PHYSICS 227: Cosmology

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Office Hours: Fri. 10-12
Homework no. 1
Due: Thurs. April 22

1

In class we computed radial timelike geodesics in a FRW metric.

(a) Do the same for lightlike geodesics.

(b) Assuming that the photon energy $h\nu = U_a p^a$, where U_a is the four velocity of a fundamental observer in the FRW metric and p^a is the 4 momentum of the photon, use your solution in (a) to show how the redshift depends on scale factor $a(t)$.

2

In class we showed that the peculiar velocity with respect to a fundamental observer, $U^\alpha \propto 1/a(t)$, where in this case U^α are the spatial components of the time-like 4 velocity of a galaxy with respect to a fundamental observer. Show that the corresponding physical observable, the 3-velocity v^α ($\equiv dx^\alpha/dt$), does not exceed the speed of light as $a(t) \rightarrow 0$, but rather saturates at a well determined value. Hint: make use of the relationship between 4-velocity and 3-velocity.

3

(a) Use coordinate transformation matrix methods to prove that in the case of homogeneous isotropic flow the functional form of the Hubble law $\mathbf{u} = H\mathbf{x}$ is invariant under rigid rotations and translations of coordinate systems.

(b) Now consider the more general case in which $u_\alpha = H_{\alpha\beta} x^\beta$ and the Hubble tensor $H_{\alpha,\beta}$ has off diagonal elements. Is the functional form of this flow invariant under the either group of coordinate transformations described in (a)?

4

(a) Compute the normalized scale factor $x \equiv a(t)/a_0$ as a function of $t - t_0$ from $x = 0$ to $x \leq 2.0$ for FRW models with $\Omega_\Lambda = 0$, $\Omega_R = 2.55 \times 10^{-5}/h^2$,and $\Omega_{\rm M}(0) = 0.2, 0.6, 1.0, 2.0, 4.0$. Here t_0 is current cosmic time where $((1/a)da/dt)_0 = H_0 = 70$ km s^{-1}Mpc^{-1}.

(b) Repeat part (a), but now let $\Omega_\Lambda + \Omega_{\rm M} + \Omega_R = 1$, and do the calculation for $\Omega_{\rm M}(0) = 0.2, 0.4, 0.6, 0.8$.

5

In class we discussed density parameters derived at the current time. We can also define density parameters at cosmic time t as $\Omega_i(t) = \rho_i(t)/\rho_{crit}(t)$ where $\rho_{crit}(t) = 3H^2(t)/(8\pi G)$ and $\rho_i(t)$ correspond to matter, radiation, and vacuum energy densities. We may also define the curvature term as $\Omega_K(t) = -c^2 K/(H(t)a(t))^2$.

(a) Derive the relationship

$$\Omega_\Lambda + \Omega_M(t) + \Omega_R(t) + \Omega_K(t) = 1$$

(b) Plot each of the $\Omega_i(t)$ as a function of redshift for the case that the present values $\Omega_K = 0$, $\Omega_\Lambda = 0.7$, $\Omega_R = 2.44 \times 10^{-5}/h^2$, and $h = 0.7$.

(c) At what redshifts are $\Omega_M(t) = \Omega_R(t)$, and $\Omega_\Lambda = \Omega_M(t)$?