Physics 2D Lecture Slides
Lecture 4

April 3, 2009

Synchronizing Clocks

After coincidence of their origins at t=0, t′ = 0
Sam and Sally agree to send light signals to each other after time t on their respective clocks. Let t_1 = time on Sam's clock at which he would receive signal if clocks ran at same time and t_2 be actual time he received it.
Then,

$$t_1 = t(1 + v/c)$$

$$t_2 = \gamma t(1 + v/c) = t\sqrt{\frac{1 + v/c}{1 - v/c}}$$

Let t'_1 = time on Sally's clock at which she would receive signal if clocks ran at same time and t'_2 be actual time she received it.
According to Sam she should get it at $(t + t_0)$ where

$c\, t_0 = vt + vt_0$ or
$t_0 = [v/(c-v)]\, t$ so

$t'_1 = t\,[1 + v/(c-v)] = t/(1 - v/c)$

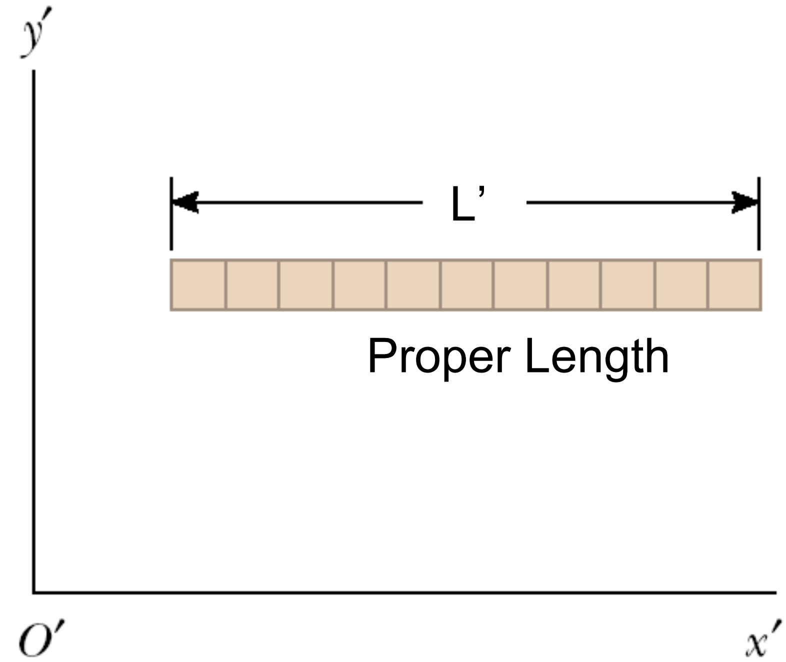
But Sally's clock shows $t'_2 = t'_1/\gamma = \quad t\sqrt{\dfrac{1+v/c}{1-v/c}}$

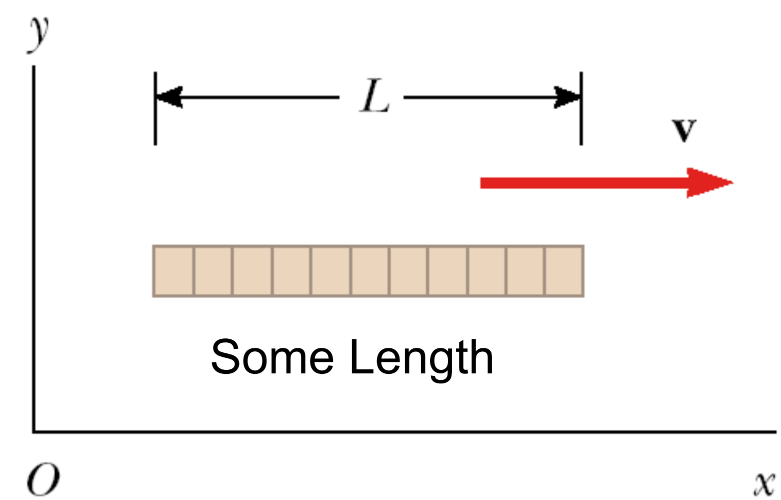
So each of them sees the others clock slow down!

Rocketman Vs The Earthling

- Earth Observer saw rocketman take time $\Delta t = (Lp/ V)$
- Rocketman says he is at rest, Star B moving towards him with speed V from right passed him by in time $\Delta t'$, so
  - $L' = \Delta t'. V$
  - But $\Delta t' = \Delta t / \gamma$ (time dilation)
  - $\Rightarrow L' = V. (\Delta t/ \gamma)$

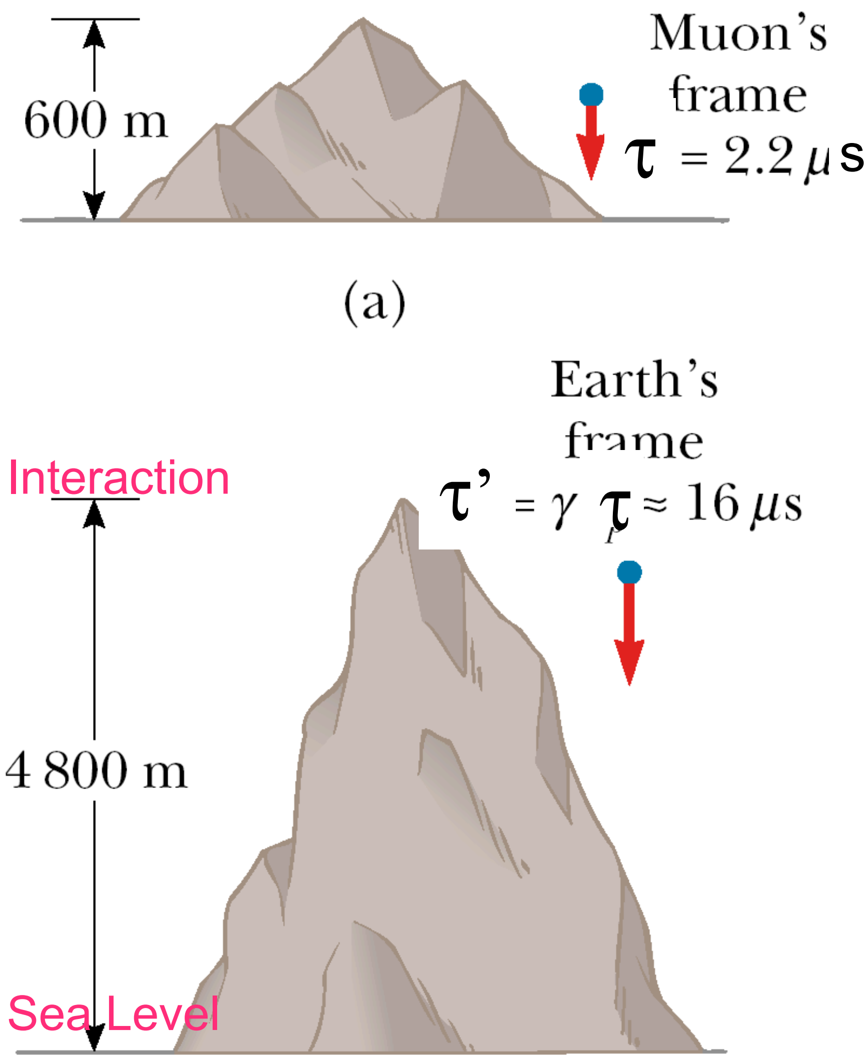
$$= Lp/\gamma = Lp\, [\, 1 - v^2/c^2]^{1/2}$$





Moving Rods Contract in direction Of relative motion

Cosmic Rays Are Falling On Earth : Example of Time Dilation



- Consider Two frames of references
  1. You Riding on the Muon Particle
  2. Your twin watching On surface of earth

– Muon Rider has "Proper Time"
  – Time measured by observer moving along with clock
  – $\Delta t' = \tau = 2.2\,\mu S$
  – $D' = v\,\Delta t' = 650m$

– Earthling watches a moving clock (muon's) run slower
  – $\Delta t' = \gamma\tau$
    – $v = 0.99c$, => $\gamma = 7.1$
    – $D = v\,\Delta t = 4700m$

Contrived Paradoxes of Relativity

A paradox is an apparently self-contradictory statement, the underlying meaning of which is revealed only by careful scrutiny. <u>The purpose of a paradox is to arrest attention and provoke fresh thought</u>

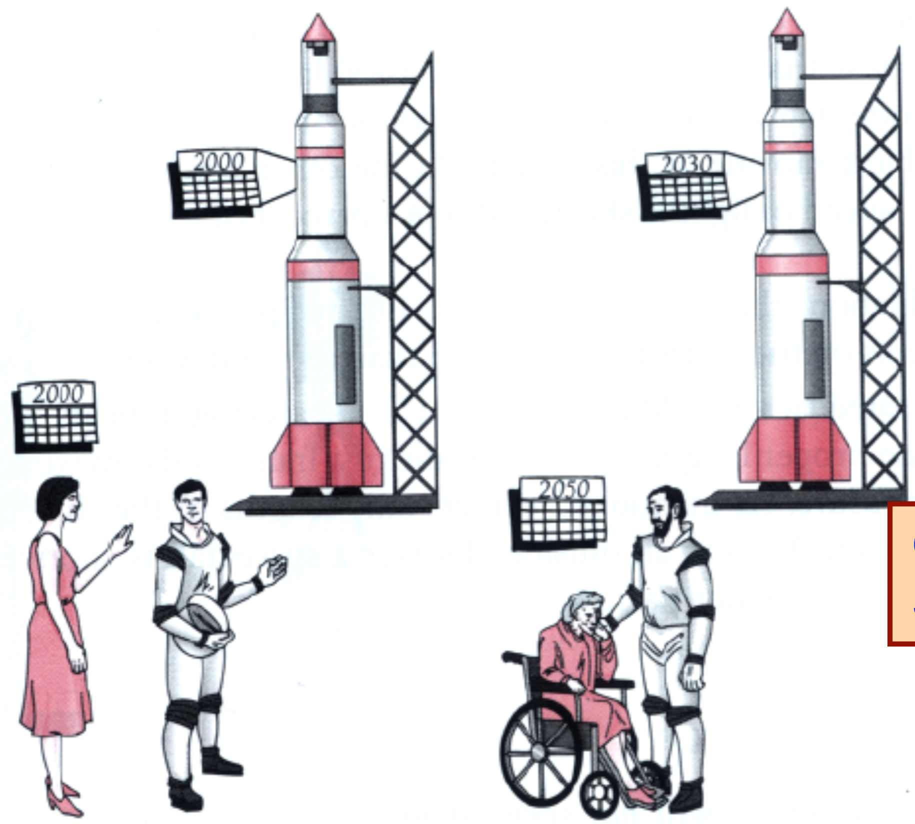
``A paradox is not a conflict within reality. It is a conflict between reality and your feeling of what reality should be like.'' – Richard Feynman

Construct a few paradoxes in Relativity & analyze them

Jack and Jill's Excellent Adventure: Twin Paradox

Jack & Jill are <u>20 yr old twins</u>, with same heartbeat
Jack takes off with V = 0.8c = (4/5)c to a star
20 light years Away.

Jill stays behind, watches Jack by telescope. They
Eventually compare notes



Jill sees <u>Jack's heart</u> slow down
Compared to her by the factor :

$$\sqrt{1-(v/c)^2}$$
$$= \sqrt{1-(0.8c/c)^2} = 0.6$$

For every 5 beats of her heart
She sees Jack's beat only 3 !

Jack has only 3 thoughts for 5 that
Jill has ! …..Every things slows!

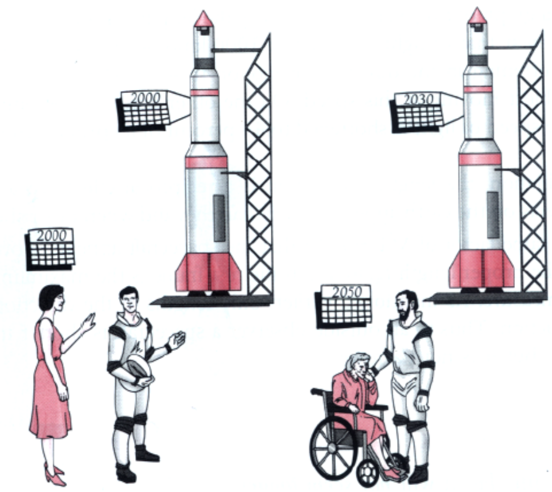
Finally Jack returns after 50 yrs
gone by according to Jill's calendar

Only 30 years have gone by Jack's calendar
SO Jack is 50 years old but Jane is 70 !

Where is the paradox ??

Twin Paradox ?

- Paradox : Turn argument around, motion is relative. Look at Jack's point of view !
- Jack claims he at rest, Jill is moving v=0.8c
- Should not Jill be 50 years old when 70 year old Jack returns from space Odyssey?



No ! …because Jack is not <u>always</u> traveling in a inertial frame of reference

TO GET BACK TO EARTH <u>HE</u> HAS TO TURN AROUND => decelerate/accelerate

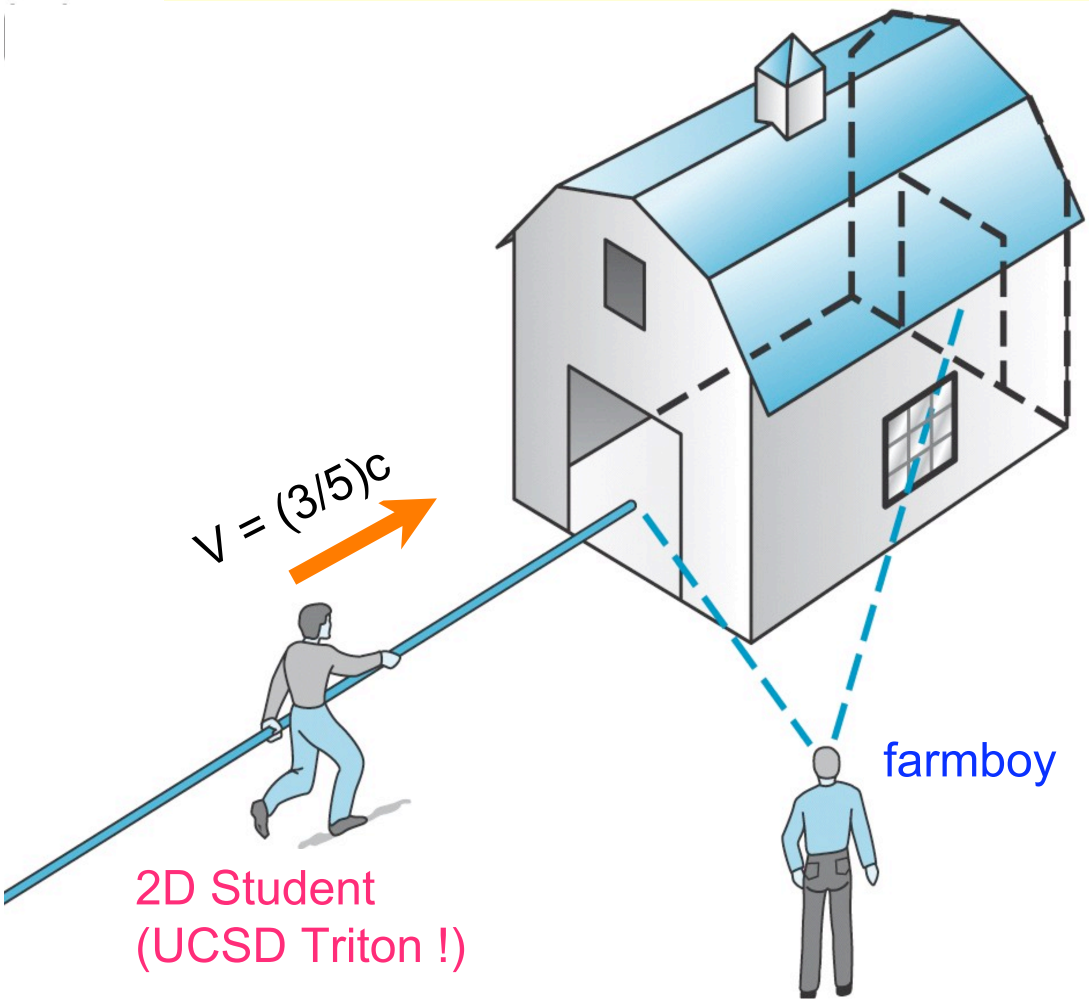
But Jill always remained in Inertial frame
Time dilation formula valid for Jill's observation of Jack <u>but not to Jack's observation of Jill !!….</u>remember this always

Non-symmetric aging verified with atomic clocks taken on airplane trip around world and compared with identical clock left behind. Observer who departs from an inertial system will always find its clock slow compared with clocks that stayed in the system

Fitting a 5m pole in a 4m Barnhouse ?

Student attends 2D lecture (but does no HW) …banished to a farm in Iowa ! Meets a farmboy who is watching 2D lecture videos online. He does not do HW either!

There is a Barn with 2 doors 4m apart ; There is a pole with proper length = 5m
Farm boy goads the student to run fast and fit the 5m pole within 4m barn
The student tells the farmboy: "Dude you are nuts!" …who is right and why ?



Sequence of Events

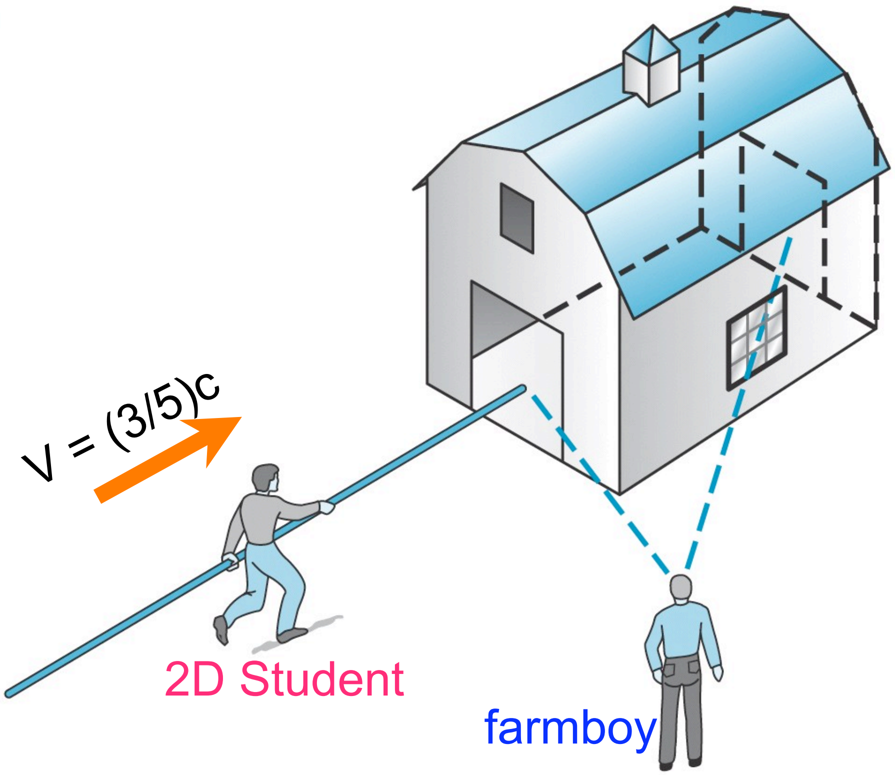
A: Arrival of right end of pole at left end of barn

B: Arrival of left end of pole at left end of barn

C: Arrival of right end of pole at right end of barn

Think Simultaneity !

Fitting a 5m pole in a 4m Barnhouse ?!!



Student with pole runs with v=(3/5)c

farmboy sees pole contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says pole just fits in the barn fully!

Student with pole runs with v=(3/5)c

Student sees barn contraction factor

$$\sqrt{1-(3c/5c)^2} = 4/5$$

says barn is only 3.2m long, too short to contain entire 5m pole !

Farmboy says “You can do it”

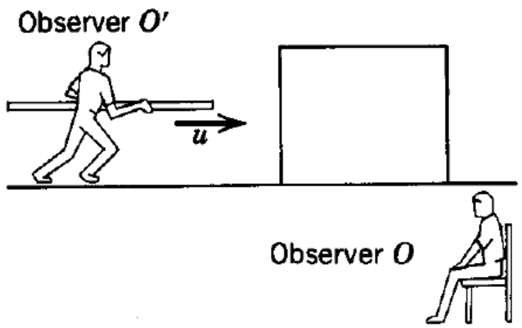
Student says “Dude, you are nuts”

Is there a contradiction ? Is Relativity wrong?

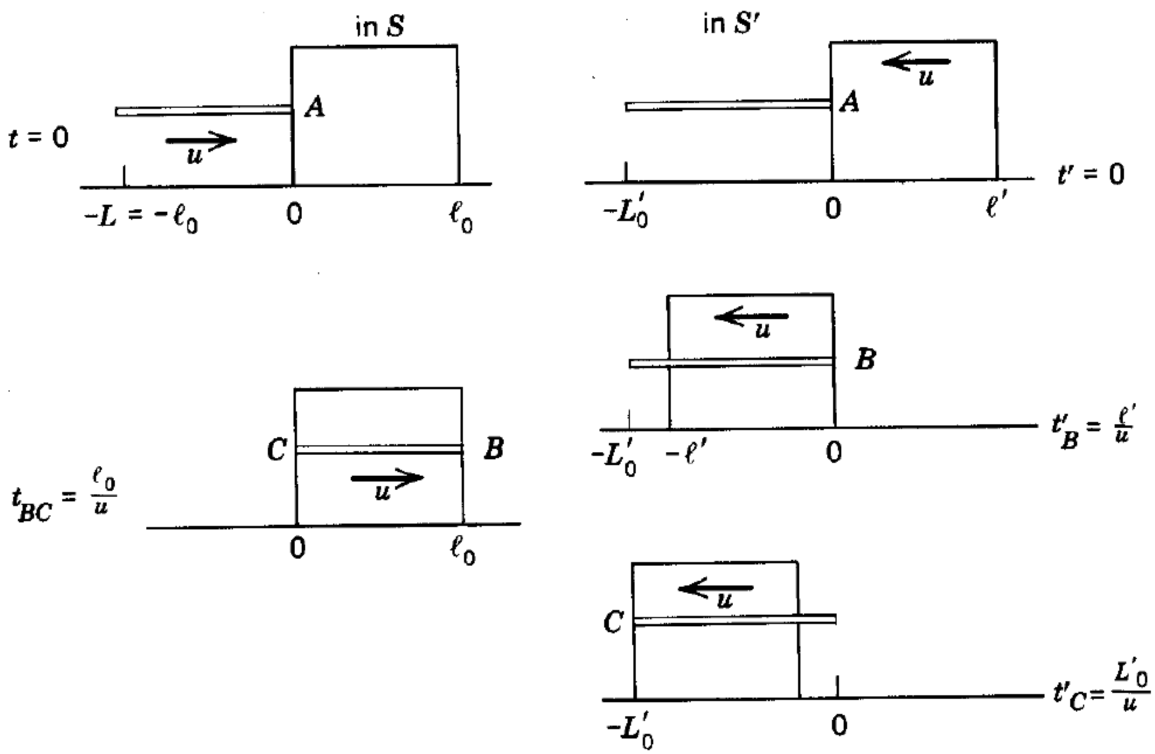
Homework: You figure out who is right, if any and why.

Hint: Think in terms of observing three events

Farmboy Vs 2D Student



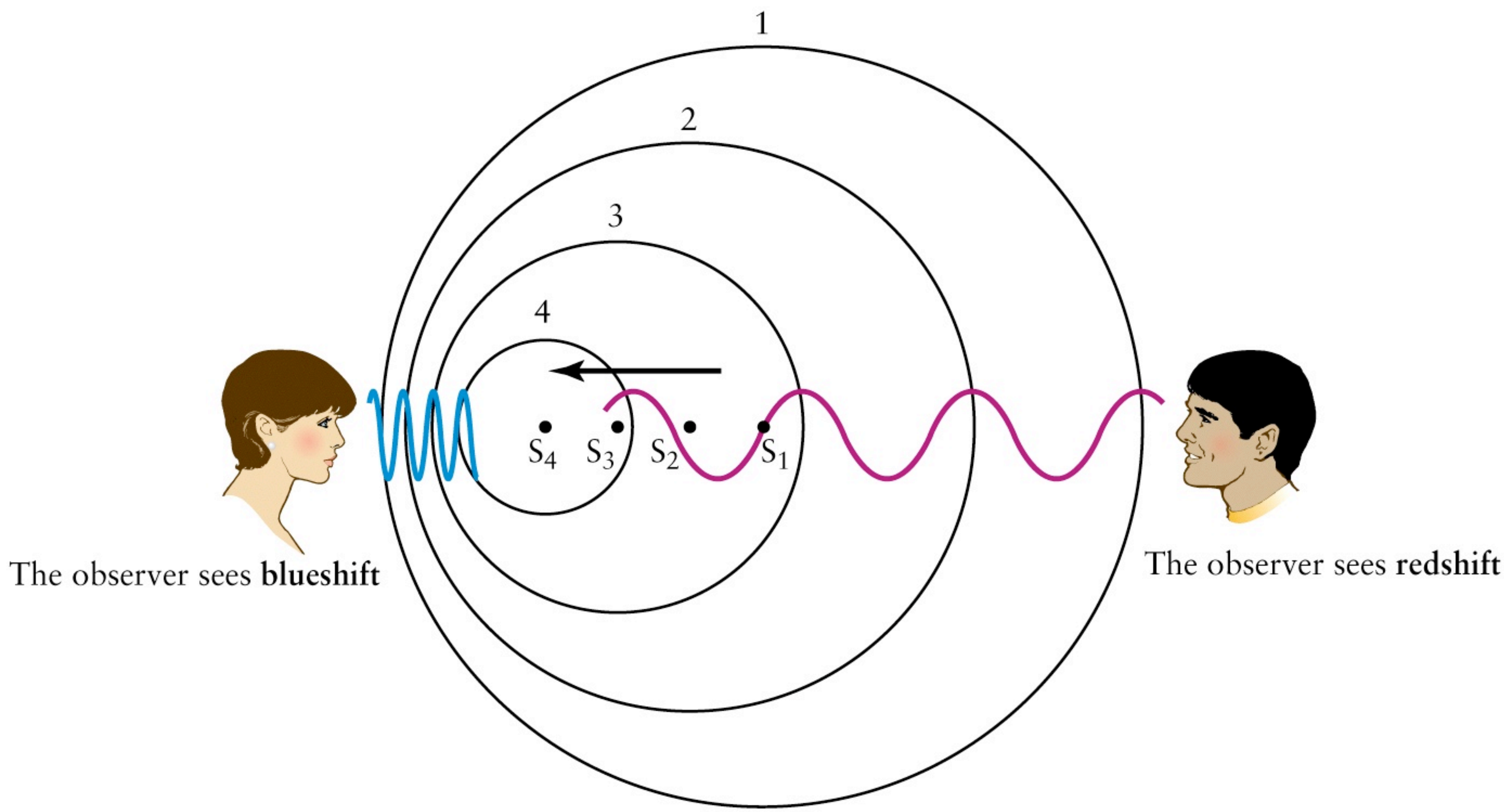
Pole and barn are in relative motion u such that lorentz contracted length of pole = Proper length of barn



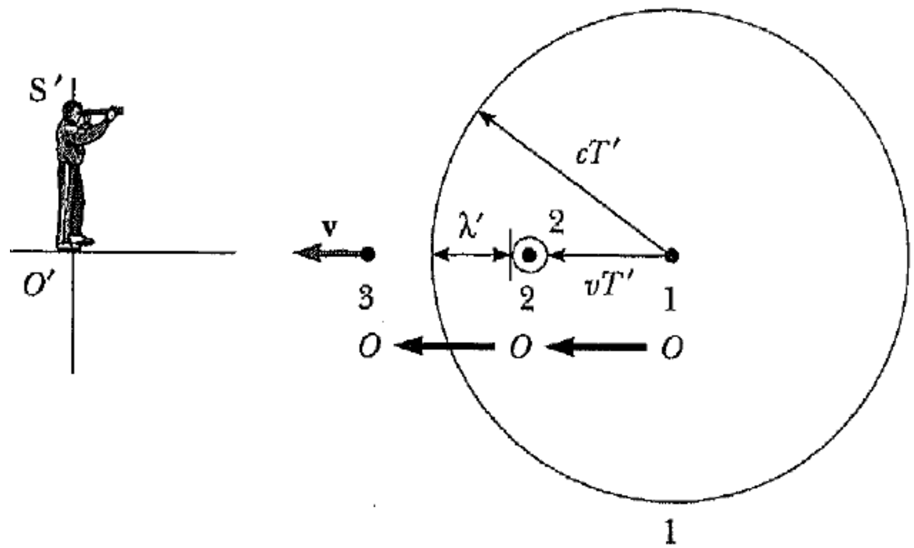
In rest frame of pole, Event B precedes C

Relativistic Doppler Shift

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} \; f_{source}$$



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T', the wavefront moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefront

$\lambda' = cT' - vT'$

use $f = c/\lambda$

$$f' = \frac{c}{(c-v)T'} \;,\; T' = \frac{T}{\sqrt{1-(v/c)^2}}$$

Substituting for T', use f=1/T

$$\Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)} f$$

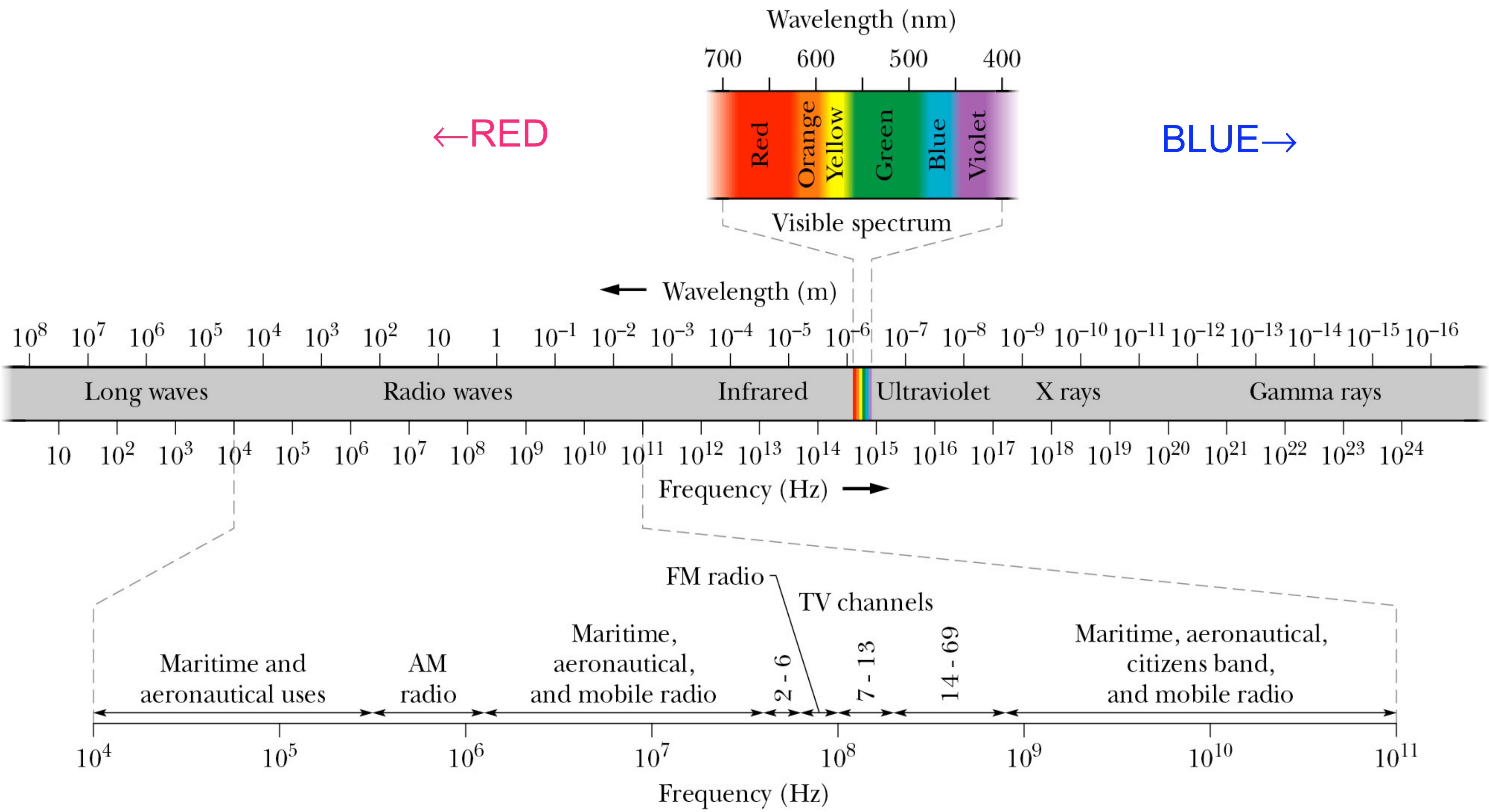
$$\Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f$$

better remembered as:

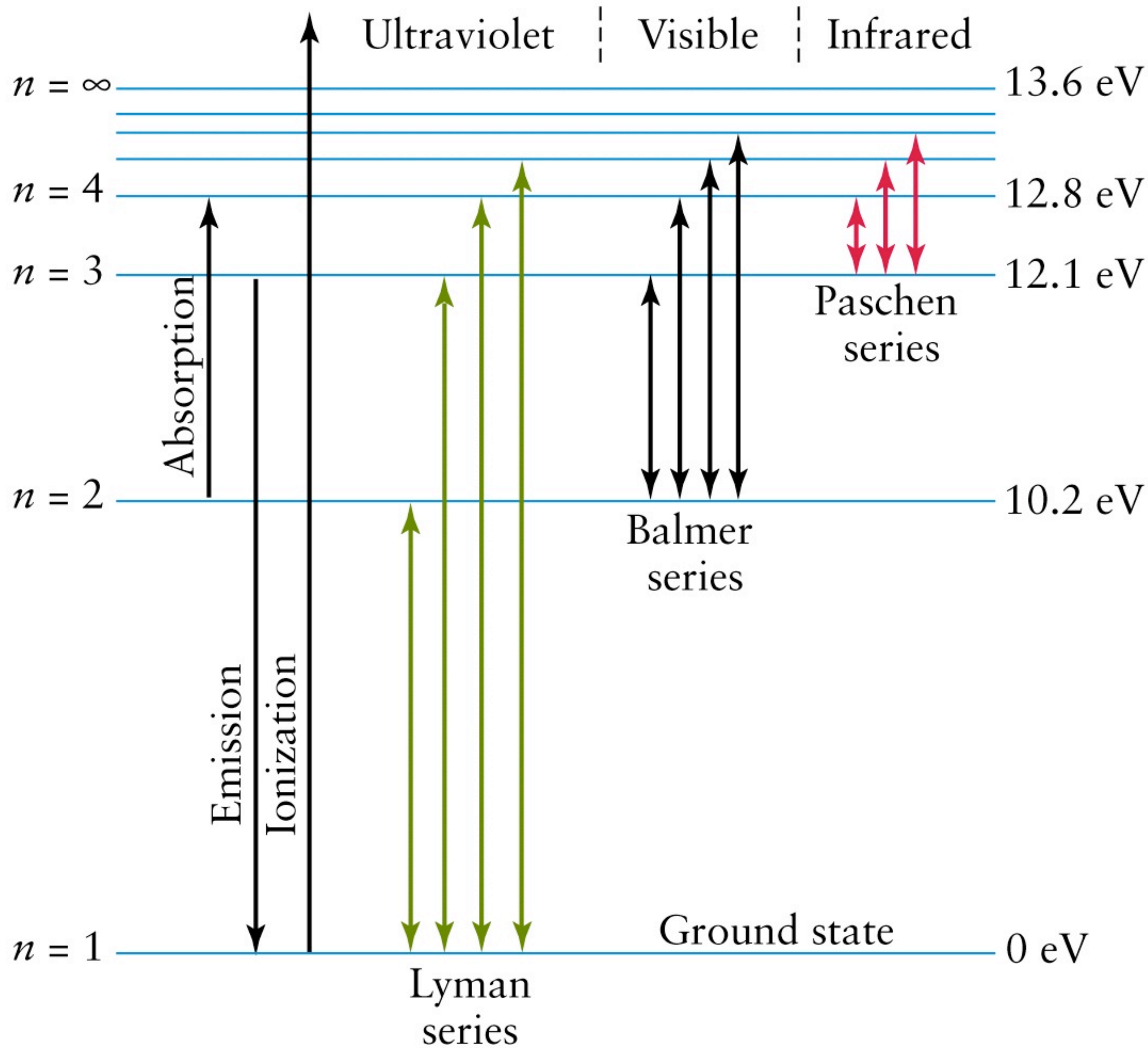
$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$

f_{obs} = Freq measured by observer approching light source

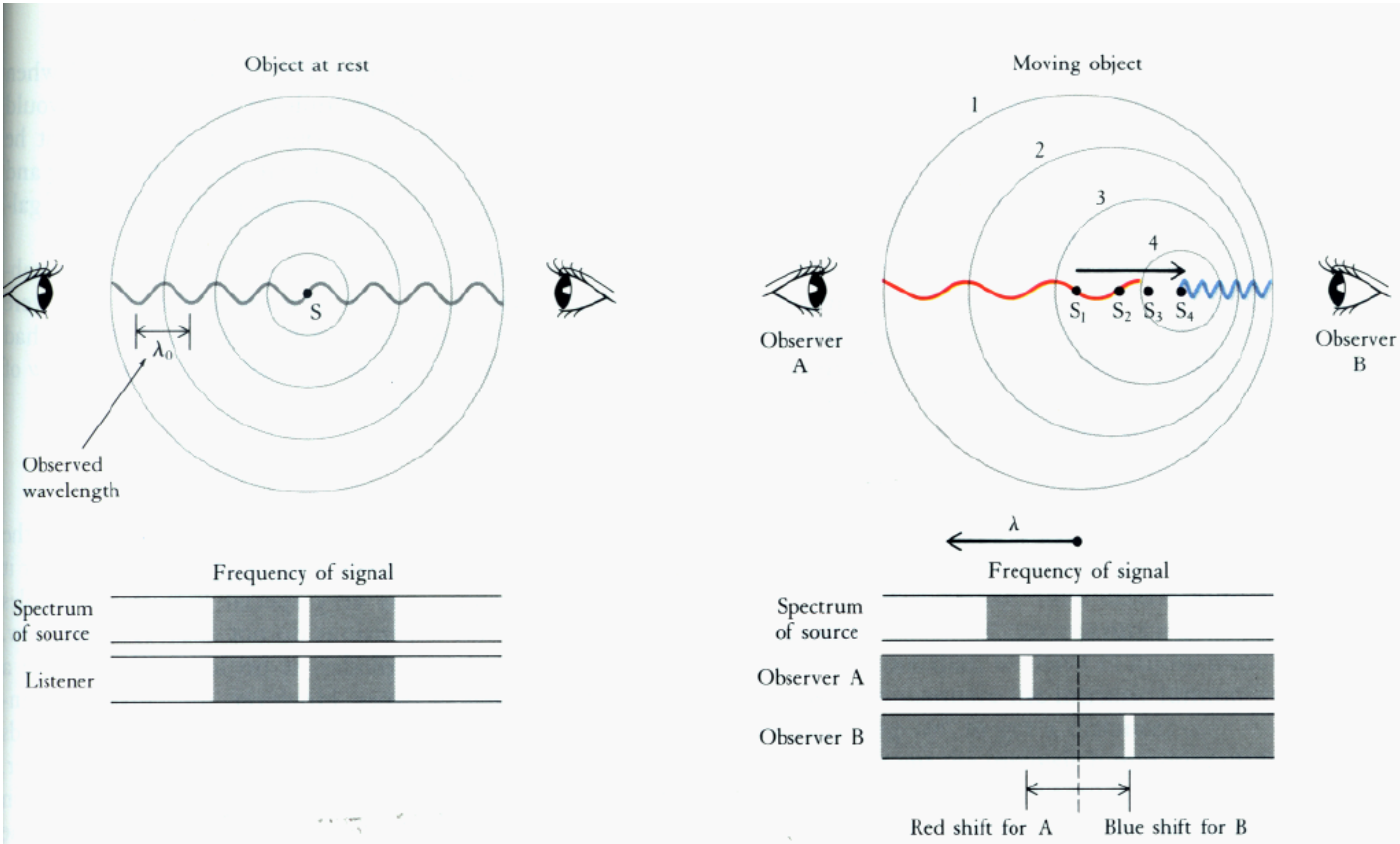
Doppler Shift & Electromagnetic Spectrum



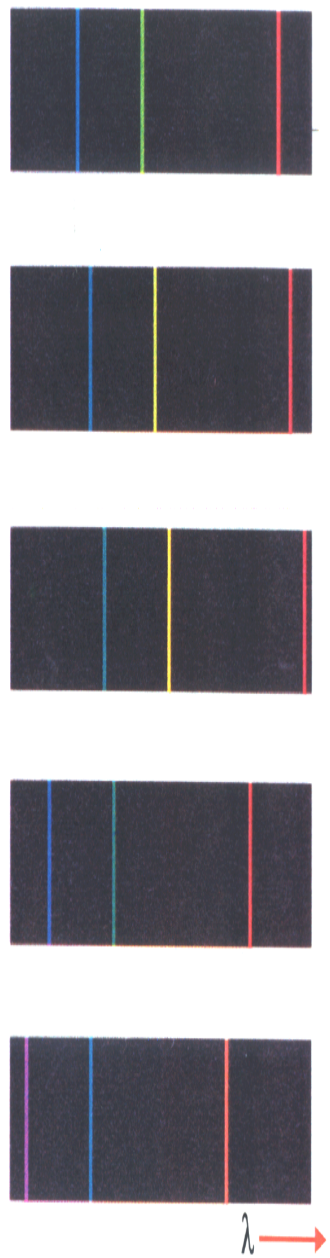
Fingerprint of Elements: Emission & Absorption Spectra



Spectral Lines and Perception of Moving Objects



Doppler Shift in Spectral Lines and Motion of Stellar Objects



Laboratory Spectrum, lines at rest wavelengths

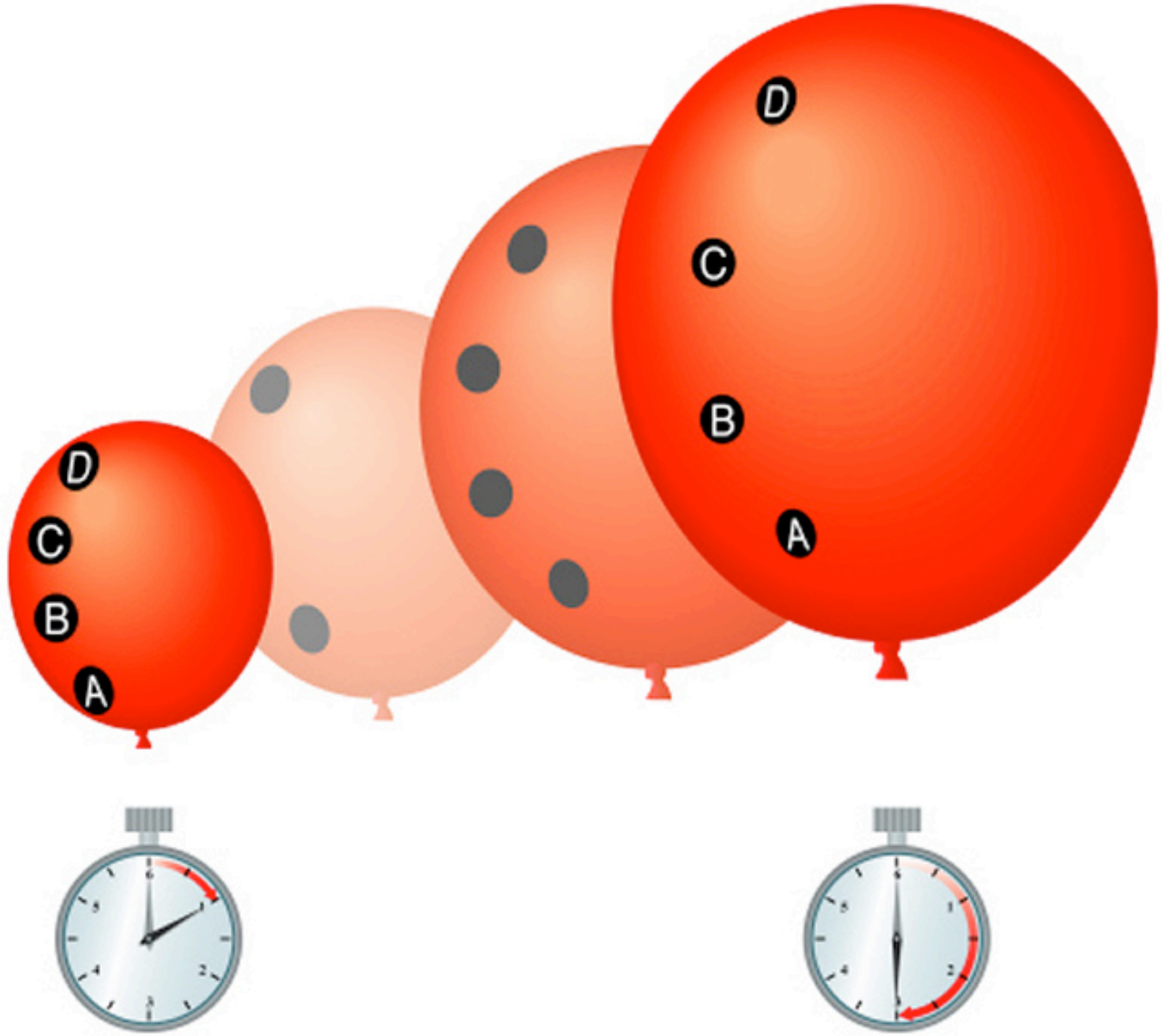
Lines Redshifted, Object moving away from me

Larger Redshift, object moving away even faster

Lines blueshifted, Object moving towards me

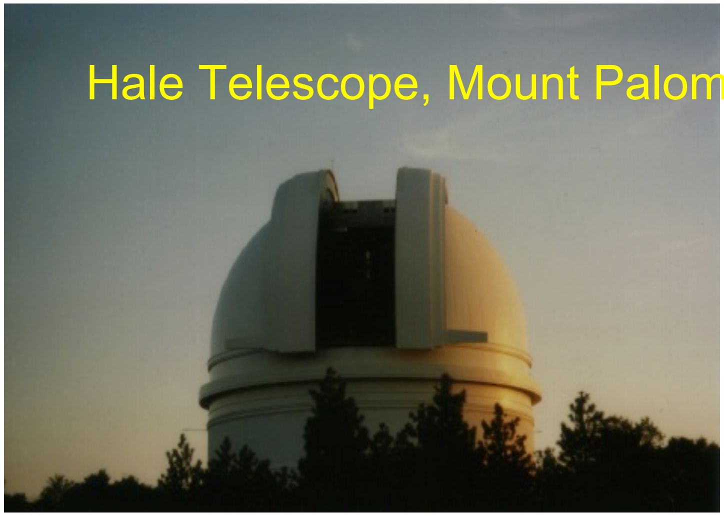
Larger blueshift, object approaching me faster

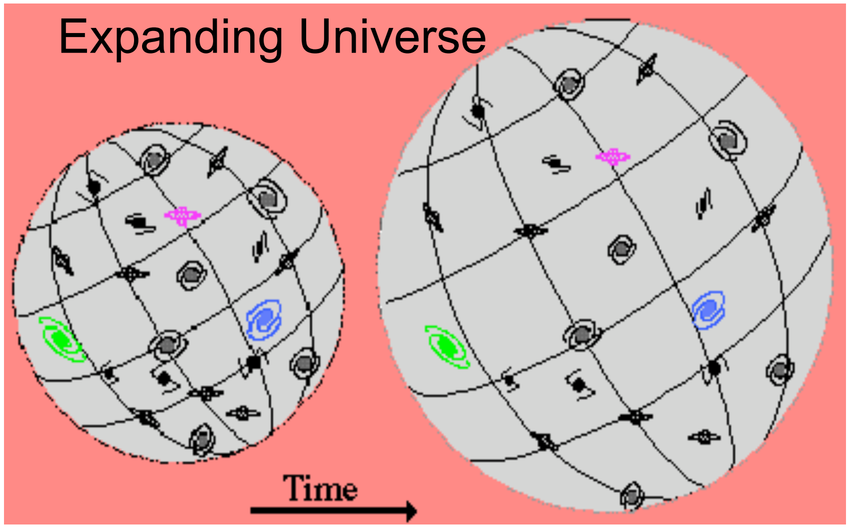
Cosmological Redshift & Discovery of the Expanding Universe:
[Space itself is Expanding]



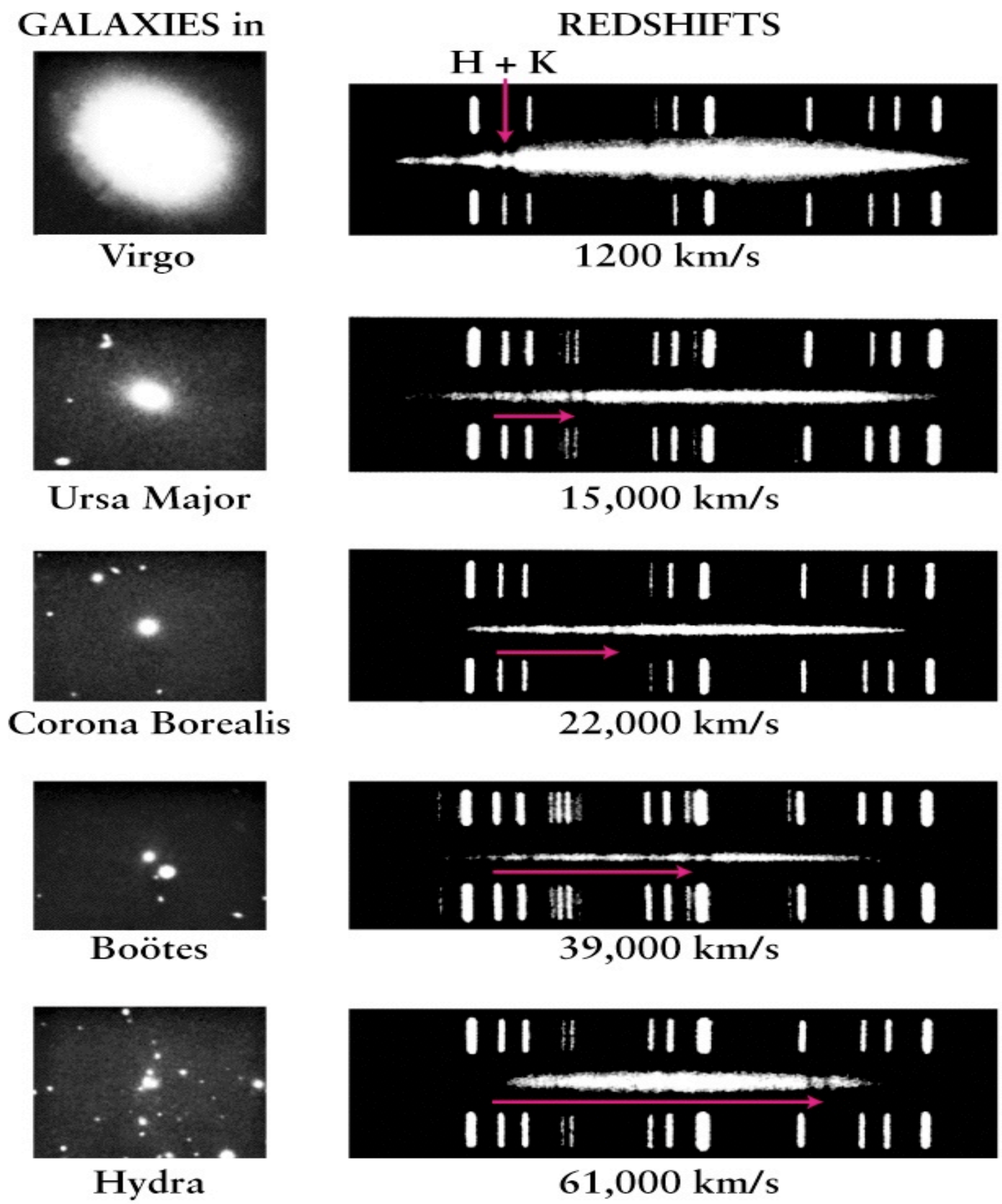
Seeing Distant Galaxies Thru Hubble Telescope

Expanding Universe, Edwin Hubble & Mount Palomar



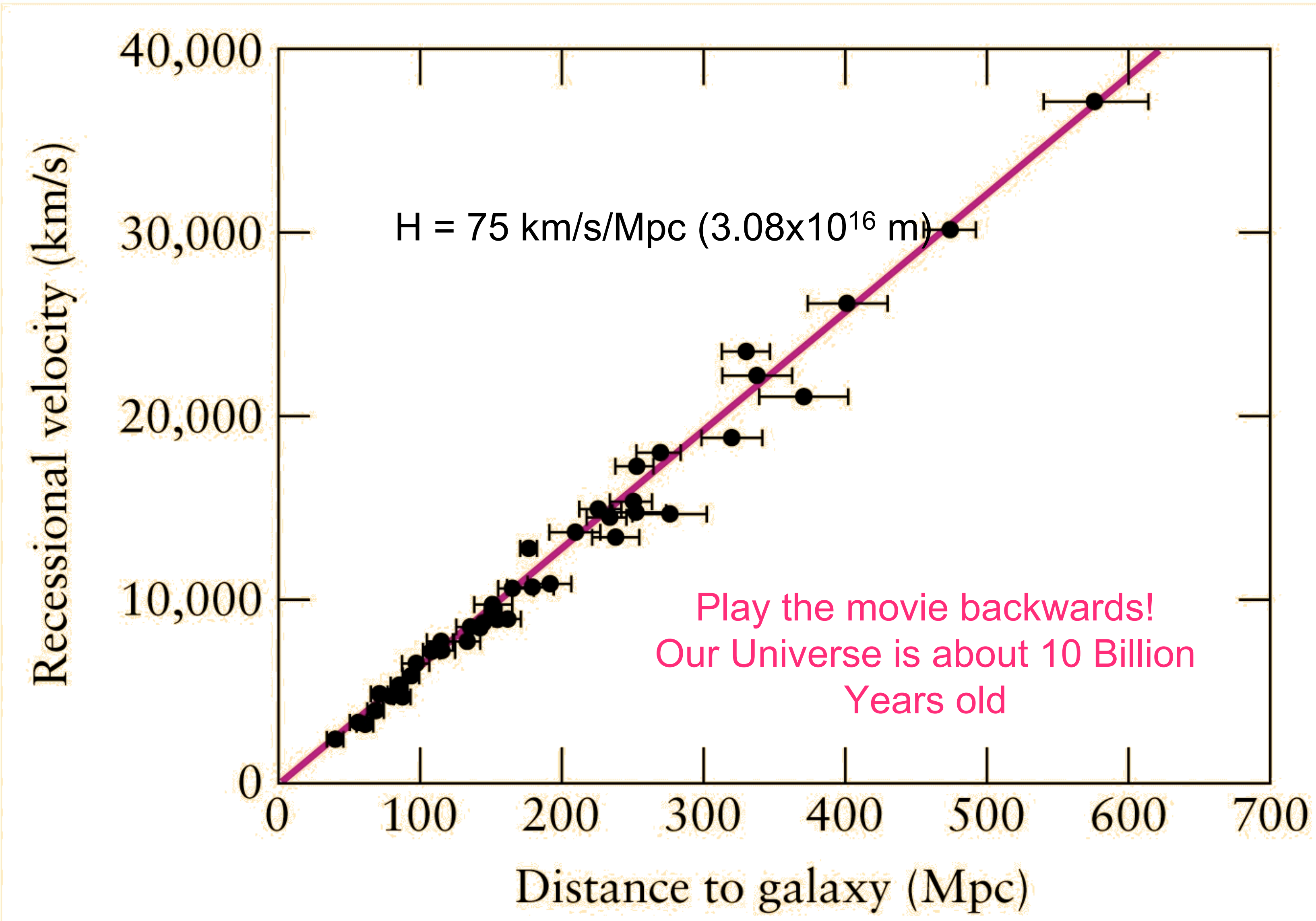


Galaxies at different locations in our Universe travel at different velocities



Hubble's Measurement of Recessional Velocity of Galaxies

V = H d : Farther things are, faster they go



New Rules of Coordinate Transformation Needed

- The Galilean/Newtonian rules of transformation could not handles frames of refs or objects traveling fast
  - $V \approx C$ (like v = 0.1 c or 0.8c or 1.0c)
- Einstein's postulates led to
  - Destruction of concept of simultaneity ($\Delta t \neq \Delta t'$)
  - Moving clocks run slower
  - Moving rods shrink
- Lets formalize this in terms of <u>general rules</u> of coordinate transformation : Lorentz Transformation
  - Recall the Galilean transformation rules
    - $x' = (x-vt)$
    - $t' = t$
      - These rules that work ok for ferraris now must be modified for rocket ships with $v \approx c$

Discovering The Correct Transformation Rule

$$x' = x - vt \qquad \text{guess} \rightarrow \quad x' = G(x - vt)$$

$$x = x' + vt' \qquad \text{guess} \rightarrow \quad x = G(x' + vt')$$

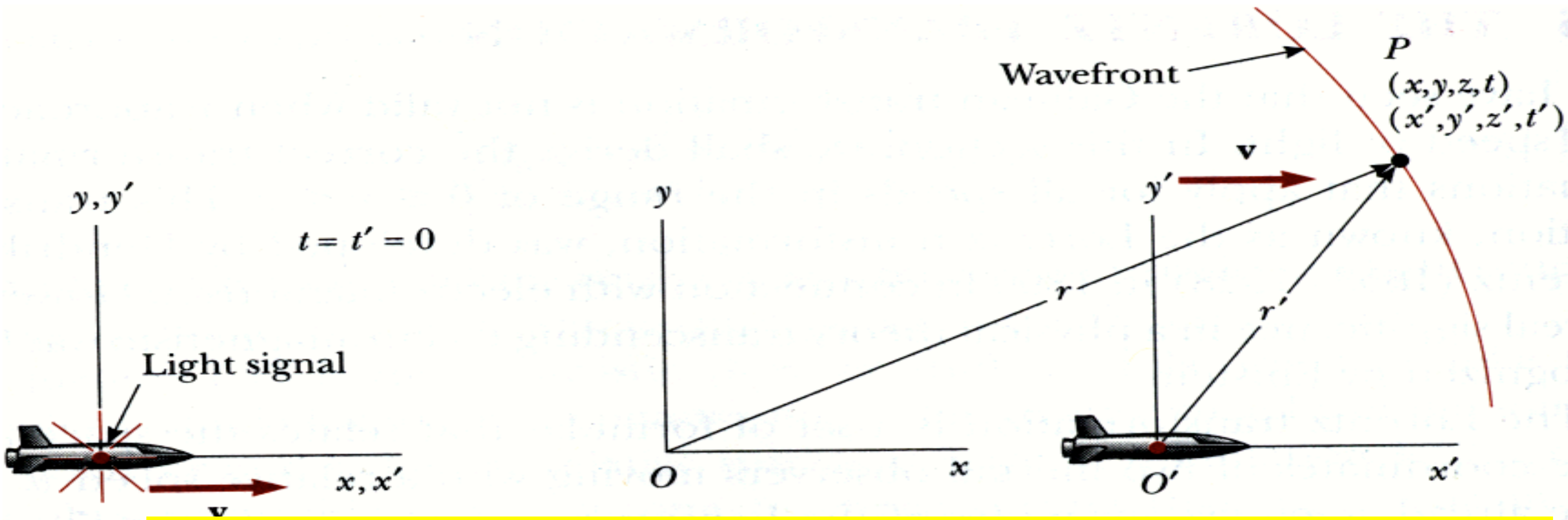
Need to figure out the functional form of G !

- G must be dimensionless
- G does not depend on x,y,z,t
- But G depends on v/c
- G must be symmetric in velocity v
- As v/c$\rightarrow$0 , G $\rightarrow$1

Guessing The Lorentz Transformation

Do a Thought Experiment : Watch Rocket Moving along x axis

Rocket in S' (x',y',z',t') frame moving with velocity v w.r.t observer on frame S (x,y,z,t)
Flashbulb mounted on rocket emits pulse of light at the instant origins of S,S' coincide
That instant corresponds to t = t' = 0 . Light travels as a spherical wave, origin is at O,O'



Speed of light is c for both observers: Postulate of SR

Examine a point P (at distance r from O and r' from O') on the Spherical Wavefront

The distance to point P from O : r = ct
The distance to point P from O : r' = ct'

Clearly t and t' must be different

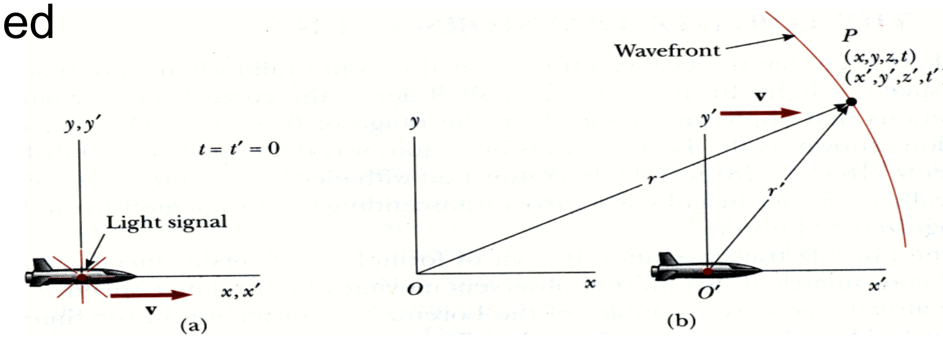
$$t \neq t'$$

Discovering Lorentz Transfromation for (x,y,z,t)

Motion is along x-x’ axis, so y, z unchanged

$$y' = y, \qquad z' = z$$

Examine points x or x’ where spherical wave
crosses the horizontal axes: x = r , x’ =r’



$$x = ct = G(x' + vt')$$

$$x' = ct' = G(x - vt) \ ,$$

$$\Rightarrow t' = \frac{G}{c}(x - vt)$$

$$\therefore \ x = ct = G(ct' + vt')$$

$$\therefore ct = G^2\left[(ct - vt) + vt - \frac{v^2}{c}t\right]$$

$$\Rightarrow c^2 = G^2[c^2 - v^2]$$

$$\text{or } G = \frac{1}{\sqrt{1 - (v/c)^2}} = \gamma$$

$$\therefore \quad x' = \gamma(x - vt)$$

$$x' = \gamma(x - vt) \ , \ x = \gamma(x' + vt')$$

$$\Rightarrow \quad x = \gamma(\gamma(x - vt) + vt')$$

$$\therefore \ x - \gamma^2 x + \gamma^2 vt = \gamma v t'$$

$$\therefore \ t' = \left[\frac{x}{\gamma v} - \frac{\gamma^2 x}{\gamma v} + \frac{\gamma^2 vt}{\gamma v}\right] = \gamma\left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right]$$

$$\therefore \ t' = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right], \ \text{since} \ \left(\frac{1}{\gamma^2} - 1\right) = -\left(\frac{v}{c}\right)^2$$

$$\Rightarrow t' = \gamma\left[t + \frac{x}{v}[1 - \left(\frac{v}{c}\right)^2 - 1\right] = \gamma\left[t - \left(\frac{vx}{c^2}\right)\right]$$

Lorentz Transformation Between Ref Frames

<u>Lorentz Transformation</u>

$x' = \gamma(x - v t)$

$y' = y$

$z' = z$

$t' = \gamma(t - v x/ c^2)$

<u>Inverse Lorentz Transformation</u>

$x = \gamma (x' + v t)$

$y = y'$

$z = z'$

$t = \gamma (t' + v x/ c^2)$

As $v \rightarrow 0$, Galilean Transformation is recovered, as per requirement

Notice : SPACE and TIME Coordinates mixed up !!!

Not just Space, Not just Time

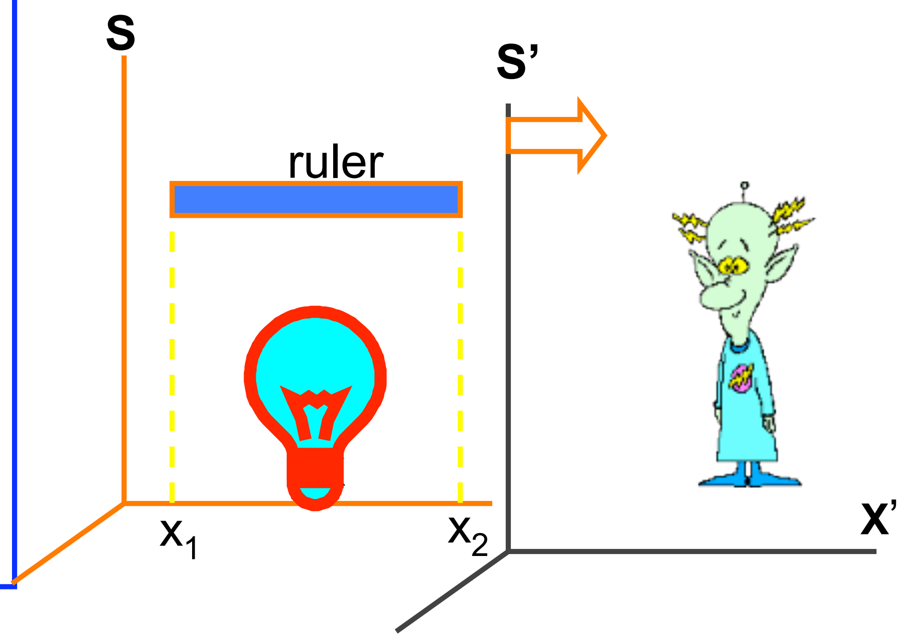
New Word, new concept !

SPACETIME

Lorentz Transform for Pair of Events

$$\left.\begin{aligned}\Delta x' &= \gamma(\Delta x - v\,\Delta t)\\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)\end{aligned}\right\} S \rightarrow S'$$

$$\left.\begin{aligned}\Delta x &= \gamma(\Delta x' + v\,\Delta t')\\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right)\end{aligned}\right\} S' \rightarrow S$$



Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at t_1 & off at t_2 : What $\Delta t'$ did S' measure ?

two events occur at same place in S frame => $\Delta x = 0$

$$\Delta t' = \gamma\,\Delta t \quad (\Delta t = \text{proper time})$$

Length Contraction: Ruler measured in S between x_1 & x_2 : What $\Delta x'$ did S' measure ?

two ends measured at same time in S' frame => $\Delta t' = 0$

$$\Delta x = \gamma\,(\Delta x' + 0\,) \Rightarrow \Delta x' = \Delta x / \gamma \qquad (\Delta x = \text{proper length})$$