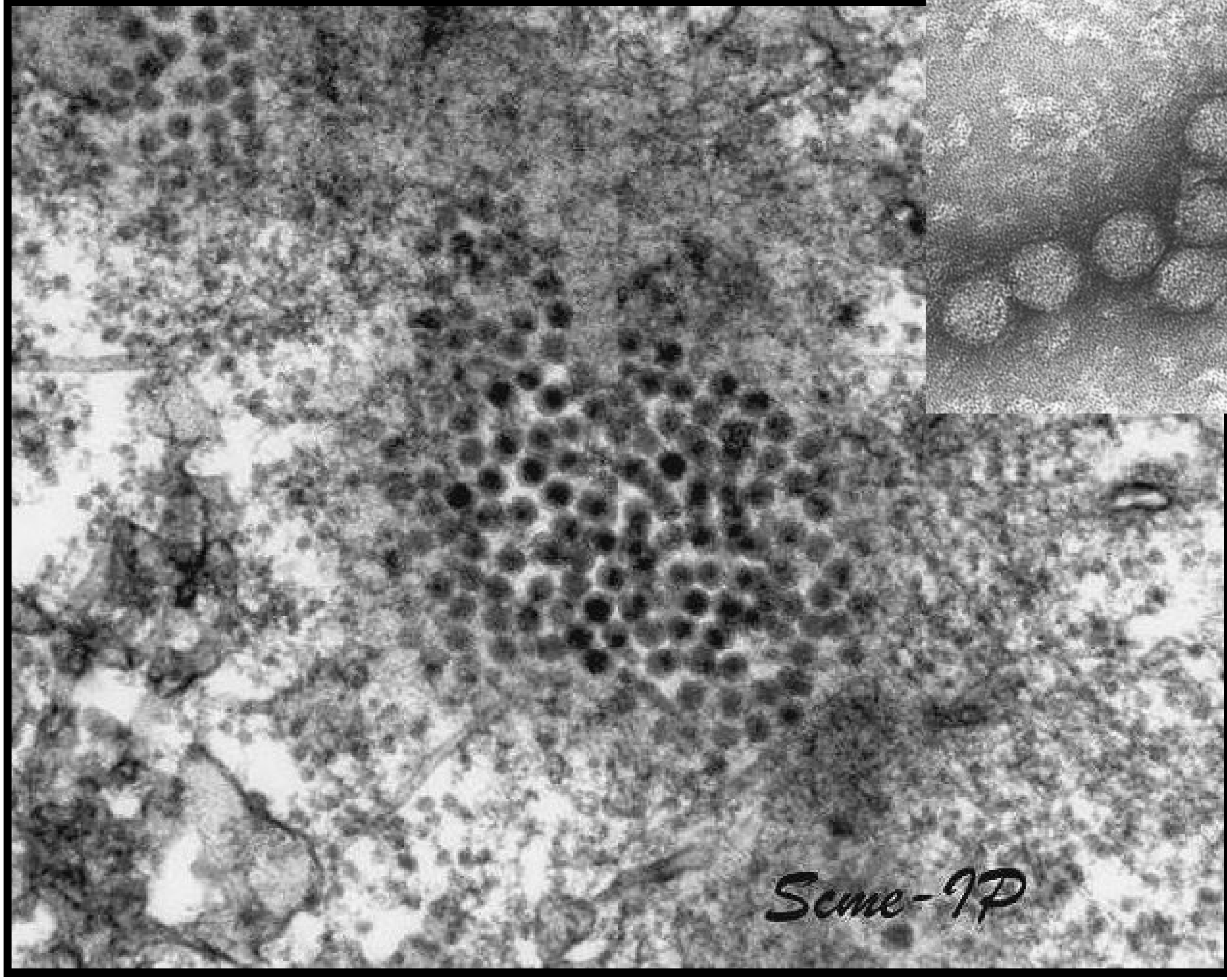
1918 Swine Flu Virus

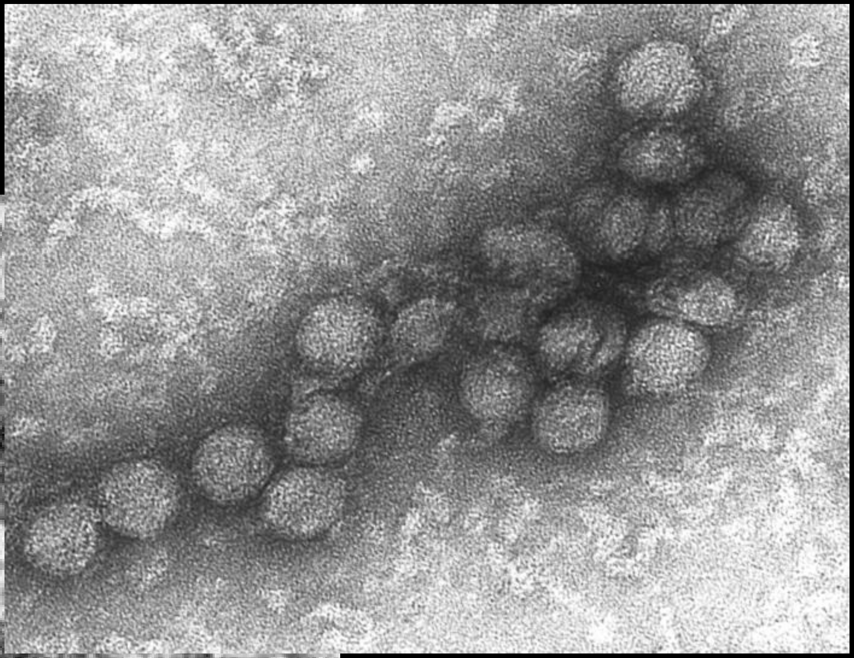


Negative stained transmission electron micrograph (TEM) showed recreated 1918 influenza virions that were collected from the supernatant of a 1918-infected Madin-Darby Canine Kidney (MDCK) cell culture 18 hours after infection.

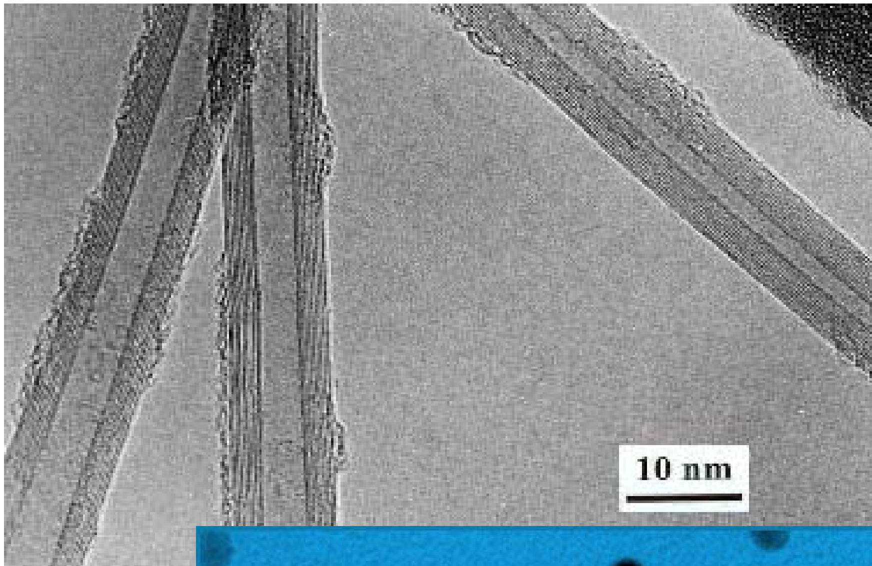
The 1918 Spanish flu epidemic was caused by an influenza A (H1N1) virus, killing more than 500,000 people in the United States, and up to 50 million worldwide.

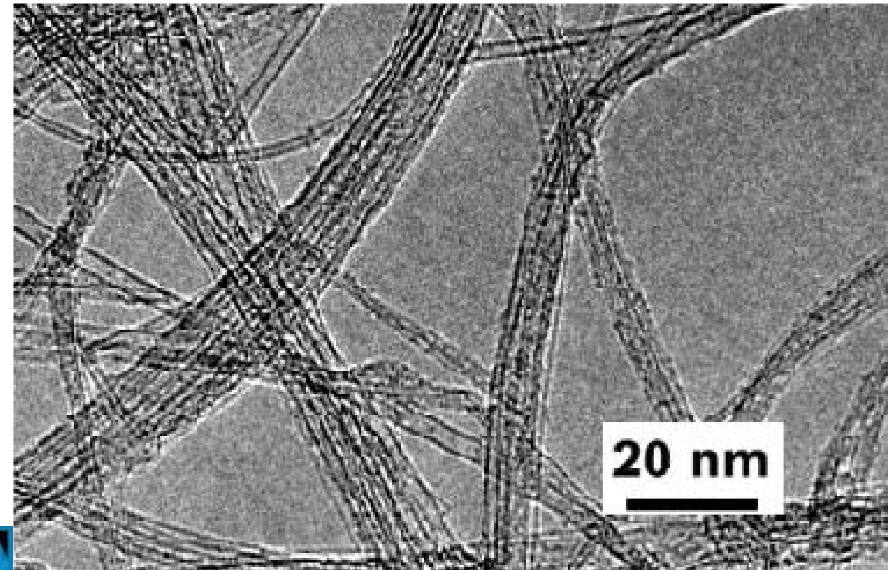
West Nile Virus extracted from a crow brain

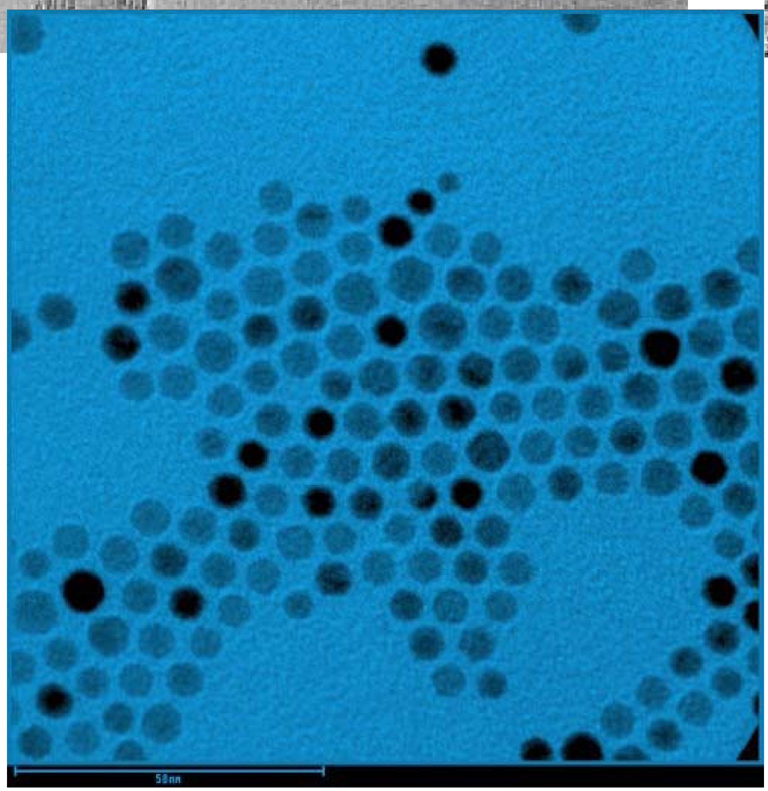




TEM pictures of Carbon Nanotubes







TEM of 6 nm Au Nanoparticles
(Shpyrko Group, UCSD)

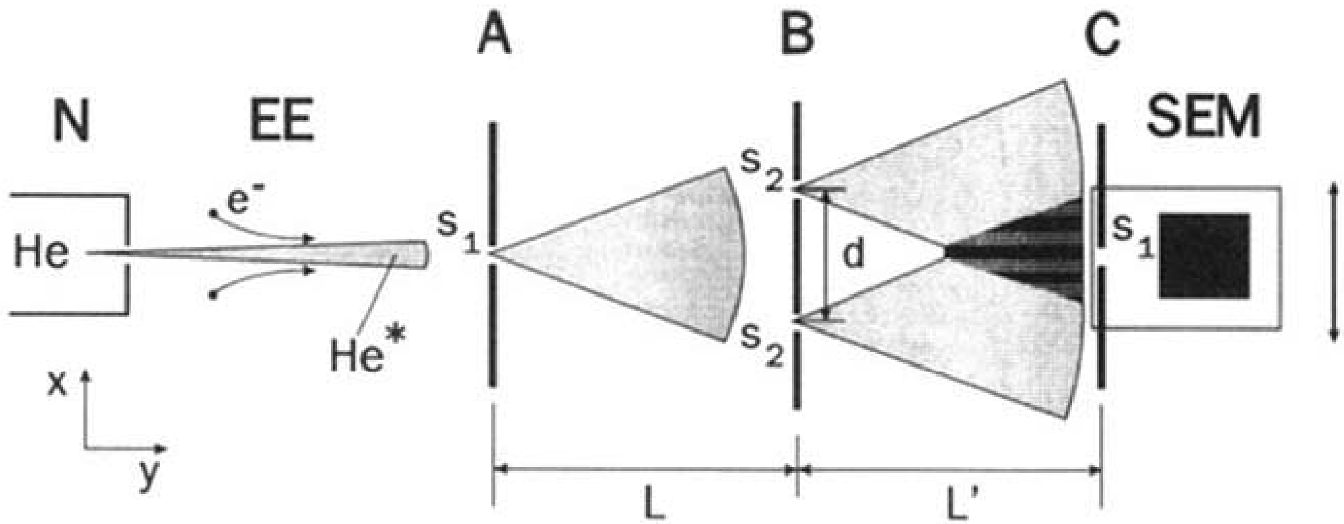
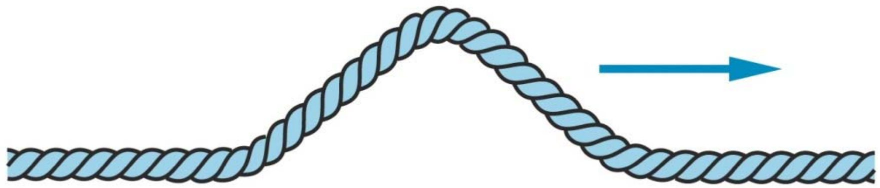


FIG. 2. Schematic representation of the experimental setup: nozzle system and gas reservoir N; electron impact excitation EE; entrance slit A, double slit B, and detector screen C; secondary electron multiplier SEM (mounted together with C on a translation stage). Dimensions: $d=8\ \mu$m, $L=L'=64$ cm; slit widths: $s_1=2\ \mu$m, $s_2=1\ \mu$m.

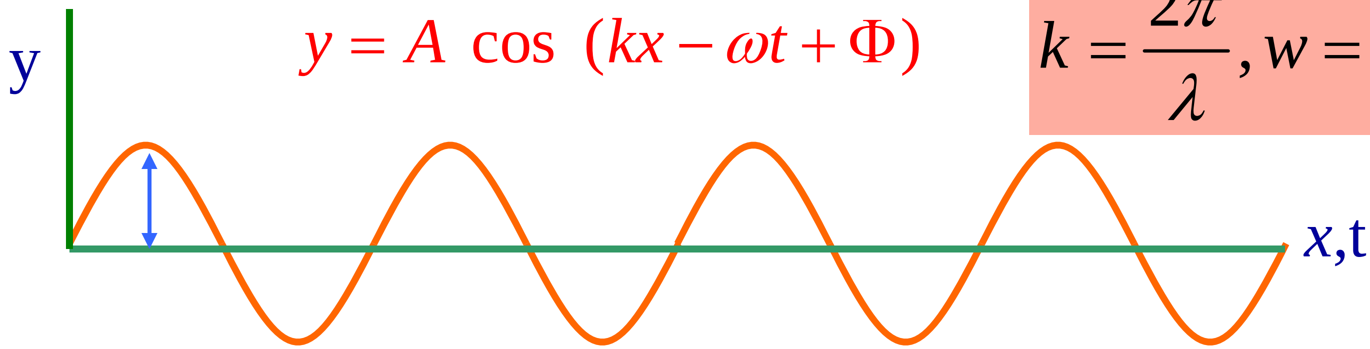
Just What is Waving in Matter Waves ?

- For waves in an ocean, it's the water that "waves"
- For sound waves, it's the molecules in medium
- For light it's the **E** & **B** vectors
- What's waving for matter waves ?
  - **It's the PROBABLILITY OF FINDING THE PARTICLE that waves !**
  - **Particle can be represented by a wave packet in**
    - **Space**
    - **Time**
    - **Made by superposition of many sinusoidal waves of different λ**
    - **It's a "pulse" of probability**

Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)



What Wave Does Not Describe a Particle



$$y = A\cos(kx - \omega t + \Phi)$$

$$k = \frac{2\pi}{\lambda}, w = 2\pi f$$

- What wave form can be associated with particle's pilot wave?
- A traveling sinusoidal wave? $y = A\cos(kx - \omega t + \Phi)$
- Since de Broglie "pilot wave" represents particle, it must travel with same speed as particle ……(like me and my shadow)

Phase velocity (v_p) of sinusoidal wave: $v_p = \lambda f$

In Matter:

$$(a)\quad \lambda = \frac{h}{p} = \frac{h}{\gamma mv}$$

$$(b)\quad f = \frac{E}{h} = \frac{\gamma mc^2}{h}$$

$$\Rightarrow v_p = \lambda f = \frac{E}{p} = \frac{\gamma mc^2}{\gamma mv} = \frac{c^2}{v} > c!$$

Conflicts with Relativity → Unphysical

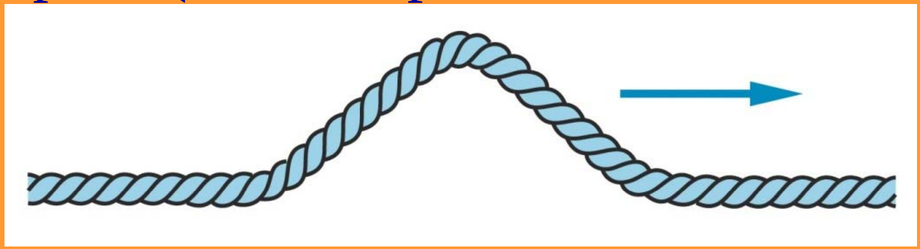
Single sinusoidal wave of infinite extent does not represent particle localized in space

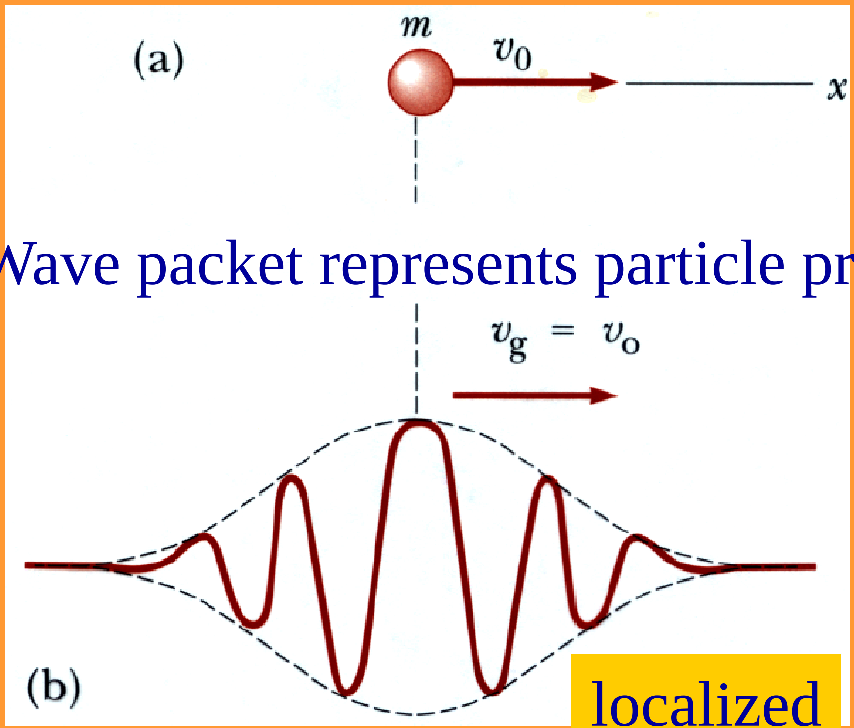
Need "wave packets" localized Spatially (x) and Temporally (t)

Wave Group or Wave Pulse

- Wave Group/packet:
  - Superposition of many sinusoidal waves with different wavelengths and frequencies
  - Localized in space, time
  - Size designated by
    - Δx or Δt
  - Wave groups travel with the speed $v_g = v_0$ of particle
- Constructing Wave Packets
  - Add waves of diff λ,
  - For each wave, pick
    - Amplitude
    - Phase
  - Constructive interference over the space-time of particle
  - Destructive interference elsewhere !

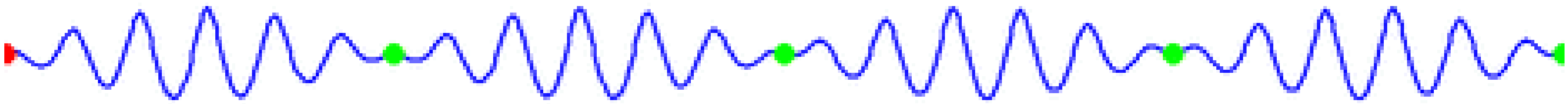
Imagine Wave pulse moving along a string: its localized in time and space (unlike a pure harmonic wave)





Wave packet represents particle prob

localized

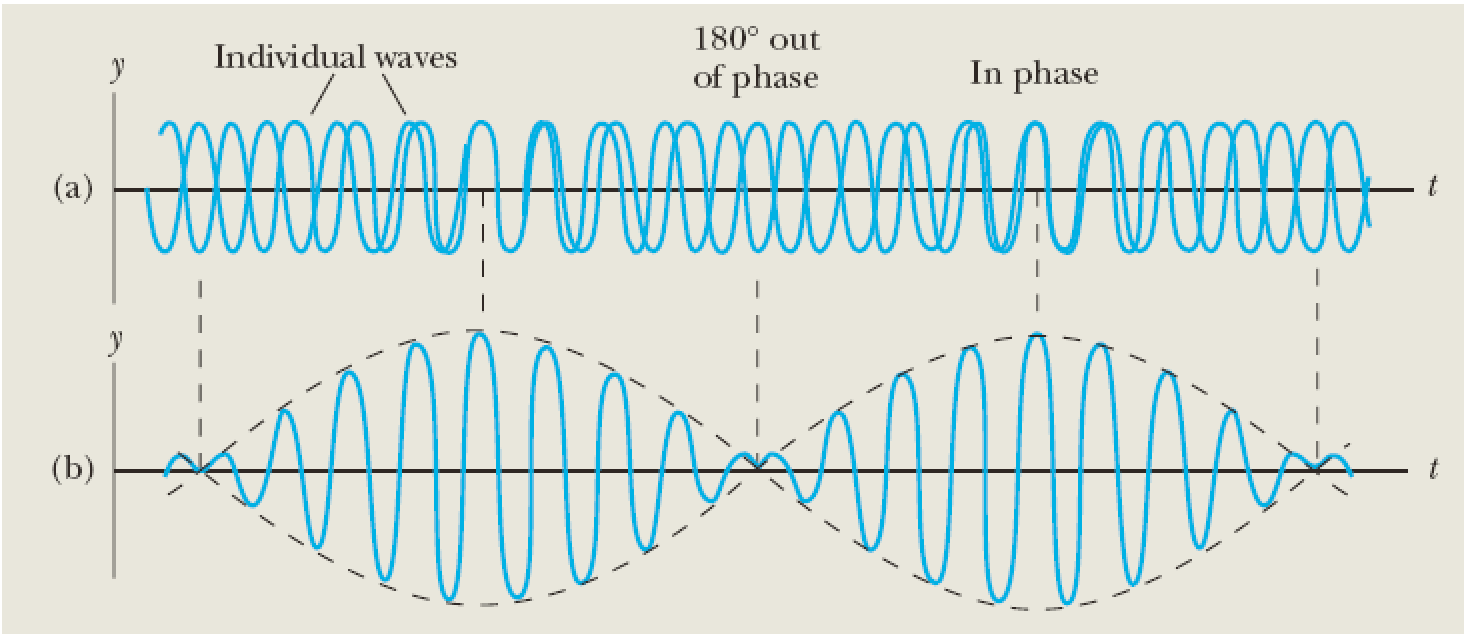


- Phase velocity $V_p = \frac{\omega}{k} = \lambda \cdot f$

- Group velocity $V_g = \frac{\partial \omega}{\partial k}$

See animation of group/phase velocity at:

http://en.wikipedia.org/wiki/Group_velocity


y
Individual waves
180° out
of phase
In phase
(a)
t
y
(b)
t

Resulting wave's "displacement" $y = y_1 + y_2$:

$$y = A\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$$

$k = \frac{2\pi}{\lambda}$

$\cos(kx - \omega t)$

Trignometry : $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$\therefore\ y = 2A\left[\left(\cos\left(\frac{k_2 - k_1}{2}x - \frac{\omega_2 - \omega_1}{2}t\right)\right)\left(\cos\left(\frac{k_2 + k_1}{2}x - \frac{\omega_2 + \omega_1}{2}t\right)\right)\right]$$

since $k_2 \cong k_1 \cong k_{ave},\ \omega_2 \cong \omega_1 \cong \omega_{ave}, \Delta k \ll k, \Delta\omega \ll \omega$

$$\therefore \boxed{y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right)\cos(kx - \omega t)\right]} \equiv\ y = A' \cos(kx - \omega t),$$ A' oscillates in x,t

$$A' = 2A\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right) = \text{modulated amplitude}$$

Phase Vel $\quad V_p = \frac{w_{ave}}{k_{ave}}$

Group Vel $\quad V_g = \frac{\Delta w}{\Delta k}$

V_g : Vel of envelope$= \frac{dw}{dk}$