Cosmic Background Radiation

This isotropic background radiation, which is a perfect black body, has current temperature $T_0 = 2.728 \pm .004$ K

Discovered by Penzias and Wilson in 1965 its discovery is comparable in importance to Hubble's earlier finding that the Universe is expanding.

Why is CMB so important? Because CMB provides us with unmistakable messages:

(A) Universe in remote past different than current Universe

(i) Past Universe was both hotter and denser than current Universe

(ii) Universe was much smoother, or less lumpy, than current Universe

(iii) Past singularity or "moment" of creation?

Reasoning: Perfect BB spectrum suggests photons relaxed to thermodynamic equilibrium via interactions with matter.

- Present Universe – this could not happen today ($z=0$) ~~because~~ or even back to $z \sim 10$, because at these "recent" epochs Universe is optically thin to mm radiation (BB peak λ)

  Evidence? –

  we can detect discrete high-z

sources at submm and mm wavelengths (quasars, Scuba sources, etc.) implies such photons propagate freely without interaction between high z and $z=0$ i.e., across distances $c/H_0 = 3000/h$ Mpc; i.e., close to horizon scales.

- Only modification is cosmic expansion, which is we shall see causes T to decrease with time.
- Signature: BB radiation signifies a hot, dense, past Universe

(B) CMB is remarkably isotropic

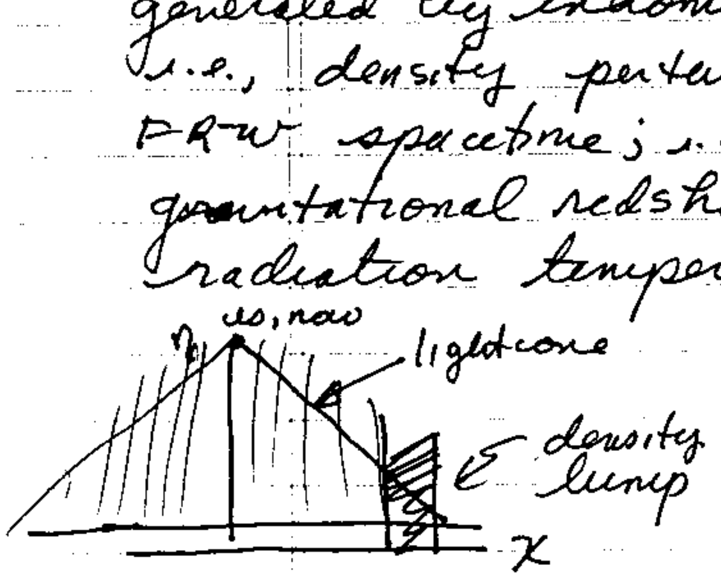
Lg. scale Dipole: Principal angular variation is due to the fact that we are not a FO. As a result we have a peculiar velocity with respect to the local FO. This generates a $\cos\theta$ variation with amplitude $\delta T/T \sim 10^{-3}$: FO sees perfect isotropy. $\delta T/T \equiv (T(\theta)-\langle T\rangle)/\langle T\rangle$.

Locally Induced

Small Scale: On smaller angular scales there are angular variations generated by gravitational effects that density inhomogeneities have on lightlike geodesics. Departures from FRW metric causes $\delta T/T \sim 10^{-5}$. These give precise information about cosmological parameters.

Cosmologically induced

SW effect: Principal cause is called Sachs-Wolfe effect: variations in grav. potential generated by inhomogeneities in mass distribution; i.e., density perturbation superposed on smooth FRW spacetime; i.e., specifically additional gravitational redshifts cause net dips in radiation temperature. Because $\delta T/T \lesssim 10^{-5}$ in relevant scales, we shall see that the implication is that matter on such



scales described by $\delta\rho/\rho \sim 10^{-5}$. Universe very smooth back then even though today it is quite lumpy. superhorizon scales relevant $\Rightarrow$ On smaller scales; i.e., compression and rarefaction "sound" waves cause peaks & troughs in density distribution, which ultimately lead to further variations in $T(\theta)$. ~~#~~ these are readily interpreted and by making use of the angular power spectrum (~~decomp~~ decomposing $\delta T/T$ into sums of spherical harmonics), measuring ~~amplitude~~ amplitudes of higher, i.e. l values of Y_l^m we obtain cosmological information.

(c) History. CMB predicted by George Gamow and his colleagues (Alpher & Herman). Remarkable that in common with Hubble expansion, the existence of

CMB, more specifically a hot big bang Universe, was predicted before its discovery, and even more remarkable in both cases by isolated Russian physicists.

(D) Significance

(i) Steady - State Universe: Before 1965 there were 2 competing cosmological models: Big-bang and Steady State -
Unique prediction of steady state: Universe has no beginning and no end. There is no hot dense phase in the past that is required to thermalize radiation. Detection of CMB caused all, but a few, astrophysicists to abandon Steady State. However, when we discuss inflation we will see that modern version of big bang incorporates a central idea of the steady state, namely a vacuum energy density with negative pressure resulting in $\ddot{a} > 0$

(2) Light-Element Synthesis:

Thermal history implies that Universe expanded and cooled through epoch in which:

- $kT \sim$ few MeV and $n > 10^8\ cm^{-3}$
- copious free neutrons

straightforward calculations that we will do show

- 90% of all baryons wind up as $_1H^1$ (75% by mass)
- 10% of all baryons " " as $_2He^4$ (25% by mass)

We shall see: there is a fundamental connection between $_2He^4$, $_1H^2$ abundance and CMB temperature.

$_1H^2$ = D

(E) Idea: (1) Brief summary of Big Bang Scenario

Far back in time, $z \gg 10^3$, long before galaxies, quasars, etc. formed, Universe closely resembled idealized FRW models. Then Universe consisted of smooth distribution of –

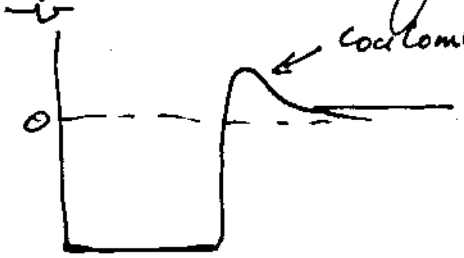
(a) matter (b) radiation

Because interaction time-scale (i.e. relaxation time) between baryonic matter and radiation was shorter than the then current age of the Universe, at a given redshift $z(t)$, radiation came into thermodynamic equilibrium with matter: stated differently, distributions lost memory of initial conditions (?), and in that case:

all distribution function relaxed to their equilibrium form:

(ii) Hot Big Bang

Gamow assumed T to be initially high. Reasoning – high temperature required for charged baryons to undergo quantum tunneling through Coulomb barriers and then undergo nuclear fusion



Of course today we know this is how fusion of ~~th~~ elements occurs in hot stellar interiors. But in the 1940s little was known about physical conditions at the centers of stars. For these reasons "Fireball" scenario as a site for fusion He, C, O, ... seemed like a plausible way to make all the heavy elements: In fact Big bang makes no element heavier than ^{7}Li

(iii) Cold Big Bang

Keep in mind there is no a priori reason why initial T had to be high: indeed Zeldovich and several other eminent theorists argued for a "cold" big bang,

(iv) Entropy

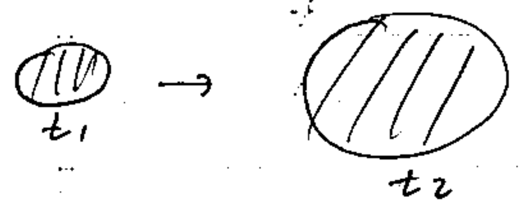
High initial $T \Rightarrow$ large entropy associated with radiation field. We shall see that as the Universe expands the entropy per baryon is conserved. If treated as a free parameter it is fixed by the He/H ratio. But there are models to explain its origin.

Temperature Evolution

assume, as I will later prove, that entropy of radiation per unit comoving volume = const.

1st Law of Thermodynamics:

Consider an expanding sphere of radiation and matter



1st Law of thermodynamics: $\frac{dU}{dt} = T\frac{dS}{dt} - P\frac{dV}{dt}$

U = internal energy
S = entropy
P = pressure
V = volume

of sphere

~~Since S = const, expansion is adiabatic~~
~~$\frac{dU}{dt} = -PdV$~~

~~Source of Pressure~~

~~Photons: $P_\gamma = \frac{1}{3} a_B T^4$; $a_B = 7.56\times10^{-15}$ erg cm^{-3} K^{-4}~~
~~Gas: $P_g = \frac{R}{\mu} \rho T$; $R = \frac{k}{m_H} = 8.3\times10^{7}$~~

In thermodynamic equilibrium, the entropy and number of baryons in a comoving volume are conserved. Thus the entropy per baryon is conserved.

Let $s \equiv k_B \sigma$ be entropy per baryon!

1st Law: $\boxed{T ds = d(\epsilon/n_B) + P d\left(\frac{1}{n_B}\right)}$

Here: n_B = # baryons / cm^3 $\Rightarrow$ $1/n_B$ = Vol per baryon

$\epsilon = \frac{U}{V}$ = internal energy density

In that case:

$$\epsilon = a_B T^4 + \frac{3}{2} n_B \mathcal{N} k_B T \quad (\mathcal{N} = \text{\# non. rel particles/bary})$$

$$P = \frac{1}{3} a_B T^4 + n_B \mathcal{N} k_B T$$

Therefore:

$$T ds = d\left[\frac{a_B T^4 + \frac{3}{2} n_B \mathcal{N} k_B T}{n_B}\right] + \left(\frac{1}{3} a_B T^4 + n_B \mathcal{N} k_B T\right)\left(\frac{-dn_B}{n_B^2}\right)$$

$$T ds = d\left(\frac{a_B T^4}{n_B}\right) + d\left(\frac{3}{2} \mathcal{N} k_B T\right) - \left(\frac{1}{3} a_B T^4\right)\frac{dn_B}{n_B^2} - \mathcal{N} k_B T \frac{dn_B}{n_B}$$

Collect terms -

$$T ds = a_B \left[\frac{4T^3 dT}{n_B} - \frac{T^4 dn_B}{n_B^2} - \frac{1}{3} T^4 \frac{dn_B}{n_B^2}\right] + \mathcal{N} k_B \left(\frac{3}{2} dT - T \frac{dn_B}{n_B}\right)$$

$$ds = a_B \left(\frac{4T^2 dT}{n_B} - \frac{4}{3} T^3 \frac{dn_B}{n_B^2}\right) + \mathcal{N} k_B \left(\frac{3}{2} \frac{dT}{T} - \frac{dn_B}{n_B}\right)$$

$$\therefore \quad ds = d\left[\frac{4}{3}\frac{a_B T^3}{n_B}\right] + Nk_B\left[d\ln\left(\frac{T^{3/2}}{n_B}\right)\right]$$

$$\boxed{s = \frac{4}{3}\frac{a_B T^3}{n_B} + Nk_B \ln\left(\frac{T^{3/2}}{n_B C}\right)} \quad : C = \text{int. const.} \quad (1)$$

~~Compare terms~~

Dimensionless entropy $\boxed{\sigma = \frac{4}{3}\frac{a_B T^3}{k_B n_B} + N \ln\left(\frac{T^{3/2}}{n_B \cdot C}\right)}$ (2)

Formulae: $I_\nu = \dfrac{2h\nu^3/c^2}{e^{\frac{h\nu}{kT}} - 1}$: intensity per bandwidth

$$u_\nu = \frac{4\pi I_\nu}{c} = \frac{8\pi h\nu^3/c^3}{e^{\frac{h\nu}{kT}} - 1}$$: energy density per bandwidth

$$n_\nu = \frac{u_\nu}{h\nu} = \frac{8\pi \nu^2/c^3}{e^{\frac{h\nu}{kT}} - 1}$$ number density per bandwidth

$$n_\gamma(t_0) = \int_0^\infty n_\nu \, d\nu = \frac{0.37 \times a_B T^3}{k_B} \approx 410 \text{ photons cm}^{-3}$$

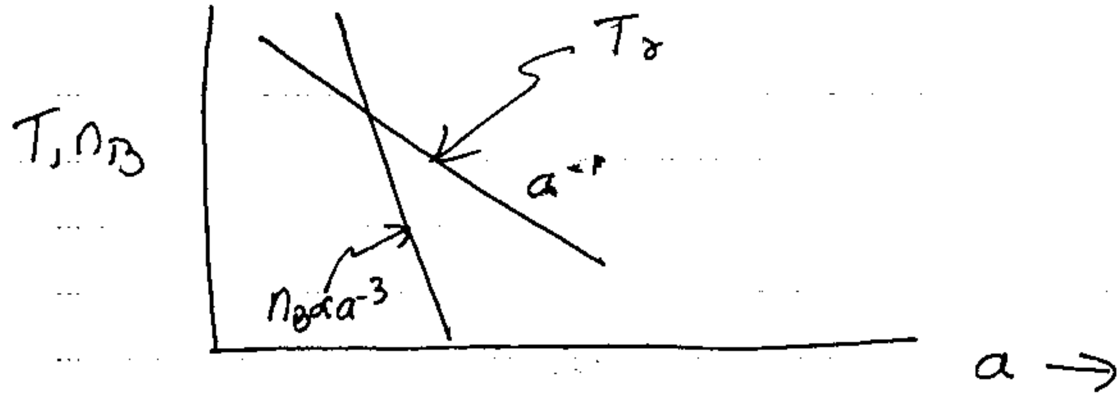
$$n_B(t_0) = \Omega_b \rho_{crit} = 1.1 \times 10^{-5} \underbrace{\Omega_b h^2}_{.02} \approx 2 \times 10^{-7}$$

1^{st} term in eq. (2) of order $\approx \dfrac{1.3 \times 10^8}{h^2 \Omega_B}$ swamps 2^{nd} term

$$\therefore \quad \sigma = \frac{4}{3}\frac{a_B T^3}{k_B n_B} = \text{entropy per baryon!}$$

Since $n_B \propto a^{-3} \Rightarrow a^3 T^3 = \text{const}$

or $\boxed{T \propto \frac{1}{a}}$



Comparison of energy density of relativistic particles (mainly photons and neutrinos) and non-relativistic matter

Today: $\rho_M = \Omega_M \rho_{crit}$ Matter

$\rho_\gamma = a_B T_\gamma^4(0) = 4.6 \times 10^{-34}\ \text{gm}^{-3}$ Radiation

$\therefore \Omega_\gamma = \frac{\rho_\gamma}{\rho_{crit}} = 2.47 \times 10^{-5} h^{-2}$

If we include 3 neutrino families (e^-, μ, τ)

$\rho_R(0) = \left[1 + 3\left(\frac{7}{8}\right)\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma(0) = 7.8 \times 10^{-34}$

$\Rightarrow \Omega_R = \frac{\rho_R(0)}{\rho_{crit}} = 4.15 \times 10^{-5} h^{-2}$

So Today $\frac{\rho_R(0)}{\rho_M(0)} = \frac{\Omega_R}{\Omega_M} = \frac{4.15\times10^{-5}h^2}{\Omega_m} \ll 1$

But $\rho_M \propto a^{-3}$ and $\rho_R \propto T^4 \propto a^{-4}$

$\therefore\ \rho_R/\rho_M \propto \frac{a^{-4}}{a^{-3}} \propto a^{-1}$

Back in time $\rho_R = \rho_M$: epoch of matter and radiation equality.

$$\therefore\ \frac{(\rho_R/\rho_M)_a}{(\rho_R/\rho_M)_{a_0}} = \left(\frac{a_0}{a}\right)$$

$$\Rightarrow\ 1+z_{eq} = 1/(\rho_R/\rho_M)_{a_0} = \frac{\Omega_m}{4.15\times10^{-5}h^2}$$

or $1+z_{eq} = \frac{2.41\times10^4\,h^2}{\Omega_m} = \frac{2.41\times10^4(.7)^2}{.3}$

$$\boxed{1+z_{eq} = 3.9\times10^4}$$

This is an important epoch since, growth of clustering (i.e., density) of non-relativistic matter is suppressed when $\rho_R > \rho_M$!

Recap: Evolution of hot big-bang Universe

(1) 1^{st} Law of thermodynamics:

$$T_\gamma(a) \propto a^{-1} \quad \text{for radiation}$$

(2) Since matter strongly coupled to radiation (e⁻ scattering)

$$T_m(a) = T_\gamma(a)$$

Matter relaxes to radiation temperature because radiation has higher heat capacity

$$C_m = \left(\frac{\partial \epsilon_m}{\partial T}\right)_V = \frac{\partial}{\partial T}\left(\frac{3}{2} n_B k_B T\right) = \frac{3}{2} n_B k_B$$

$$C_\gamma = \left(\frac{\partial \epsilon_\gamma}{\partial T}\right)_V = \frac{\partial}{\partial T}\left(a_B T^4\right) = 4 a_B T^3$$

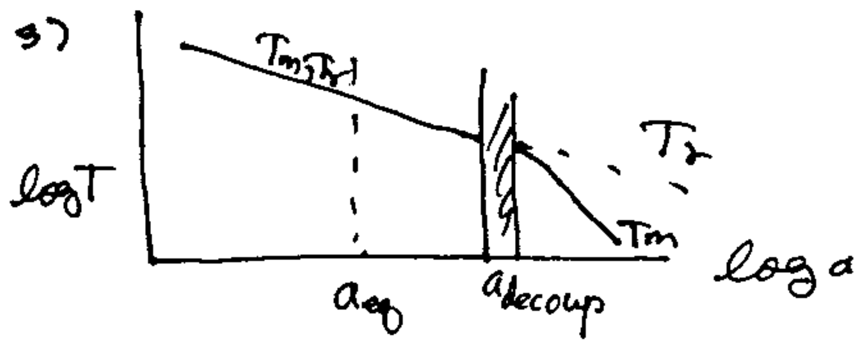
$$\frac{C_m}{C_\gamma} = \frac{\frac{3}{2} n_B k_B}{4 a_B T^3} \simeq \frac{1}{2}\sigma^{-1} \;;\; \sigma = \text{entropy/baryon}$$

$$\simeq 10^{-10}$$

In TE, radiation maintains BB spectrum despite heat flow to matter:

$$dQ = C_\gamma dT \Rightarrow dT = \frac{dQ}{C_\gamma} \ll 1 \left(\begin{matrix}\text{since} \\ C_\gamma \text{ so lg.}\end{matrix}\right)$$

(3)



(4) Matter Temperature: $a > a_{dec}$ non-rel.

Recall peculiar motions of particles wrt FO

$$v(t) \propto \frac{1}{a(t)}$$

(ii) Prior to decoupling : e^-, H^+, H^0 described by Maxwellian velocity distribution

$$dN(v,x) = d^3x\, d^3v\; n \cdot \exp\left[-\frac{mv^2}{2kT}\right]$$

As universe expands:

$$d^3x\, d^3v \propto a^{+3} \times a^{-3} = \text{const}.$$

Collision rates so high that $t_{relax} \ll H^{-1}$. So species maintain Maxwellian (equilibrium) form.

(iii) As a result: $t_1 \to t_2$

$$\exp\left[-\frac{mv^2(t_2)}{2kT(t_2)}\right] = \exp\left[-\frac{mv^2(t_1)}{2kT(t_1)}\right]$$

But $$\frac{v^2(t_2)}{T(t_2)} = \left(\frac{a(t_1)}{a(t_2)}\right)^2 \frac{\cancel{v^2(t_1)}}{T(t_2)} = \frac{v^2(t_1)}{T(t_1)}$$

Implication

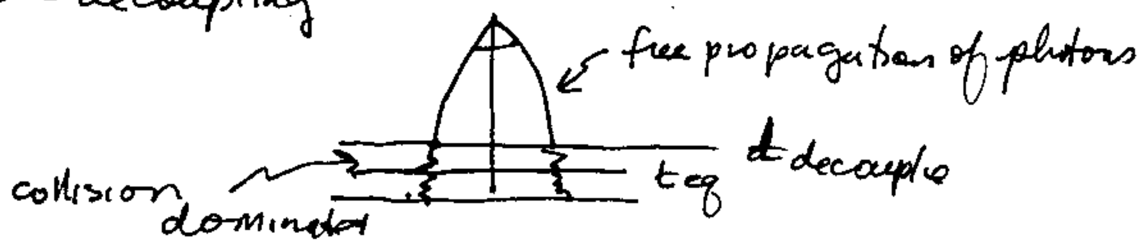
$$T(t_2) = \left(\frac{a(t_1)}{a(t_2)}\right)^2 T(t_1)$$

In other words $\boxed{T(a) \propto \frac{1}{a^2}}$ not surprising $T \propto \langle v^2 \rangle$

(iv) After decoupling, non-relativistic matter temperature of non-interacting baryons falls off more rapidly than radiation temperature.

(4) Radiation Temperature

What happens to radiation temperature after decoupling? Since radiation propagates freely, we cannot use 1st law of thermodynamic argument, which is only valid when $t < t_{decoupling}$, at $t > t_{decoupling}$



Since $k_\nu, j_\nu = 0$ for CMB radiation we can use transfer equation:

$$c(1+z)^3 \frac{d}{c\,dt}\left[\frac{I_\nu}{(1+z)^3}\right] = 0$$

Integrate from $t_{decouple}$ to t_0.

$$z = z_d \longrightarrow z = 0$$

Solution: $$\boxed{I_{\nu_0}(\nu_0, t_0) = \frac{I_\nu[(1+z)\nu_0, t_d]}{(1+z)^3}} \quad (1)$$

Initial BB: $I_\nu(\nu, t_d) = \frac{2h\nu^3}{c^2}\left[e^{\frac{h\nu}{kT_d}} - 1\right]^{-1}$

since $\nu = (1+z_d)\nu_0$

$$I_\nu[(1+z)\nu_0, t_d] = \frac{2h(1+z_d)^3\nu_0^3}{c^2}\left[e^{\frac{h(1+z_d)\nu_0}{kT_d}} - 1\right]^{-1}$$

Using eq. (1) we find:

$$I_{\nu_0}(\nu_0, t_0) = \frac{2\cancel{(1+z_d)^3}\nu_0^3/c^2}{\cancel{(1+z_d)^3}}\left[e^{\frac{h(1+z_d)\nu_0}{kT_d}} - 1\right]^{-1}$$

So observed intensity at t_0:

$$I_{\nu_0}(\nu_0, t_0) = \frac{2h\nu_0^3}{c^2}\left[\exp\left(\frac{h\nu_0}{k(T_d/(1+z_d))}\right) - 1\right]^{-1}$$

Thus $$T_0 = \frac{T_d}{1+z_d} \Rightarrow \frac{T(z)}{1+z} = \text{const}$$

$$\text{or } T(z) \propto (1+z) \propto \frac{1}{a}$$

So temperature decreases as Universe expands like $1/a$, but for completely different reasons than for epochs when $t < t_{decouple}$

Rough outline of Gamow (1942, 1946) argument

Key insight of Gamow was that abundance of $_2He^4$ ($_Z X^A$) and $_1H^2$, $_3Li^6$ result from non-equilibrium process:

Picture:

(1) $T \gtrsim 3\times10^{10}$ (since $T(z) = (1+z_0)T_0 \Rightarrow z \simeq 10^{10}$)

- Characteristic photon energy $k_B T \sim 3$ MeV. T high enough to photo-dissociate complex nuclei: Universe dominated by
- $e^-, e^+; \nu, \tilde{\nu}$

  at these temperatures, e^-, e^+ and $\nu, \tilde{\nu}$ pairs are present and couple n to p via

  $$e^- + p \rightleftarrows \nu_e + n$$

  Reaction rate high enough to maintain n, p equilibrium:

  $$n/p = \exp[-(m_n - m_p)c^2/k_B T]$$

(2) $_1H^2$ production

at $T \approx 10^{10}$ K no residual $_1H^2$ due to photodissociation.

But at $T \sim 10^9$ K, T too low for photodissociation and $_1H^2$ accumulates

Then $_1H^2 \rightarrow {}_2He^4$

(3) $_2He^4$ production : Gamow insight

$T \simeq 10^9$ K $\begin{cases} \text{(i) } H^{-1} \ll t_{reaction} : \text{no } {}_1H^2 \rightarrow {}_2He^4 \text{ production: too much } {}_1H^1 \\ \text{(ii) } H^{-1} \gg t_{reaction} : \text{no } {}_1H^1 \text{ left. Everything becomes } {}_2He^4 \end{cases}$

Thus Gamow concluded that at $T \simeq 10^9 K$
$t_{reaction} \simeq H^{-1} \Rightarrow$

$$\frac{1}{n\langle \sigma v\rangle} \sim H^{-1}$$

(4) <u>Connection between t and $T(t)$</u>

Friedmann eq. $H^2(t) = \frac{8\pi G\rho(t)}{3}$ {curvature irrelevant}

at $t < t_{eq}$: $\rho(t) = \frac{\epsilon_\gamma}{c^2} = \frac{a_B T^4}{c^2}$

Therefore: $\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{a_B T^4}{c^2}\right)$

$$\left(\frac{\dot a}{a}\right) = \left(\frac{8\pi G a_B}{3c^2}\right)^{1/2} T^2$$

But during these epochs $T \propto 1/a$
or $a(t)T(t) = \text{const}$.
Let's normalize to t_9: time at which $T = 10^9 K$.

Thus $\left(\frac{\dot a}{a}\right) = \left(\frac{8\pi G a_B}{3c^2}\right)^{1/2} \frac{a^2(t_9)\,T^2(t_9)}{a^2(t)}$

$$a\dot a = \left(\frac{8\pi G a_B}{3c^2}\right)^{1/2} \left[a(t_9)T(t_9)\right]^2$$

integrate to $10^9 K$ $\Big\}$ $$\int_0^{a(t_9)} a\,da = \left[\frac{8\pi G a_B}{3c^2}\right]^{1/2}\left[a(t_9)\,T(t_9)\right]^2 t_9$$

$$\frac{\cancel{a^2(t_9)}}{2} = \left[\frac{8\pi G a_B}{3c^2}\right]^{1/2} \cancel{a^2(t_9)}\,T^2(t_9)\,t_9$$

<u>Consequently</u>:

$$t_9 = \left[\frac{3c^2}{32\pi G a_B}\right]^{1/2} \times \frac{1}{T^2(t_9)}$$

Put in the numbers:

$$t_q = \left[\frac{3\times(3\times10^{10})^2}{32\times3.14\times6.7\times10^{-8}\times7.6\times10^{-15}}\right]^{\frac{1}{2}}\times\frac{1}{(10^9)^2}$$

$$t_q = 230\,s \quad (\simeq 3.8\,\text{minutes})$$

(5) Compute Density

Gamow condition: $n(t_q) = \dfrac{1}{\langle\sigma v\rangle t_q}$
at $T \simeq 10^9\,K$

- $v_{thermal} \simeq 5\times10^8\,cm/s$ (p, n)
- $\sigma \simeq 10^{-29}\,cm^2$

$$\Rightarrow n(t_q) \simeq \frac{1}{10^{-29}\times5\times10^8\times230} \sim 10^{18}\,cm^{-3}$$

$$n(t_q) \simeq 10^{18}\,cm^{-3}$$

(6) Gamow prediction of present temperature

$\dfrac{T^3}{n} = \text{const.}$ (holds from $t \ll t_q \to t_0$)

$$\frac{T^3(t_0)}{n(t_0)} = \frac{T^3(t_q)}{n(t_q)} \Rightarrow T_0 = T_q\left(\frac{n_0}{n_q}\right)^{1/3}$$

Recall: $n_0 = \dfrac{\rho_c\,\Omega_B}{\mu m_H} \simeq 4\times10^{-7}$

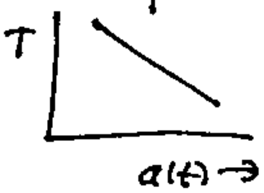
$$\Rightarrow T_0 \approx 10^9\left(\frac{4\times10^{-7}}{10^{18}}\right)^{1/3} \sim 7\,K$$

not bad!

More Comprehensive look at thermal history

Two crucial effects

As we go back in t, $T(t)$ increases. Physics is ultimately determined by 2 effects.



(1) Competition between time-scales:
Reaction time: $t_{react} = \frac{1}{n\langle\sigma v\rangle}$

when $t_{react} < t$: species relax to TE
$t_{react} > t$: "freeze-out" occurs

(2) Rest-mass energies
(a) when $k_B T > m_i c^2$: significant pair creation of species with mass m_i can occur. Detailed balance between creation and annihilation keeps particle/antiparticle pairs in equilibrium
(b) when $k_B T < m_i c^2$: Processes cease
(c) asymmetry between $\frac{n-\bar{n}}{\frac{1}{2}(n+\bar{n})} = \epsilon$

Temperature Regimes

(1) Planck Epoch: Time at which quantum corrections to GR render GR invalid.
Specifically: Time when horizon shrinks such that its physical size = Compton wavelength of mass within the horizon.
(a) Schwarzschild radius of $M(R_H)$:

$$M(R_H) = \rho \times 4\pi R^3/3 = \left(\frac{3H^2}{8\pi G}\right)(4\pi/3) R_H^3 = H^2 R_H^3 / 2G$$

$$R_{\text{Schwarzschild}} = \frac{2GM(R_H)}{c^2} = \frac{H^2 R_H^3}{c^2}$$

But since $R_H \simeq \frac{c}{H} \Rightarrow R_{\text{Schwarzschild}} \doteq R_H$

(b) Compton Wavelength : Planck Mass.

Epoch at which $\left\{ R_H \simeq \frac{G M_{\text{planck}}}{c^2} \sim \frac{\hbar}{M_{\text{planck}} c} \Rightarrow M_{\text{planck}} = \sqrt{\frac{\hbar c}{G}} \right.$

GR ~ QM

	Planck Quantity	Numeric
Planck Mass:	M_p	$\sim 10^{-5}$ g
Planck Radius	$R_p \sim \frac{GM_p}{c^2} (= R_H)$	$\sim 4 \times 10^{-33}$ cm
Planck Time	R_p / c	$\sim 10^{-45}$ s
Planck Energy	$\sim M_p c^2$	6×10^{16} erg $\sim 10^{22}$ MeV
Planck Temperature	$\frac{M_p c^2}{k_B}$	$\sim 10^{32}$ K

Major Regimes

(A) 10^{26} K $< T < 10^{32}$ K

Universe was a relativistic plasma consisting of quarks, leptons, massless bosons (no photons)

(B) SSB: Spontaneous Symmetry breaking : $T < T_{GUTS} = 10^{26}$

Expansion cools temperature to $T < T_{GUTS}$. where Universe undergoes a number of SSB phase transitions.

$T > T_{GUTS}$: Strong, weak, electromagnetic interactions are unified

$T < T_{GUTS}$: Massless particles acquire mass and interactions no longer unified.

Summary of Subsequent Epochs

(A) $T > 10^{12}K$ ($kT > 100$ MeV)

Hadron era - wide range of particles in TE. At these times, photons are a minor constituent. Rather universe dominated by
- hadron anti-hadron pairs
- mesons anti-meson pairs
- leptons anti-lepton pairs

(B) $T = 10^{12}K$ ($kT = 100$ MeV)

Hadrons annihilate $\Rightarrow$ $\mu^-, \mu^+, \gamma, e^+, e^-, \nu, \tilde{\nu}$ all in TE

(C) $T < 10^{12}K$

μ^+, μ^- pairs annihilate, leading to $\nu, \tilde{\nu}$ decoupling. Only e^+, e^-, γ and some nucleons are in TE

(D) $T < 10^9 K$

e^-, e^+ pairs annihilate leading to $\gamma, \nu, \tilde{\nu}$ as dominant species. In this epoch $(n/p) = 0.2$

(E) $T \sim 10^9 K$

n, p reactions lead to $_2He^4$ production.
Ionized gas of $_1H^1, {}_1H^2, {}_2He^4, {}_2He^3, {}_3Li^7$.

(F) $4 \times 10^3 < T < 10^9 K$

Gas and radiation locked in same T. But at some T, $e^- + H^+ \to H^0 + \gamma$ recombinations occur at $T \sim 4000 K$. We saw that in this T range, matter dominated era begins

Details:

First we need $t = t(T)$.

Recall: analysis of Friedmann eq. at early times gave us:

$$a\dot{a} = \left(\frac{8\pi G a_B}{c^2}\right)^{1/2} [a(t_q) T(t_q)]^2$$

But since $a(t)T(t) = a_0 T_0$, let

$$a\dot{a} = \left(\frac{8\pi G a_B}{c^2}\right)^{1/2} (a_0 T_0)^2$$

Integrate

$$\frac{a^2}{2} = \left(\frac{8\pi G a_B}{c^2}\right)^{1/2} (a_0 T_0)^2 t$$

or $t = \left(\frac{c^2}{32\pi G a_B}\right)^{1/2} \left(\frac{a(t)}{a_0 T_0}\right)^2 = \left(\frac{c^2}{32\pi G a_B}\right)^{1/2} \frac{1}{T^2}$

$t = 1 \times 10^{20} / T^2$ sec (a little wrong since other rel. species left out)

Implications of entropy conservation:

. 2st law.. $$T\,dS = d(\rho V) + P\,dV \quad (1)$$

$$T\,dS(V,T) = V\,d\rho + \rho\,dV + P\,dV$$

$$\boxed{dS = \frac{V}{T}\frac{d\rho}{dT}dT + \frac{1}{T}(\rho + P)\,dV} \quad (2)$$

where
$$\rho = \sum_{i=1}^{n_{species}} \int E\, n_i(p,T)\,dp \quad (a)$$
$$P = \sum_{i=1}^{n_{species}} \int \frac{c^2 p^2}{3E}\, n(p,T)\,dp \quad (b)$$
(3)

From the entropy equation we have:

$$\frac{\partial S}{\partial V} = \frac{1}{T}(\rho + P)\,; \quad \frac{\partial S}{\partial T} = \frac{V}{T}\frac{d\rho}{dT}$$

Since $\dfrac{\partial^2 S}{\partial T\,\partial V} = \dfrac{\partial^2 S}{\partial V\,\partial T}$ we have

$$\frac{\partial}{\partial T}\left[\frac{1}{T}(\rho + P)\right] = \frac{\partial}{\partial V}\left(\frac{V}{T}\frac{d\rho}{dT}\right)$$

$$-\frac{1}{T^2}(\rho + P) + \frac{1}{T}\left(\cancel{\frac{d\rho}{dT}} + \frac{dP}{dT}\right) = \cancel{\frac{1}{T}\frac{d\rho}{dT}}$$

or $$\boxed{\frac{dP}{dT} = \frac{1}{T}(\rho + P)} \quad (4)$$

Get expression for s:

Rewrite (1): $T dS = d[(\rho+P)V] - V dP$

subst. in (3)

$$dS = \frac{1}{T} d[(\rho+P)V] - \frac{V}{T}\left[\frac{1}{T}(\rho+P)dT\right]$$

$$= d\left[\frac{(\rho+P)V}{T}\right] = d(sV)$$

$$\Rightarrow \boxed{s = \frac{\rho+P}{T}} \quad (5)$$

Relativistic Matter:

at $kT \gg m_i c^2$ species will be relativistic. Since $E = \sqrt{p^2c^2+m^2c^4}$, in this limit $E \simeq pc$

From eqs. (3a) and (3b) we have

$$\rho = \int pc\, n(p,T)dp \quad ; \quad P = \frac{1}{3}\int pc\, n(p,T)dp$$

$$\Rightarrow P = \frac{1}{3}\rho$$

Therefore from eq. (4): $\frac{dP}{dT} = \frac{1}{T}(\rho+P) \Rightarrow$

$$\frac{1}{3}\frac{d\rho}{dT} = \frac{1}{T}\cdot\frac{4}{3}\rho \quad \Rightarrow \quad \frac{d\rho}{\rho} = 4\frac{dT}{T} \Rightarrow$$

$$\boxed{\rho = (\text{const})T^4} \quad (5)$$

{valid for any relativistic species}

where const. depends on spin + statistics.

Entropy conservation:

$$S = \left[V\left(\frac{\rho+P}{T}\right)\right] = \left[\frac{V\frac{4}{3}\rho}{T}\right] \propto (a^3T^3) = \text{const}$$

$\Rightarrow T \propto \frac{1}{a}$ for all relativistic species: not just photons.

Equilibrium Conditions

(1) Thermal history decided by determining which particles are in TE at given $T(t)$, and which are not.

(2) At given T only those particles with $kT \gg m_i c^2$ can be in TE, since pair creation is essential for maintaining TE (detailed balancing). [though p, n equilibrium is an exception]

Example: $kT < m_\pi c^2$ where $T_\pi \equiv \frac{m_\pi c^2}{k} = 1.5 \times 10^{12}$ K
$\pi, \tilde{\pi}$ pairs are gone. Only $\mu^\pm, e^\pm, \bar{\nu}_e, \nu, \gamma$ (plus small n, p component) are present

Thermalization Process

(i) pair production and annihilation
(ii) Compton scattering of e^-, e^+ couples chg. particles to γ's.
(iii) μ's are crucial for coupling ν's to matter; i.e., thermalizing ν's

How are ν's thermalized? $e^+ + e^- \rightleftarrows \nu + \tilde{\nu}$

$e^- + \mu^+ \rightleftarrows \nu_e + \tilde{\nu}_e$ $\quad$ $e^+ + \mu^- \rightleftarrows \tilde{\nu}_e + \nu_\mu$

$\nu_e + \mu^- \rightleftarrows \nu_\mu + e^-$ $\quad$ $\tilde{\nu}_e + \mu^+ \rightleftarrows \nu_\mu + e^+$

$\nu_\mu + \mu^+ \rightleftarrows \nu_e + e^+$ $\quad$ $\nu_\mu + \mu^- \rightleftarrows \tilde{\nu}_e + e^-$

Clearly muons are crucial for thermalizing ν's

To evaluate fate of ν's, etc we need momentum space distributions:

<u>number</u> $$n_i(p)\,dp = g \times \frac{4\pi p^2 dp}{h^3} \times \left(\exp \frac{\sqrt{(pc)^2 + m^2c^4}}{k_B T} \pm 1\right)^{-1}$$

Fermions $+$, Bosons $-$

g = no. spinstates

From eq. (3)

energy densities $$\rho_i = \int_0^\infty \sqrt{(pc)^2 + m^2c^4}\; n_i(p)\,dp$$

$$P_i = \int \frac{c^2 p^2 \quad n_i(p)\,dp}{3\sqrt{(pc)^2 + m^2c^4}}$$

Relativistic limit: $pc \gg mc^2$ $$\rho_i = g \int_0^\infty \frac{4\pi p^3 c}{h^3}\left(\exp\left(\frac{pc}{k_B T}\right) \pm 1\right)^{-1}$$

$$\rho_i = \begin{cases} g\, a_B T^4/2 & \text{Bosons} \\ g\, 7 a_B T^4/16 & \text{Fermions} \end{cases}$$

Fate of ~~[illegible]~~ Neutrinos:

- weak interaction cross-section (e^-, ν), (μ, ν)

  $\sigma_{wk} \propto T^2$, density $n \propto \frac{1}{a^3} \propto T^3$

  $t_{interact} = \frac{1}{n \sigma_{wk} c} \propto \frac{1}{T^5}$: $kT \gg m_e c^2$

Since $t \propto \frac{1}{T^2}$ we find that

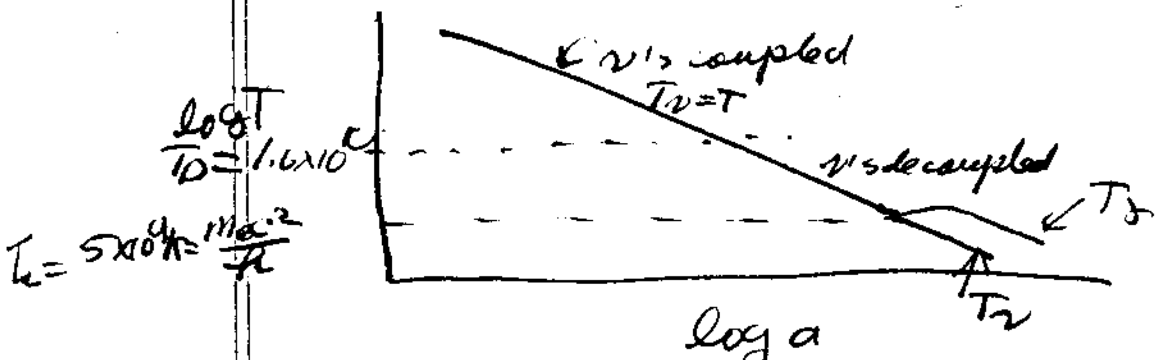
$$\frac{t}{t_{interact}} = \left(\frac{T}{1.6\times10^{10}}\right)^{3}$$

$\nu\bar{\nu} \leftrightarrow e\bar{e}$
$\nu e \leftrightarrow \nu e$

Thus

(A) $T \gg 1.6\times10^{10}$: ν's remain in equilibrium, $T_\nu = T \propto \frac{1}{a}$: $t_{interact} < t$

(B) $T < T_D = 1.6\times10^{10}$; $t_{interact} > t$. ν's decouple. But distribution frozen in, and just as in case of collisionless photons $T_\nu \propto \frac{1}{a}$ even though ν's not in thermal contact.



(C) Now T decreases to $m_e c^2$
$e\bar{e} \to \gamma\gamma$, But $\gamma\gamma \to e\bar{e}$ is severely suppressed.

Before: $T > T_e$: species in equilibrium.

Entropy: $S_- = \frac{a^3}{T_-}(\rho + P) = \frac{a^3}{T_-}\left[\rho_{e^-} + \rho_{e^+} + \rho_\gamma + P_{e^-} + P_{e^+} + P_\gamma\right]$

$$= \frac{4}{3}\frac{a^3}{T_-}\left(\rho_{e^-} + \rho_{e^+} + \rho_\gamma\right)$$

$$= \frac{4}{3}\frac{a^3}{T_-}\left(2\left(\frac{7}{16}\right) + 2\left(\frac{7}{16}\right) + \overset{2\times\frac{1}{2}}{\cancel{2}}\right) a_B T_-^{4}$$

Therefore $s_- = \frac{4}{3} \frac{a^3}{T_-} \left[1 + 2 \times \frac{7}{8}\right] \overbrace{h}^{a_B T_-^4} = \frac{4}{3} a^3 \left(\frac{11}{4}\right) a_B T_-^3$

After

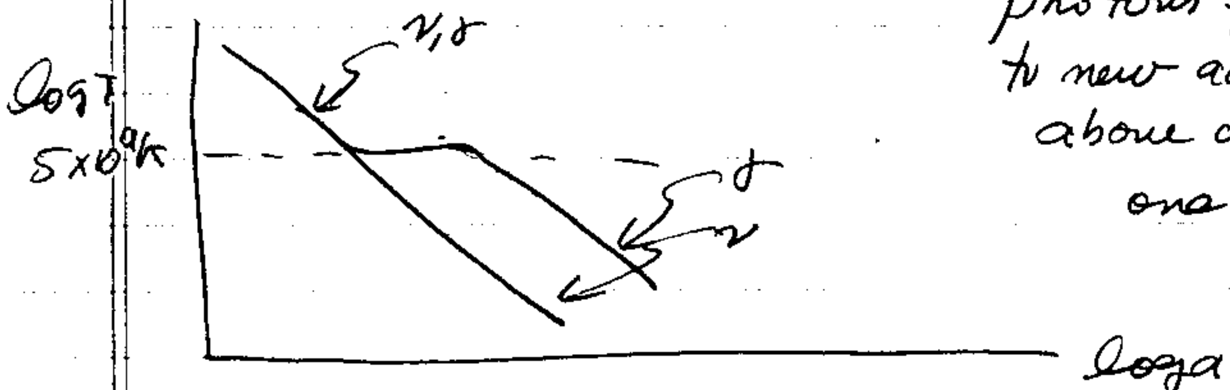
2 polarization states ($g=2$)

$$S_+ = \frac{a^3}{T_+}(\rho_\gamma + P_\gamma) = \frac{4}{3}\frac{a^3}{T_+}\left(\frac{2\times 1}{2}\right) a_B T_+^4 = \frac{4}{3} a^3 (1) a_B T_+^3$$

Since $S_+ = S_-$ (entropy conserved)

$$(T_+ a)^3 = \frac{11}{4}(T_- a)^3$$

$$\boxed{T_+ = \left(\frac{11}{4}\right)^{1/3} T_-}$$



photons jump to new adiabat above original one!

Comments

(1) Neutrinos not heated by $e^\pm$ annihilation

~~Neutrinos~~ Neutrinos $T_\nu = (aT)_-/a$

(2) But photons are heated and

$T_\gamma = (aT)_+/a$ at $T < 5\times10^9$K

where $(aT)_+ = (aT_-)(11/4)^{1/3}$

(3) at lower temperatures

$$\frac{T_\gamma(a)}{T_\nu(a)} = \frac{(Ta)_+/a}{(Ta)_-/a} = \left(\frac{(Ta)_+}{(Ta)_-}\right) = \left(\frac{11}{4}\right)^{1/3}$$

Today: $T_\nu(0) = T_\gamma(0)\left(\frac{4}{11}\right)^{1/3} = 0.714 \times T_\gamma(0)$

Neutrino background $T_\nu(0) = 1.95\,K$

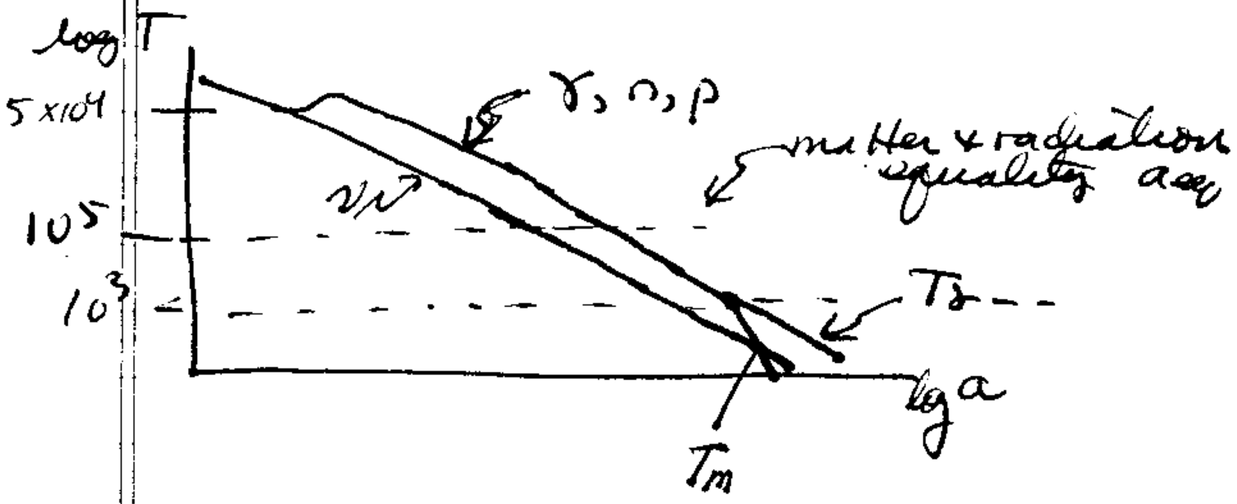
Number Density

$$n_\nu = \frac{3\zeta(3)}{2\pi^2}\left(\frac{k_B T_\nu}{\hbar c}\right)^3 \; ; \; n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T_\gamma}{\hbar c}\right)^3$$

Therefore: $\frac{n_\nu}{n_\gamma} = \frac{(3/2)}{2}\left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{3}{4} \times \frac{4}{11} = \frac{3}{11}$

$n_\gamma = 410(1+z)^3$
$n_\nu = 113(1+z)^3$ for each ν family
$(n_\nu)_T = 3n_\nu = 339(1+z)^3$

Further history



Recall $1+z_{eq} = 3.9\times10^4$ or $T \simeq 1\times10^5\,K$
at $a > a_{eq}$, matter still coupled to radiation by e^- scattering of photons

Recombination Epoch

As T decreases further neutral fraction of the gas starts to increase. Let's see why.

Crucial reaction: (atomic) $e^- + H^+ \rightleftarrows H^0 + \gamma$

In TE. no. of particles in a box with volume V is given by

$$\frac{N_p N_e}{N_{H^0}} = \frac{Z_p Z_e}{Z_{H^0}}$$ where Z is the partition function

In TE Z's are maximum no. of occupancy of available energy states.

$$Z = Z_{translation} \times Z_{internal}$$

(a) Free particles (no bound states)

$$Z_e = g_s \iint \frac{d^3x\, d^3p}{h^3} \exp\left(-\frac{p^2}{2m_e k_B T}\right) = \frac{2V\,(2\pi m_e k_B T)^{3/2}}{h^3}$$

$$Z_p = g_p \iint \frac{d^3x\, d^3p}{h^3} \exp\left(-\frac{p^2}{2m_p k_B T}\right) = \frac{2V\,(2\pi m_p k_B T)^{3/2}}{h^3}$$

(b) Particles with bound states (H^0)

$$Z_{H^0} = \frac{2V\,(2\pi m_p k_B T)^{3/2}}{h^3} \times \sum_{n=1}^{\infty} g_n e^{-E_n/k_B T}$$

Dominant term in sum is ground state

$$g_1 = 2\,,\quad E_1 = -\chi_H = -13.6\,\text{eV}$$

$$\therefore\ Z_{H^0} = Z_p \cdot 2\, e^{+\frac{\chi_H}{k_B T}}$$

Back to Saha equation

First Let $N_e = N_p$

$$\therefore \quad \frac{N_e^2}{N_{H^0}} = \frac{\cancel{Z_p} \cdot Z_e}{\cancel{Z_p} \cdot 2e^{+\chi_H/k_B T}} = \frac{Z_e}{2} \exp\left(-\frac{\chi_H}{k_B T}\right)$$

As a result. $$\frac{N_e^2}{N_{H^0}} = \frac{\cancel{2} V (2\pi m_e k_B T)^{3/2} e^{-\frac{\chi_H}{k_B T}}}{\cancel{2} h^3}$$

But $N_e = \underset{\text{density}}{n_e} V$, $N_{H^0} = n_{H^0} V$

Thus $$\frac{N_e^2}{N_{H^0}} = \frac{n_e^2 V^2}{n_{H^0} V} = \frac{n_e^2}{n_{H^0}} V$$

Consequently: $$\frac{n_e^2 \cancel{V}}{n_{H^0}} = \cancel{V} \frac{(2\pi m_e k_B T)^{3/2}}{h^3} \exp\left(-\frac{\chi_H}{k_B T}\right)$$

Define electron fraction: $x = n_e/n$; $1-x = n_{H^0}/n$

In that case $$\frac{n_e^2}{n_{H^0}} = \frac{x^2 n^2}{(1-x) n} = \frac{x^2}{1-x} n$$

Combine with previous equation.

$$\boxed{\frac{x^2}{1-x} = \frac{(2\pi m_e k_B T)^{3/2}}{n h^3} \exp\left(-\frac{\chi_H}{k_B T}\right)}$$

Recall: Baryon density $n_b^{(0)} = 1.1 \times 10^{-5} \Omega_b h^2$

$$n(z) = (1+z)^3 n(0) = \left(\frac{T}{2.72}\right)^3 n(0)$$

$$n(z) = 5.5 \times 10^{-7} \Omega_b h^2 \times T^3$$

$$\frac{X^2}{1-X} = \left[\frac{(2\pi m_e k_B)^{3/2}}{h^3}\right] \times \frac{1}{5.5\times10^{-7}\, 2_n R^2 T^{3/2}} \exp\left(-\frac{\chi_H}{k_B T}\right)$$

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$$\frac{X^2}{1-X} = \frac{4.4\times10^{21}}{\left(\frac{2_n R^2}{.02}\right)(.02)\,T^{3/2}} \exp\left[-\frac{155,000}{T}\right]$$

$$\boxed{\frac{X^2}{1-X} = \frac{2.2\times10^{23}}{\left(\frac{2_n R}{.02}\right)T^{3/2}} \exp\left[-\frac{155,000}{T}\right]}$$

Z	T	$\frac{X^2}{1-X}$	X
1000	2725		
1100	2998		
200	3270		
1300	3543		
1400	3815		
1500	4088		