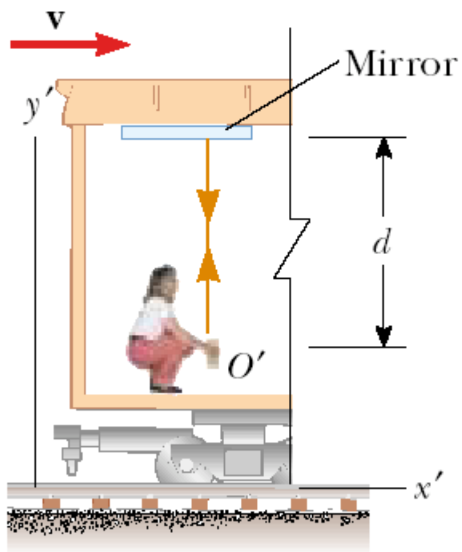
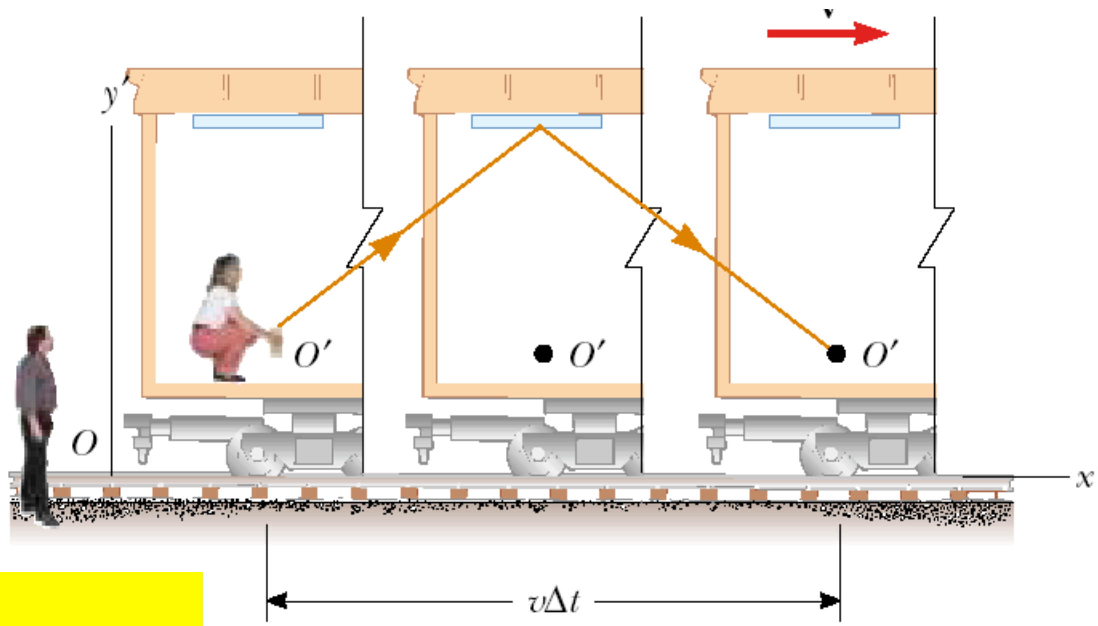
Physics 2D Lecture Slides
Lecture 3

April 1, 2009

Time Dilation and Proper Time

Watching a time interval (between 2 events) with a simple clock





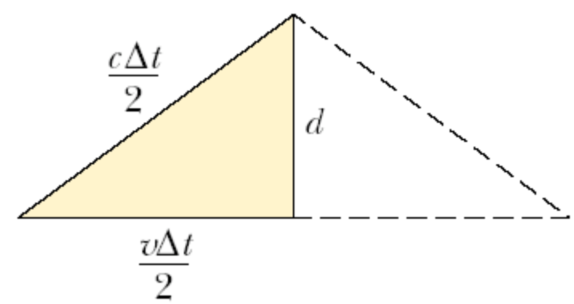
Observer O′ : $\Delta t' = \dfrac{2d}{c}$

Observer O : Apply Pythogoras Theorem

$$\left(\frac{c\Delta t}{2}\right)^2 = (d)^2 + \left(\frac{v\Delta t}{2}\right)^2, \text{ but } d = \left(\frac{c\Delta t'}{2}\right)$$

$$\therefore \quad c^2(\Delta t)^2 = c^2(\Delta t')^2 + v^2(\Delta t)^2$$

$$\therefore \quad \Delta t = \frac{\Delta t'}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma\,\Delta t', \quad \Delta t > \Delta t'$$

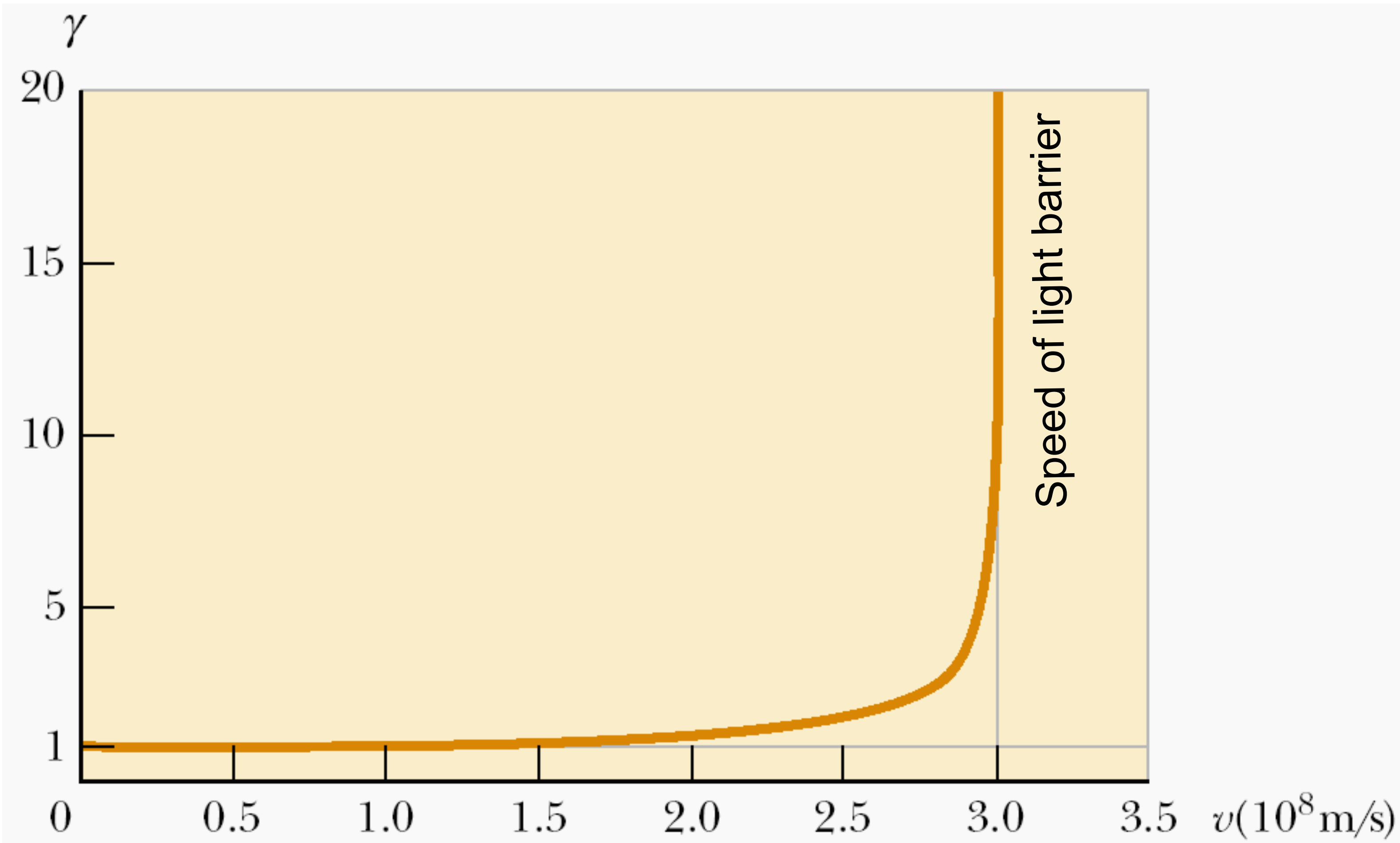


The γ factor

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

as $v \rightarrow 0, \ \gamma \rightarrow 1$

as $v \rightarrow c, \ \gamma \rightarrow \infty$



Pop Quiz !

- What happens when I reverse the clocks being watched ?
  - Sally now watches Sam’s clock
  - Sally is moving w.r.t. Sam’s clock. Sam is at rest w.r.t the clock.
  - What does she make of time intervals as measured by his clock ?

Measuring Time: Period of a Pendulum

- Period of a pendulum is 3.0 s in the rest frame of the pendulum
- What is period of the pendulum as seen by an observer moving at v=0.95c

Answer:

- Proper time T' = 3.0s
- Since motion is relative and time dilation does not distinguish between
  - relative motion →→ (V) from relative motion ← ← (-V)
- lets reformulate the problem like this (??)
  - A pendulum in a rocket is flying with velocity V =0.95c past a stationary observer
  - Moving clocks runs slower [w.r.t clock in observer's hand (rest)] by factor γ
  - ➔ Period T measured by observer = γ T'

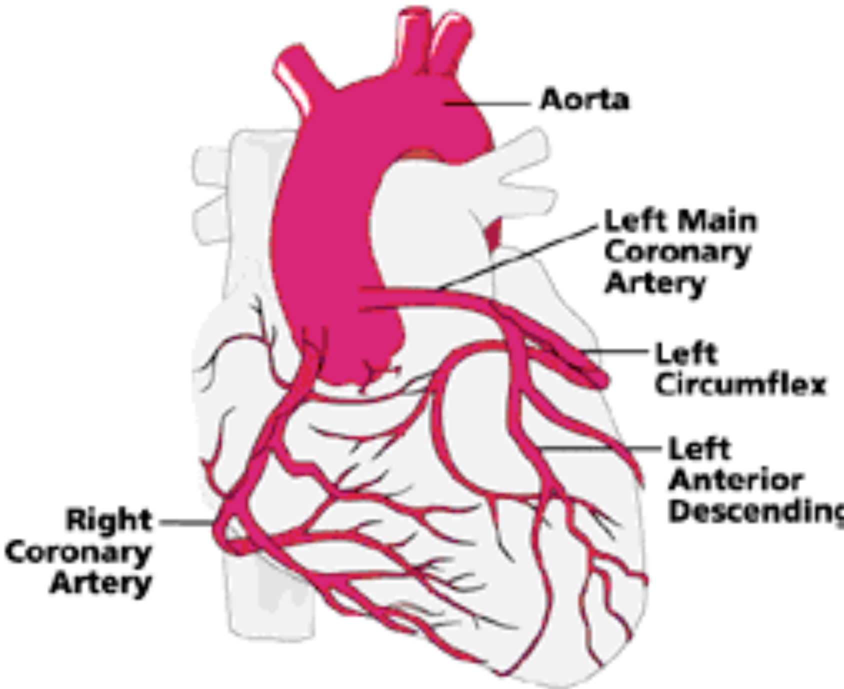
$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.95)^2}} = 3.2$$

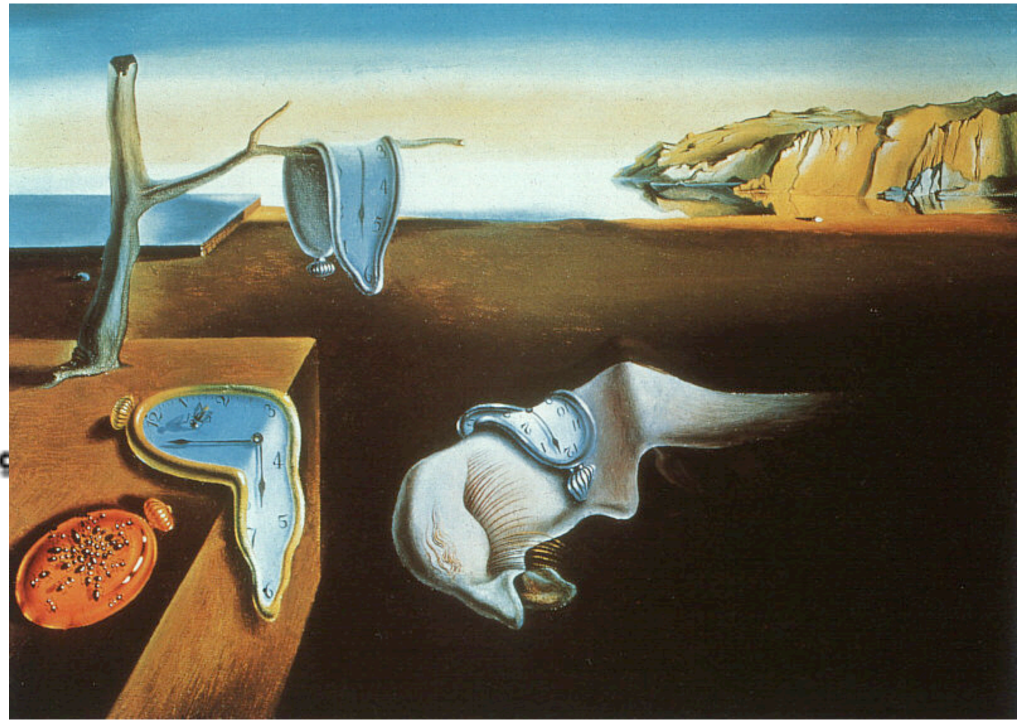
$$\Rightarrow T = \gamma T' = 3.2 \times 3.0s = 9.6s$$

Moving pendulum slows down ➔ takes longer to complete a period

All Measures of Time Slow down from a <u>Moving Observer's</u> Perspective !

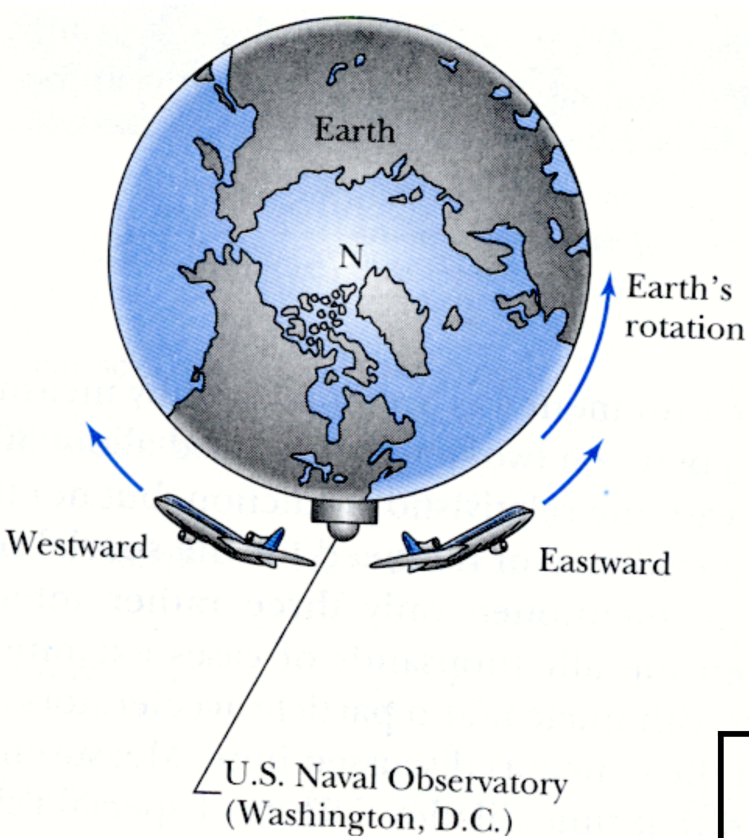
- Your heartbeat or your pulse





- Mitosis and Biological growth
- Growth of an inorganic crystal
- '...Watching the river flow''
- …all measures of time interval

Round The World With An Atomic Clock !

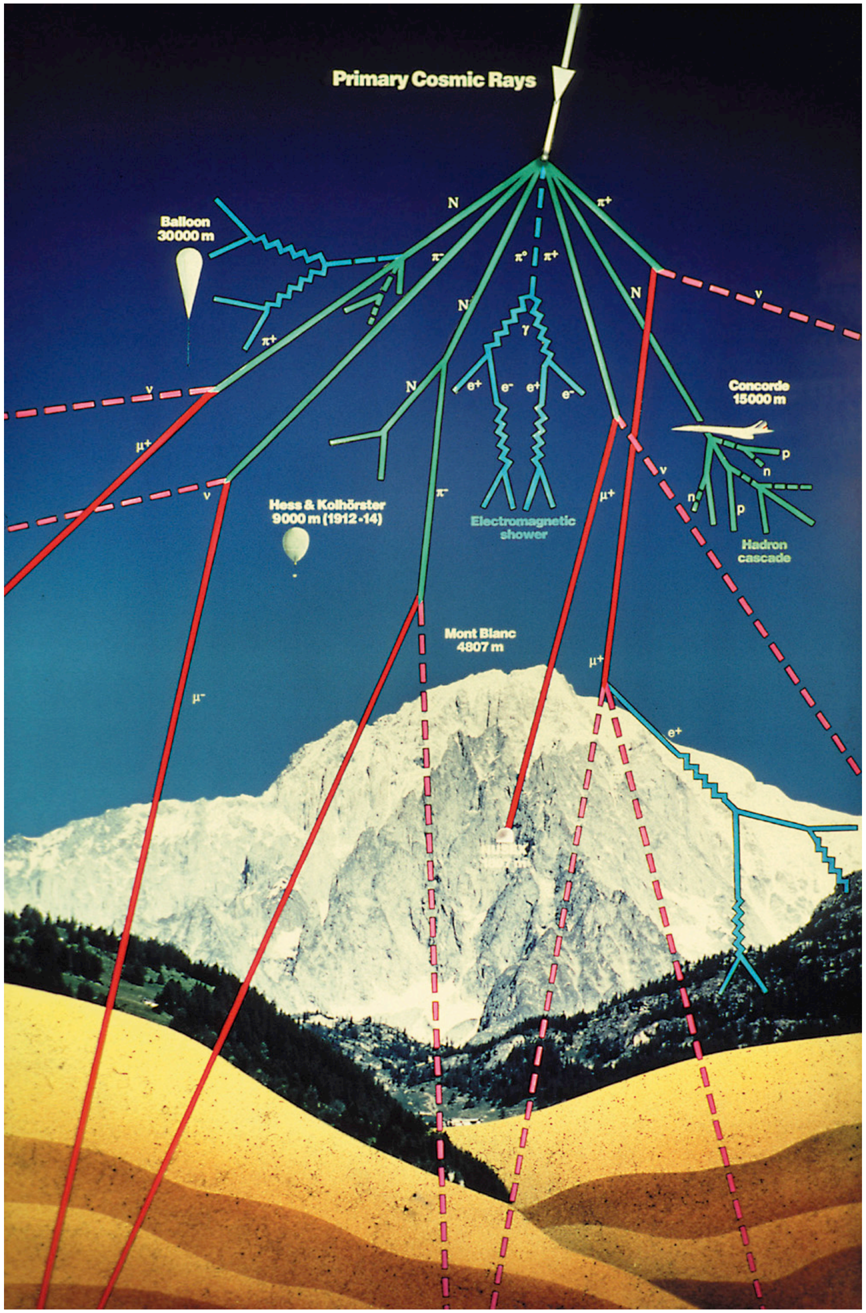


- Atomic Clock : measure time interval for certain atomic level transitions in Cesium atom
- Two planes take off from DC, travel east and west with the atomic clock
  - Eastward trip took 41.2 hrs
  - Westward trip took 48.6
- Atomic clocks compared to similar ones kept in DC
- Need to account for Earth's rotation + GR etc

Travel	Predicted	Measured
Eastward	-40 ± 23 ns	-59 ± 10 ns
Westward	275 ± 21 ns	273 ± 7 ns

Flying clock ticked faster or slower than reference clock. Slow or fast is due to Earth's rotation

Cosmic Rain !



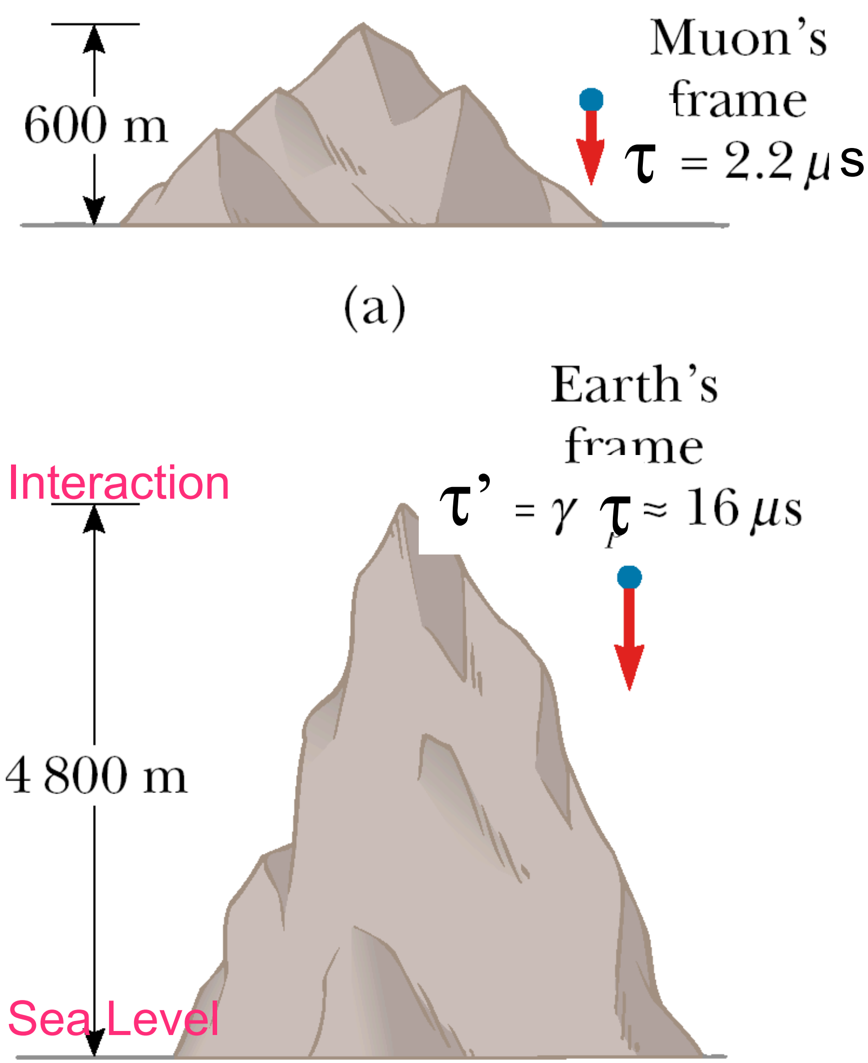
- Cosmic “rays” are messengers from space
- **Produced in violent collisions in the cosmos**
- **Typical Kinetic energy ~ 100 GeV**
- **Smash into Earth’s outer atmosphere**
  - **4700 m from sea level**
- **Sometimes produce short lived Muons (μ)**

- **Muon is electron like charged particle**
  - **~ 200 times heavier , same charge**
  - **Lifetime τ= 2.2μs = 2.2 x10^{-6} s**
  - **Produced with speed v $\equiv$ c**
  - **Distance traveled in its lifetime**

$$d = c\tau = 650m$$

  - **Yet they seem to reach the surface!!**
    - **Why => Time Dilation**
    - **Must pay attention to frames of references involved**

Cosmic Rays Are Falling On Earth : Example of Time Dilation



- Consider Two frames of references
  1. You Riding on the Muon Particle
  2. Your twin watching On surface of earth

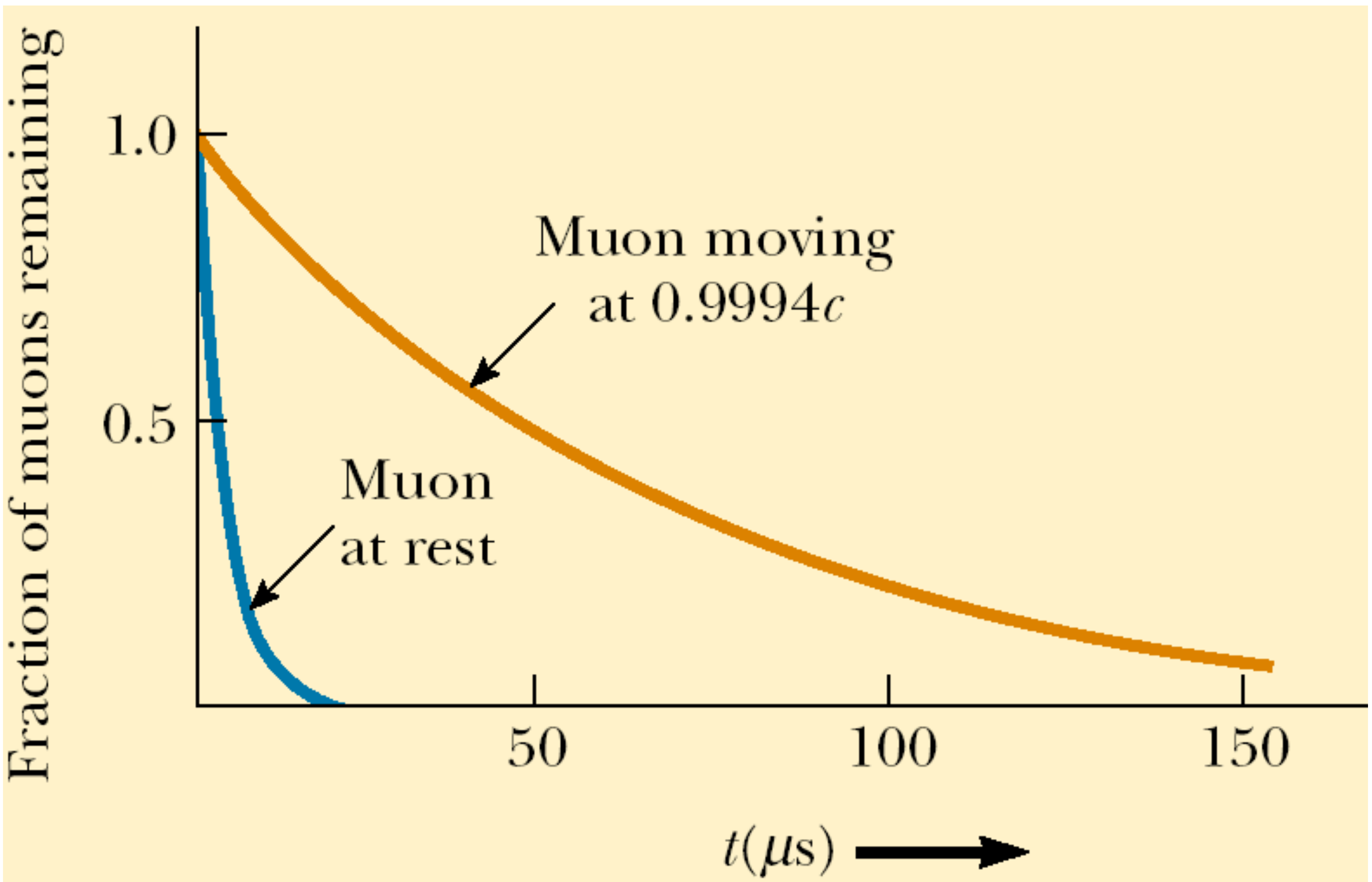
- Muon Rider has "Proper Time"
  - Time measured by observer moving along with clock
  - $\Delta t' = \tau = 2.2\,\mu S$
  - $D' = v\,\Delta t' = 650m$
- Earthling watches a moving clock (muon's) run slower
  - $\Delta t' = \gamma\tau$
    - $v = 0.99c$, => $\gamma = 7.1$
    - $D = v\,\Delta t = 4700m$

Muon Decay Distance Distribution

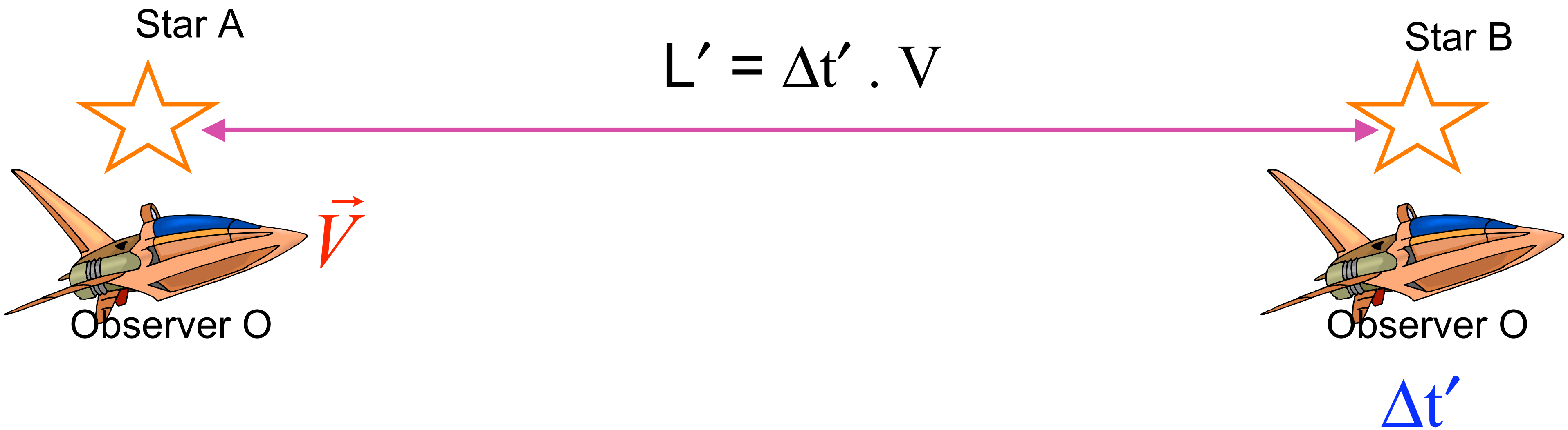
Relative to Observer on Earth Muons have a lifetime

$$t = \gamma\tau = 7.1\ \tau$$

Exponential Decay time Distribution : As in Radioactivity



Offsetting Penalty : Length Contraction



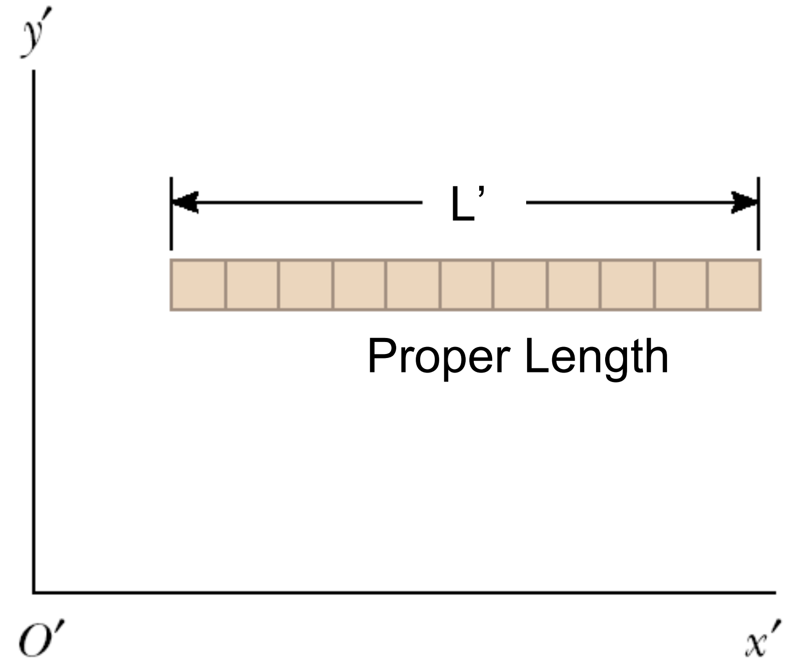
$$\Delta t = L_p/V$$

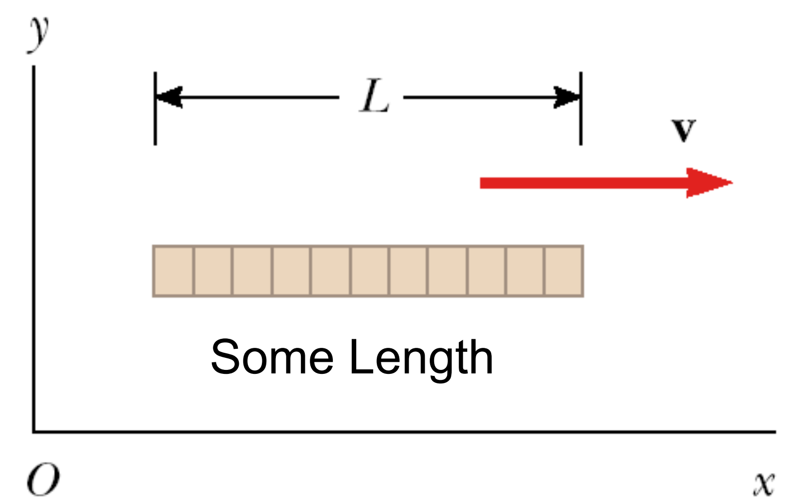
Observer O'
At rest w.r.t stars A & B
Watches rocketship cross from
Star A to Star B in time Δt

Rocketman Vs The Earthling

- Earth Observer saw rocketman take time $\Delta t = (Lp / V)$
- Rocketman says he is at rest, Star B moving towards him with speed V from right passed him by in time $\Delta t'$, so
  - $L' = \Delta t' . V$
  - But $\Delta t' = \Delta t / \gamma$ (time dilation)
  - $\Rightarrow L' = V . (\Delta t / \gamma)$

    $= Lp/\gamma = Lp\,[1 - v^2/c^2]^{1/2}$



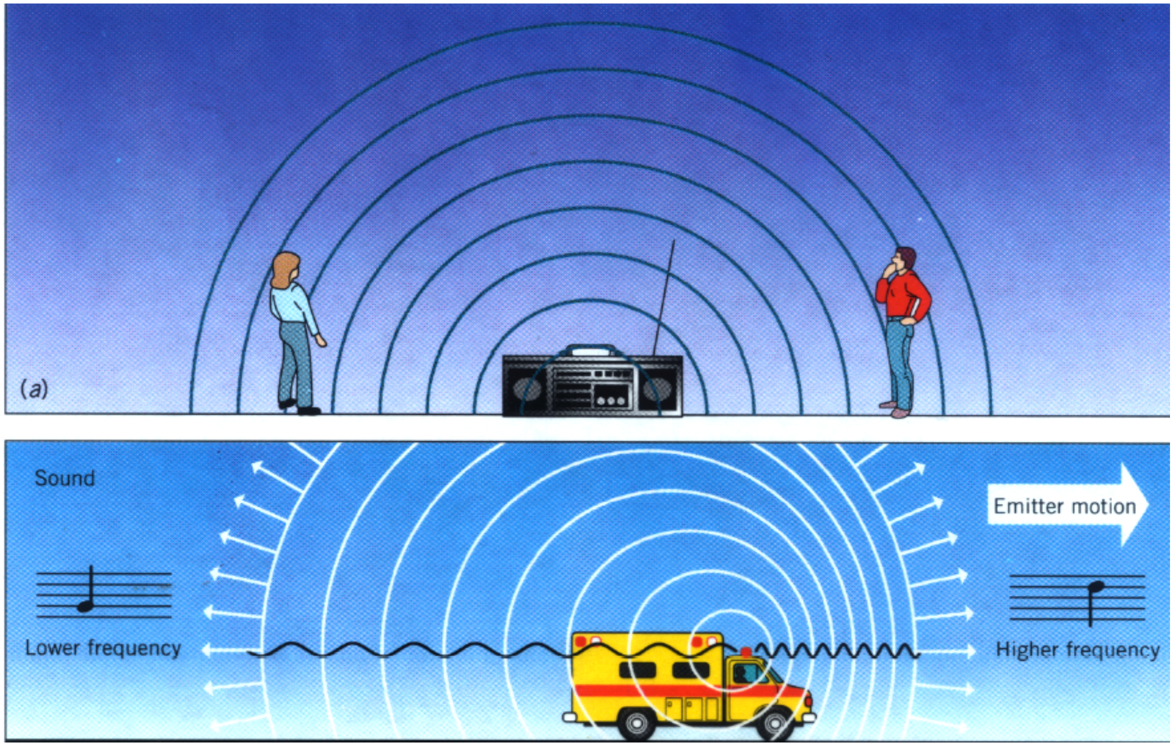


Moving Rods Contract in direction Of relative motion

Immediate Consequences of Einstein’s Postulates: Recap

- Events that are simultaneous for one Observer are not simultaneous for another Observer in relative motion
- Time Dilation : Clocks in motion relative to an Observer appear to slow down by factor γ
- Length Contraction : Lengths of Objects in motion appear to be contracted in the direction of motion by factor γ^{-1}
- New Definitions :
  - Proper Time (who measures this ?)
  - Proper Length (who measures this ?)
  - Different clocks for different folks !

Doppler Effect In Sound : Reminder from 2C

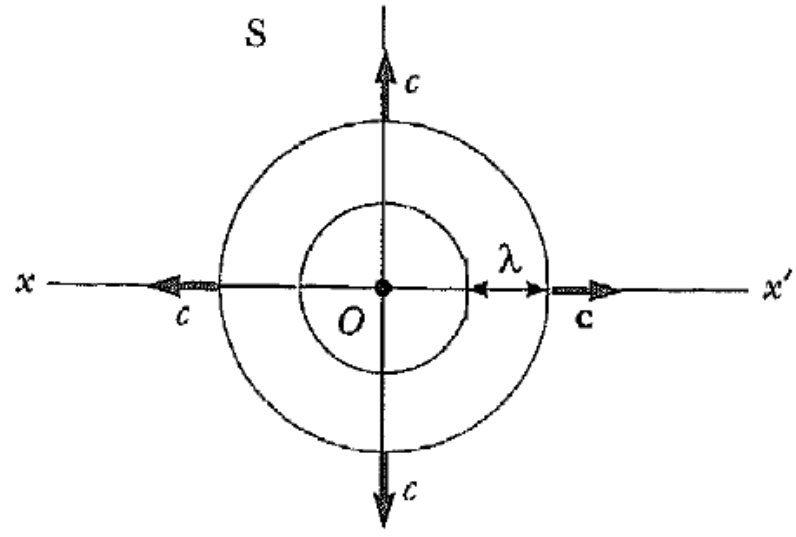


Observed Frequency of sound INCREASES if emitter moves towards the Observer
Observed Wavelength of sound DECREASES if emitter moves towards the Observer

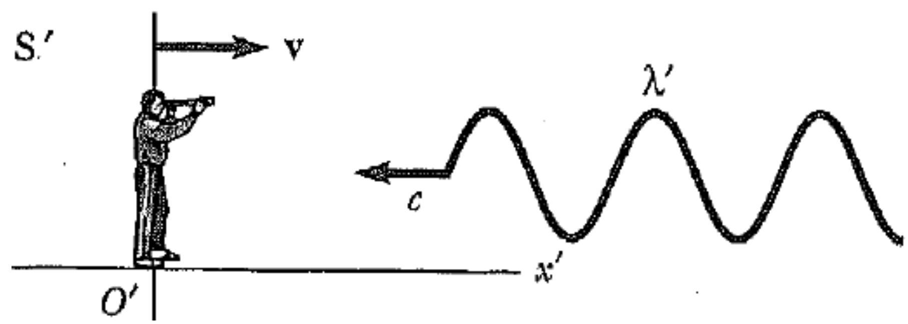
$$v = f \lambda$$

Time Dilation Example: Relativistic Doppler Shift

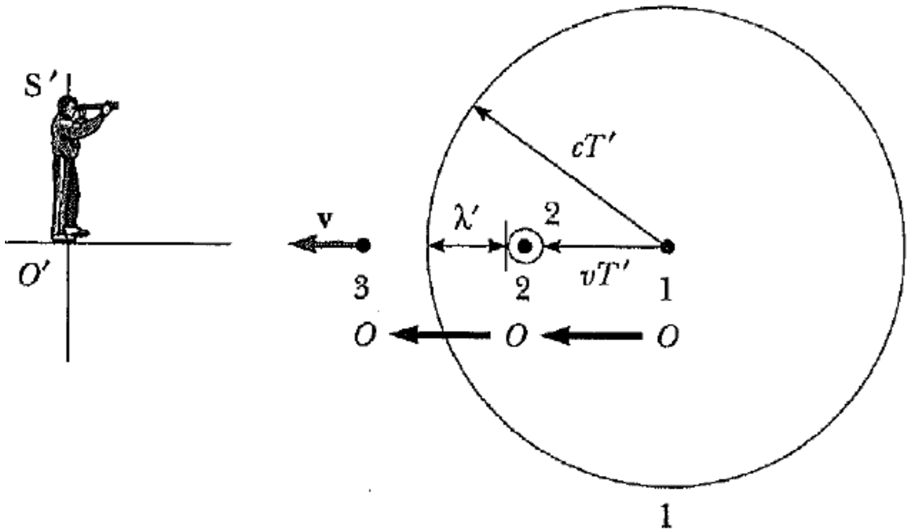
- Light : velocity c = f λ, f=1/T
- A source of light S at rest
- Observer S′approches S with velocity v
- S′ measures f′ or λ′, c = f′λ′
- Expect f′ > f since more wave crests are being crossed by Observer S’due to its approach direction than if it were at rest w.r.t source S



(a)



Relativistic Doppler Shift



Examine two successive wavefronts emitted by S at location 1 and 2

In S' frame, T' = time between two wavefronts

In time T', the wavefront moves by cT' w.r.t 1

Meanwhile Light Source moves a distance vT'

Distance between successive wavefront

$\lambda' = cT' - vT'$

use $f = c/\lambda$

$$f' = \frac{c}{(c-v)T'}, \quad T' = \frac{T}{\sqrt{1-(v/c)^2}}$$

Substituting for T', use f=1/T

$$\Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)} f$$

$$\Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f$$

better remembered as:

$$f_{obs} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} f_{source}$$

f_{obs} = Freq measured by observer approching light source