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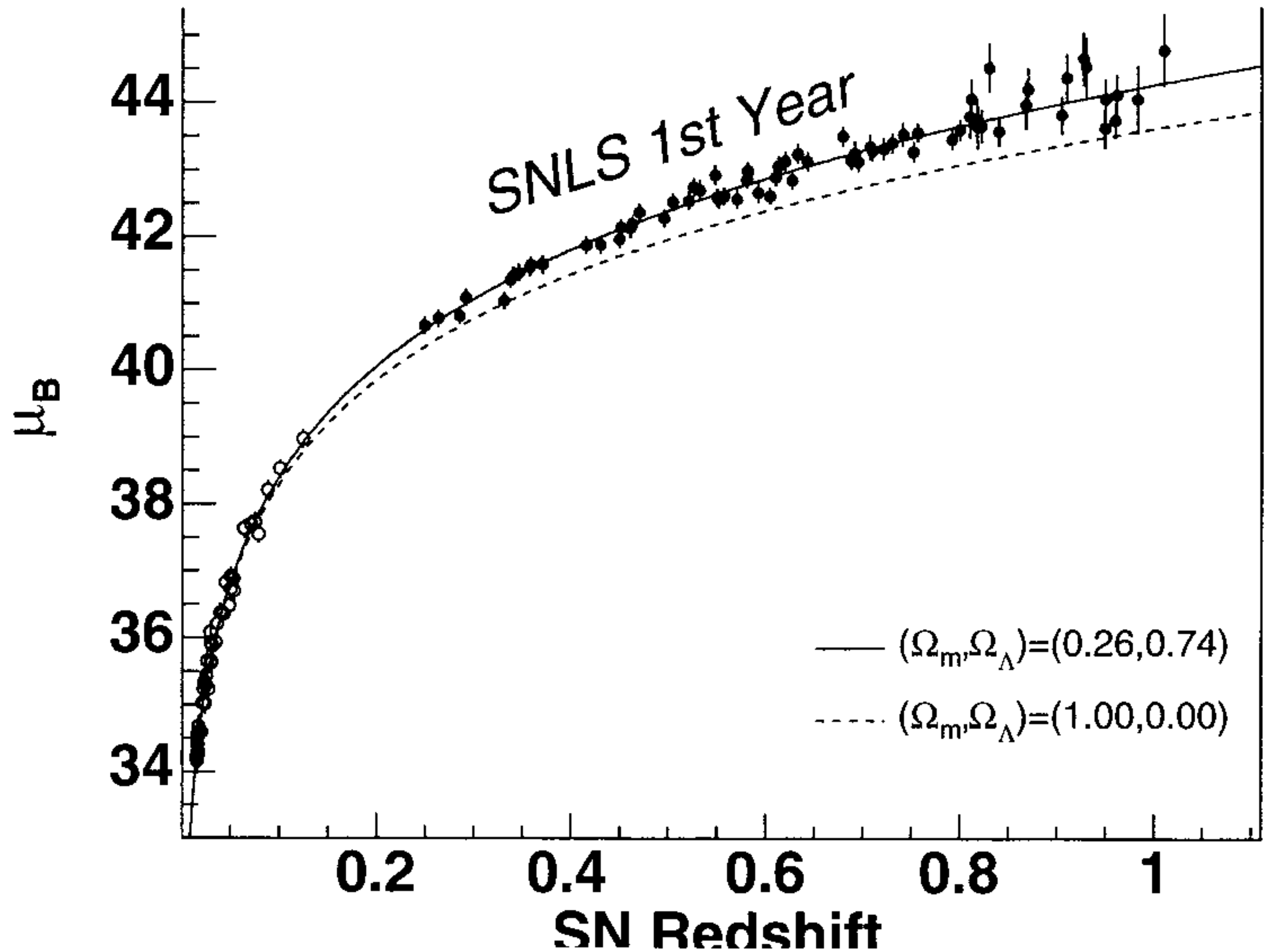


Integrate eq. (1) from t_E to present epoch, t_0.

$$\int_{I_{\nu_E}(t_E)}^{I_{\nu_0}(t_0)} \frac{dI_\nu}{I_\nu} = -3\int_{t_E}^{t_0} \frac{\dot{a}}{a}\,dt = -3\int_{a(t_E)}^{a(t_0)} \frac{da}{a}$$

$$\ln\left[I_{\nu_0}(t_0)/I_{\nu_E}(t_E)\right] = -3\ln\left[\frac{a(t_0)}{a(t_E)}\right] = \ln\left[\left(\frac{a(t_E)}{a(t_0)}\right)^3\right]$$

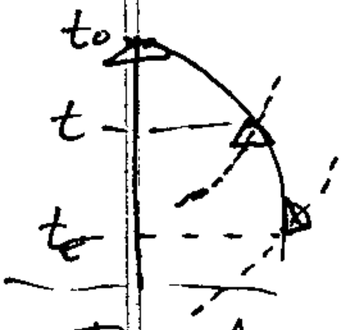
$$\therefore \quad \frac{I_{\nu_0}(t_0)}{I_{\nu_E}(t_E)} = \left(\frac{a(t_E)}{a(t_0)}\right)^3 = \frac{1}{(1+z_E)^3}$$

where $1+z_E = \frac{a(t_0)}{a(t_E)}$; $\nu_0 = (1+z_E)^{-1}\nu_E$

Implications:

- Expansion alone decreases intensity, even in absence of opacity
- Without expansion and opacity I_ν = const along light rays (Liouville theorem)

Consider a second observer at t where $t_E < t < t_0$



Observer detects same photons emitted at t_E with freq. ν_E

Obviously $I_\nu(t) = I_{\nu_E}(t_E)\left(\frac{a(t_E)}{a(t)}\right)^3$

If $(1+z_{rel}(t)) \equiv a(t)/a(t_E)$ then

$$I_\nu(t) = I_{\nu_E}(t_E)/(1+z_{rel})^3 \; : \; \nu = (1+z_{rel})^{-1}\nu_E$$

Note: $\frac{a(t_0)}{a(t_E)} = \frac{a(t_0)}{a(t)} \times \frac{a(t)}{a(t_E)}$

Therefore: $1+z_E = (1+z(t))(1+z_{rel})$

$$1+z_{rel}^{(t)} = \frac{1+z_E}{1+z(t)} \quad : z(t) \text{ is redshift at time } t.$$

$$\therefore \frac{I_\nu(t)}{I_{\nu_0}} = \frac{I_{\nu_E}[(1+z_{rel})^{-3}]}{I_{\nu_E}[(1+z_E)^{-3}]} = \left(\frac{1+z_E}{1+z_{rel}}\right)^3 = (1+z(t))^3$$

At redshift z intensity of given source is a factor of $(1+z)^3$ higher than seen at $z=0$. This is principal reason why background radiation at high-z is so much stronger than at $z=0$

Effect of emission and extinction:

These are short-range processes. ~~[illegible]~~ ~~[illegible]~~ Processes that enter rhs of Boltzmann eq. for photons:

Free-space: $\dfrac{dI_\nu}{dt} + \dfrac{3\dot a}{a} I_\nu = 0$ (just derived)

Light path: $\dfrac{dI_\nu}{d(ct)} + \dfrac{3\dot a}{c\cdot a} I_\nu = 0$

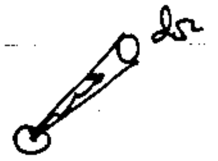
Sources & sinks:

$$\boxed{\frac{dI_\nu}{d(ct)} + \frac{3\dot a}{ca} I_\nu = \underbrace{-k_\nu I_\nu}_{\text{sink}} + \underbrace{j_\nu}_{\text{source}}} \quad (1)$$

opacity

• $k_\nu \equiv n\sigma_\nu$ where n is volume density of absorbers and σ_ν is interaction cross secti[on] at frequency ν.

$$[k_\nu] = \frac{1}{l_\nu}$$: l_ν is photon mean-free-path



Volume emissivity

$$j_\nu = \frac{dE_{em}}{dt\,dV\,d\nu\,d\Omega}$$

Radiation emitted per unit time, volume, frequency, solid angle.

Static Media : redshift-loss term vanishes.

Let $dx = c\,dt \Rightarrow$

$$\frac{dI_\nu}{dx} = j_\nu - k_\nu I_\nu$$

Divide by $-k_\nu$

$$\frac{dI_\nu}{-k_\nu\,dx} = I_\nu - \frac{j_\nu}{k_\nu}$$

Define optical depth: $d\tau_\nu \equiv -k_\nu\,dx$ ($d\tau_\nu$ no. of photon m.f.p in dx)

$$\frac{dI_\nu}{d\tau_\nu} - I_\nu = -\frac{j_\nu}{k_\nu}$$

General solution (using integrating factors)

$$I_\nu(0) = I_\nu(\tau_\nu)\, e^{-\tau_\nu} + \int_0^{\tau_\nu} dt\, e^{-t} j_\nu(t)/k_\nu$$

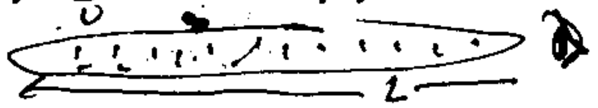
↑ incident radiation

Olber's Paradox

Consider a uniform slab of stars <u>not</u> subject to incident radiation: no dust, no gas; only stars block light

$$j_\nu = \text{const}\,; \quad k_\nu = \text{const}\,, \quad I_\nu(\tau_\nu) = 0$$

In that case $I_\nu(0) = \frac{j_\nu}{k_\nu} \int_0^{\tau_\nu} dt\, e^{-t} = \frac{j_\nu}{k_\nu}\left(1 - e^{-\tau_\nu}\right)$



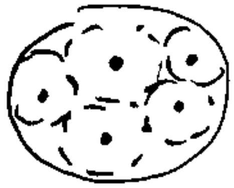
Limits:

(A) Optically thin: $\tau_\nu \ll 1$

Interpretation: $\tau_\nu = \sigma_\nu n L$ (total optical depth)

$$\sigma_\nu \ll \frac{1}{nL} = \frac{1}{\#\,\text{absorbers/area}} = \frac{\text{Area occup}}{\text{absorber}}$$

Physical stellar cross-section ≪ Area occupied per star



In that case $I_\nu = \frac{j_\nu}{k_\nu}\tau_\nu = j_\nu L$

(B) Optically thick: $\tau_\nu \gg 1$

Here $\sigma_\nu \gg \frac{1}{nL}$

Stars blanket the sky

assume $L_\nu = (L_\nu)_* (L/L_*)$: spectral shape independent of L

In general absolute mag. of L_* galaxy:
$M_* = -19.5 \to -20$!

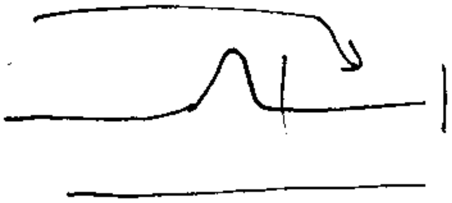
Simple case $\alpha = -1$.

$$j_\nu(z) = \begin{cases} \frac{(1+z)^3}{4\pi} L_{\nu *} \Phi_* & : \nu < \nu_{Ryd} \\ 0 & : \nu \geq \nu_{Ryd} \end{cases}$$

<u>Background Only</u>

$$\therefore I_{\nu_0}(t_0) = \int_{t_{min}}^{t_0} \frac{(\cancel{1+z})^3 L_{\nu *}\Phi_*}{4\pi} \times \frac{e^{-\tau_{\nu_0(1+z')}}}{(1+\cancel{z})^3} \, c\,dt'$$

Suppose we observe at $\nu_0 < \nu_\alpha (1+z_{max})^{-1}$: no opacity!



$\leftarrow \nu_0$

switch to z

$$c\frac{dt}{dz} = -\frac{c}{(1+z)H(z)}$$

$$\therefore I_{\nu_0}(t_0) = -\int_{z_{cut}(\nu_0)}^{0} \frac{L_{\nu *}\Phi_*}{4\pi} \frac{c\,dz'}{(1+z)H(z)} = \int_0^{z_{cut}(\nu_0)} \frac{L_{\nu *}\Phi_*}{4\pi} \frac{c\,dz}{(1+z)H(z)}$$

where $1 + z_{cut} = \frac{\nu_{Ryd}}{\nu_0} = \frac{\lambda_0}{\lambda_{Ryd}}$

In that case $I_\nu = \frac{j_\nu}{k_\nu} = j_\nu l_\nu$

$\uparrow$ 1mfp

we can only "see" 1 m.f.p. into medium.

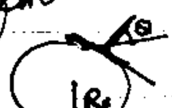
Stars $\quad j_\nu \equiv n_* L^*_\nu / 4\pi$ (Isotropic emission)

n_* = star density; L^*_ν = luminosity per unit bandwidth of star.

Assume spherical star with radius R_*.

$$L^*_\nu = 4\pi R_*^2 \cdot S^s_\nu$$

flux at surface



But $\quad S^s_\nu = \int I^s_\nu \cos\theta \, d\Omega$

$$S^s_\nu = I^s_\nu \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta = \pi I^s_\nu$$

Therefore: $\quad j_\nu = \dfrac{n_* \cdot 4\pi R_*^2 S^s_\nu}{4\pi} = n_* R_*^2 \pi I^s_\nu$

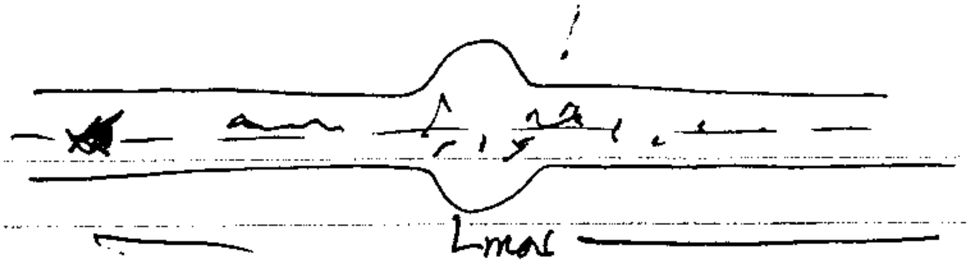
Since $k_\nu = n_* \pi R_s^2$ we have

$$I_\nu = \frac{n_* R_*^2 \cdot \pi I^s_\nu}{n_* \pi R_*^2} = I^s_\nu$$

Background "night sky" should have same intensity as surface of a star. Night sky should as bright as solar surface.

Realistic: $\quad$ milkway stellar density: $n_* \simeq 0.05\,pc^{-3}$

$$\pi R_*^2 \simeq \pi (7\times10^{10})^2 \simeq 1.5\times10^{22}\,cm^2 = \frac{1.5\times10^{22}}{(3\times10^{18})^2} = 1.6\times10^{-15}\,pc^2$$



Brightest we could get (ignoring dust opacity) is to look through entire Milky Way

$L_{max} \simeq 20$ kpc

$$\tau_\nu = \pi R_*^2 \cdot n_* \cdot L_{max}$$

$$\tau_\nu = 1.6\times10^{-15}\,pc^2 \times 0.05 \times 2\times10^4 = 1.6\times10^{-12}$$

But Olbers considered $L_{max} \to \infty$

Expanding Universe Again

Go back to p. 81

$$\frac{dI_\nu}{d(ct)} + \frac{3\dot{a}}{c\cdot a} I_\nu = -k_\nu I_\nu + j_\nu \qquad (1)$$

Change variables from $a \to z$

$$\ln(1+z) = \ln a_0 - \ln a$$

$$\frac{d\ln(1+z)}{dt} = -\frac{\dot{a}}{a} \qquad \left\{ \frac{-d(1+z)}{(1+z)dt} = H(t) \right\}$$

Eq. (1) becomes:

$$\frac{dI_\nu}{d(ct)} - \frac{3\,d\ln(1+z)}{d(ct)} I_\nu = -k_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d(ct)} + \left[\frac{d\ln(1+z)^{-3}}{d(ct)}\right] I_\nu = -k_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d(ct)} + \left[\frac{1}{(1+z)^{-3}} \frac{d}{d(ct)}\left(\frac{1}{(1+z)^3}\right)\right] I_\nu = -k_\nu I_\nu + j_\nu$$

Recap: General soln. to transfer equation

Class next Monday

$$c(1+z)^3 \frac{d}{d(ct)}\left[\frac{I_\nu}{(1+z)^3}\right] = -k_\nu I_\nu + j_\nu$$

- observer at $z=0$, $t=t_0$
- " detects photon with frequency ν_0

$$I_{\nu_0}(t_0) = \frac{I_{\nu_0(1+z_{max})}(t_{min})}{(1+z_{max})^3}\exp[-\tau_{\nu_0}(t_{min})]$$
$$+ \int_{t_{min}}^{t_0} \frac{j_{\nu_0(1+z')}\exp[-\tau_{\nu_0(1+z')}(t')]\,c\,dt'}{(1+z')^3}$$

First term

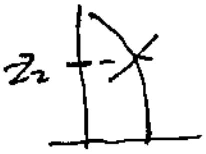
Looking at single source at $z=z_{max}$. Soln. same as in empty space $I_{\nu_0} = \frac{I_{\nu_0(1+z_{om})}}{(1+z_{om})^3}$

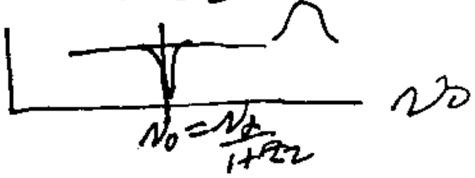
except for additional $e^{-\tau_{\nu_0}}$ attenuation term

$$\tau_{\nu_0}(t_{min}) = \int_{t_{min}}^{t_0} k_{\nu_0(1+z')}\,c\,dt'$$

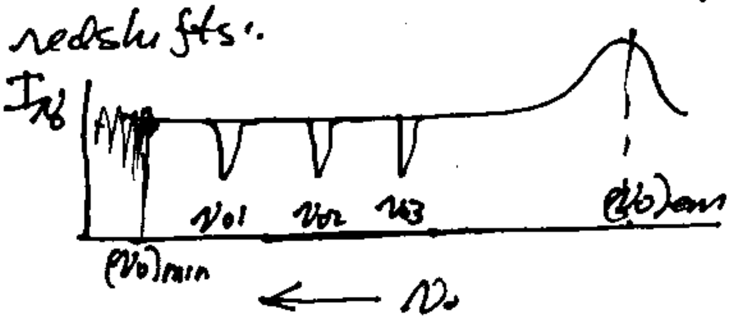
Suppose opacity due to absorption line like Lyα with rest frame frequency ν_α. Cloud at z_2

$\nu_0(1+z) = \nu_\alpha$





Of course there could be multiple clouds at several redshifts.



$$\boxed{\begin{aligned} \nu_{01} &= \nu_\alpha/(1+z_1) \\ \nu_{02} &= \nu_\alpha/(1+z_2) \\ &\vdots \\ (\nu_0)_{em} &= \nu_\alpha/(1+z_{em}) \end{aligned}}$$

High redshift spectra exhibit such high line density that continuous absorption not bad approximation

$$k_\nu = n_{HI} \times \frac{\pi e^2}{m_e c} f_{Ly\alpha}\ \delta(\nu - \nu_\alpha')$$

where ν is in rest-frame of absorber. [with respect to observer's frequency] $\nu = (1+z)\nu_0$. Therefore

$$k_{\nu_0(1+z)} = n_{HI}(z) \times \frac{\pi e^2}{m_e c} f_{Ly\alpha}\ \delta[(1+z)\nu_0 - \nu_\alpha]$$

Since Lyα-forest gas is highly ionized, introduce neutral fraction x_{HI}: $\underset{\text{neutral H}}{n_{HI}} = x_{HI} \cdot \underset{\text{total H}}{n_H}$

As a result:

$$\underbrace{\tau_{\nu_0}}_{\text{optical depth at observed frequency } \nu_0} = \int_{t(z_{em})}^{t(z_{min})} k_{\nu_0(1+z)}\, c\,dt\ ; \qquad \left\{ \begin{array}{l} z_{min} = \text{minimum redshift} \\ \text{at which Ly}\alpha \text{ detected} \\ \text{from ground } (z_{min} \simeq 1.6) \end{array} \right.$$

$$\tau_{\nu_0} = \frac{\pi e^2}{m_e c} \times f_{Ly\alpha} \int_{t(z_{em})}^{t(z_{min})} n_H(z)\, x_{HI}(z)\ \delta[(1+z)\nu_0 - \nu_\alpha]\, c\,dt$$

Recall: $dt/dz = -1/[H(z)(1+z)]$; $n_H(z) = n_H(0)(1+z)^3$

Therefore $$\tau_{\nu_0} = \frac{\pi e^2}{m_e c} \times f_{Ly\alpha} \times n_H(0) \int_{z_{em}}^{z_{min}} (1+z)^3 \cdot x_{HI}(z)\ \delta[(1+z)\nu_0 - \nu_\alpha] \left(\frac{-c\,dz}{H(1+z)}\right)$$

$$\tau_{\nu_0} = \frac{\pi e^2}{m_e c} f_{Ly\alpha}\, n_H(0) \int_{z_{min}}^{z_{em}} \frac{x_{HI}(z)\ \delta[z - z_\alpha]}{\left|\frac{dg}{dz}\right|_{z_\alpha}} \left[\frac{(1+z)^3\, c}{(1+z) H(z)}\right]$$

where $g(z) = (1+z)\nu_0 - \nu_\alpha$

$$\frac{dg}{dz} = \nu_0 \quad \text{and} \quad \nu_0 = \frac{\nu_\alpha}{1+z_\alpha}$$

Therefore:
$$\tau_{\nu_0} = \frac{\pi e^2}{m_e c} f_{Ly\alpha}\, n_H(0) \int_{z_{min}}^{z_{max}} \frac{x_{HI}(z)}{\nu_\alpha} \frac{(1+z)^3}{H(z)} \cdot c \cdot \delta(z - z_\alpha)\, dz$$

$$\tau_{\nu_0} = \frac{\pi e^2}{m_e c} f_{Ly\alpha} n_H(0) \frac{(1+z_\alpha)^3}{\nu_\alpha} \cdot x_{HI}(z_\alpha) \cdot \frac{c}{H(z_\alpha)}$$

Recall: $H(z) = H_0 \left[\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_R (1+z)^4 + \Omega_K (1+z)^2\right]^{1/2}$

Current parameters: $\Omega_K = 0$

at $z > 1.6 \to 8$ $\Omega_m (1+z)^3$ dominates

$\Rightarrow \quad H(z) = H_0 (\Omega_m)^{1/2} (1+z)^{3/2}$

Thus:
$$\boxed{\tau_{\nu_0} = \frac{\pi e^2}{m_e c} f_{Ly\alpha}\, n_H(0) \cancel{\frac{(1+z_\alpha)^3}{\nu_\alpha}} \frac{x_{HI}(x_\alpha)}{\nu_\alpha} \frac{(1+z_\alpha)^{3/2} c}{\Omega_m^{1/2} H_0}}$$

Parameters:

- $n_H(0) = \dfrac{\rho_b(0)}{\mu m_H} = \dfrac{\Omega_b \rho_{crit}}{\mu m_H}$ — baryon density

Let $\mu = 1$; $\rho_{crit} = 1.88 \times 10^{-29} h^2 \ g\,cm^{-3}$

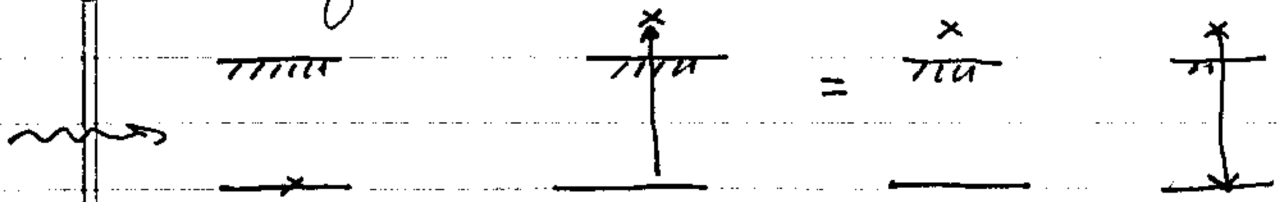
$$\Rightarrow n_H(0) = 1.125 \times 10^{-5} \Omega_b h^2 \ cm^{-3}$$
$$= 2.25 \times 10^{-7} \left(\frac{\Omega_b}{.02}\right) h^2 cm^{-3}$$

- $$\tau_{\nu_0} = \frac{2.65 \times 10^{-2} \cdot (.416) \cdot 225 \times 10^{-7}}{2.47 \times 10^{15}} \left(\frac{\Omega_b}{.02}\right) h^2 \left(\frac{9.29 \times 10^{27}}{h}\right) \frac{(1+z_\alpha)^{3/2} x_{HI}}{\Omega_m^{1/2}}$$

$$\tau(z) = \frac{1.8\times10^5}{h\,\Omega_m^{1/2}}\left(\frac{\Omega_b h^2}{.02}\right)\left(\frac{1+z}{7}\right)^{3/2} X_{HI}(z)$$

Neutral Fraction

Ionization equilibrium



Rate of photoionization = Rate of recombinations

If Γ is ionization rate per unit time per atom

$$n_{HI}\,\Gamma = n_e\, n(H^+)\,\alpha(T) \quad \alpha = \langle\sigma v\rangle \text{ rec. rate constant}$$

where $\Gamma = \int_{\nu_{H}}^{\infty} d\nu \frac{4\pi I_\nu}{h\nu}\sigma_\nu$ (photo-ionization crosssection)

Conditions.

(1) IGM is highly ionized: $n(H^+) \simeq n_H$

(2) $n_e = n(H^+)$

$$\therefore \quad n_{HI}\,\Gamma = n_H^2\,\alpha \qquad \alpha = 4.2\times10^{-13}\,(T/10^4K)^{-.7}$$

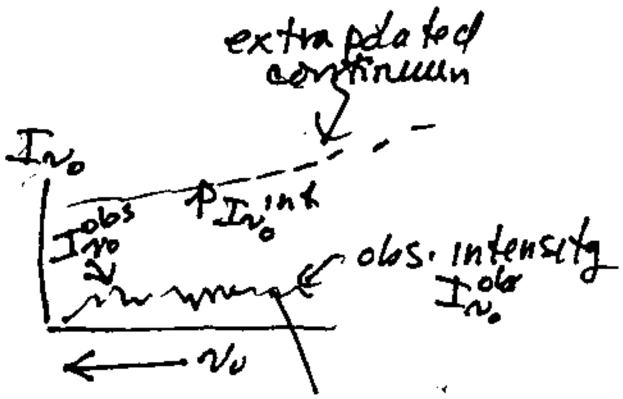
$$\Rightarrow X_{HI} = \frac{n_{HI}}{n_H} = \frac{n_H\,\alpha}{\Gamma} \qquad \Gamma = 10^{-12}\,\Gamma_{12}$$

where $\Gamma_{12} = 2.5\, I_{21}$

Clearly $x_{HI}(z) \propto \dfrac{\Omega_b h^2 (1+z)^3}{\Gamma_{21}}$

Consequently

$$\tau(z) \propto \frac{(1+z)^{9/2}}{\Gamma_{-12}}$$



Procedure

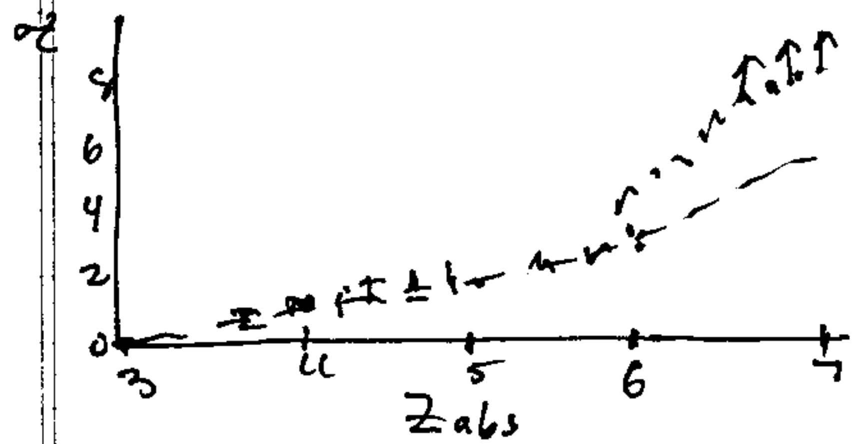
(1) Measure τ by

Transmission $\langle T \rangle = \left\langle \frac{I_{\nu_0}^{obs}}{I_{\nu_0}^{int}} \right\rangle$

Fan et al. 06, ApJ, 132, 117

Recall: $I_\nu^{obs} = (I_\nu)^c e^{-\tau_\nu}$

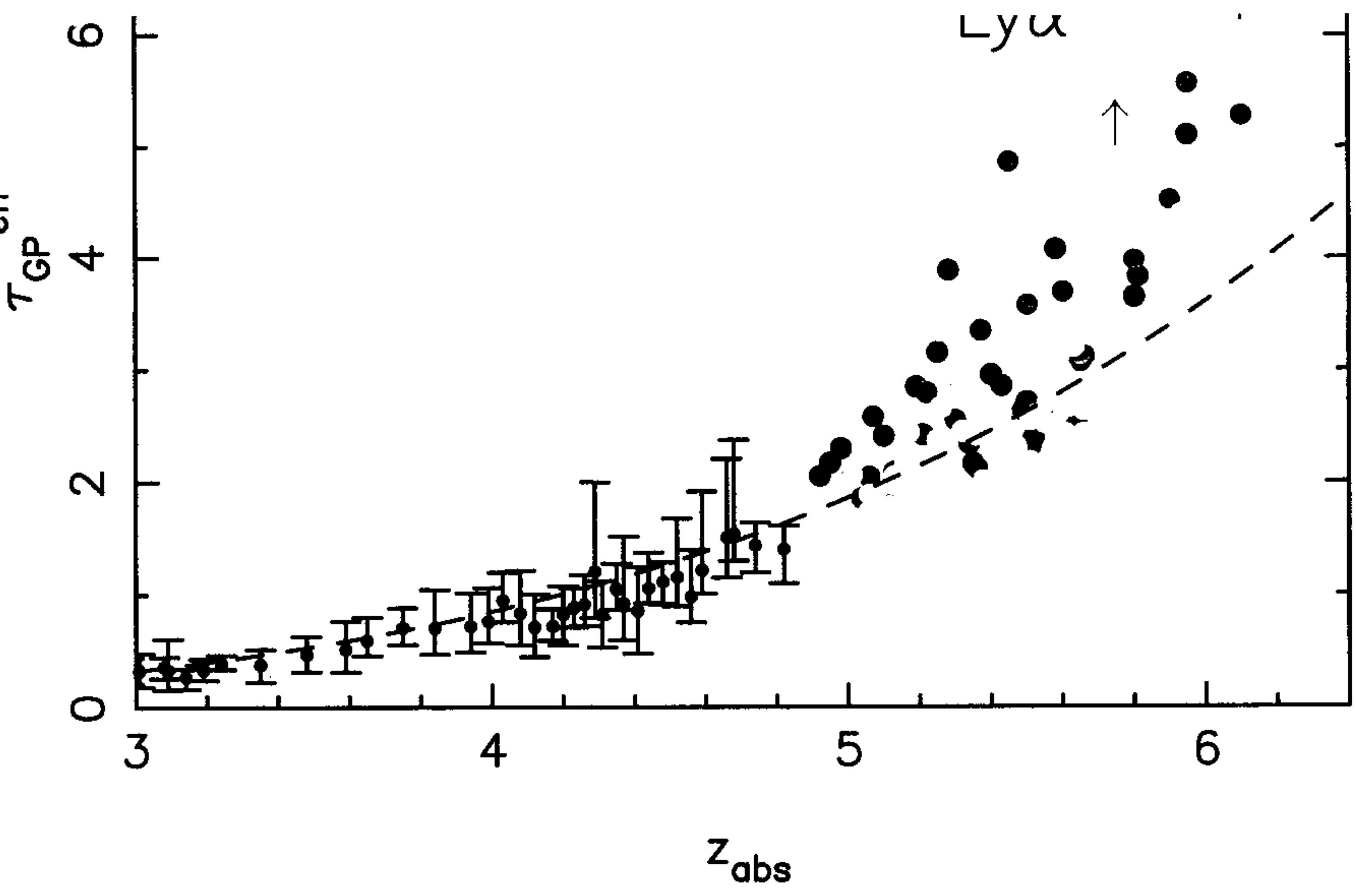
$$\Rightarrow \tau(z) = -\ln \langle T \rangle$$



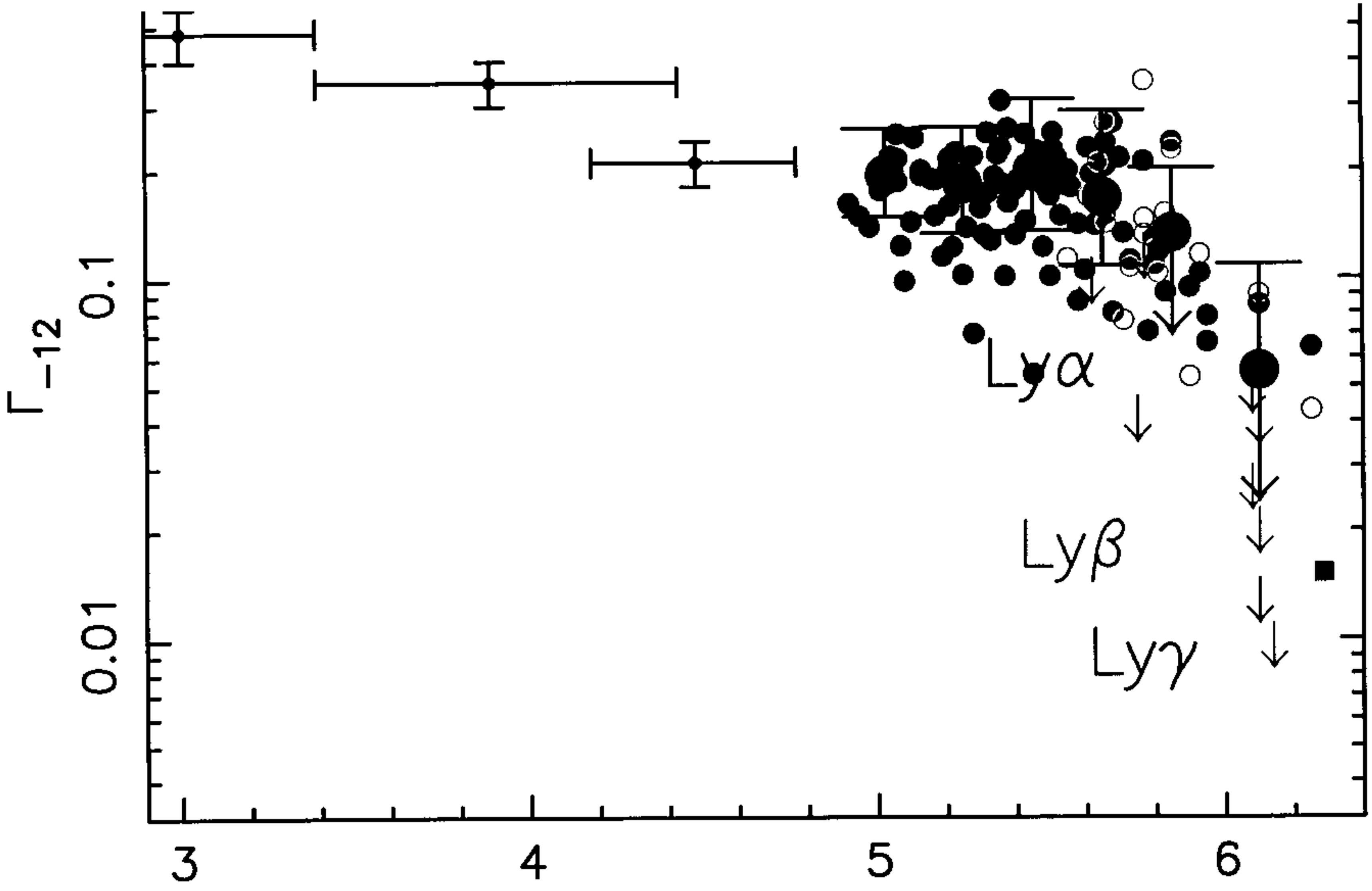
at $z < 6$: $\tau = (.85 \pm .06)\left(\frac{1+z}{5}\right)^{4.3}$ good fit

$z > 6$: obs. τ goes above extrapolation

significant decrease in Γ_{-12} at $z >$ 5.7



Lyα
τ_{GP}^{eff}
z_{abs}
0
2
4
6
3
4
5
6



Γ_{-12}
0.1
0.01
3
4
5
6
Lyα
Lyβ
Lyγ

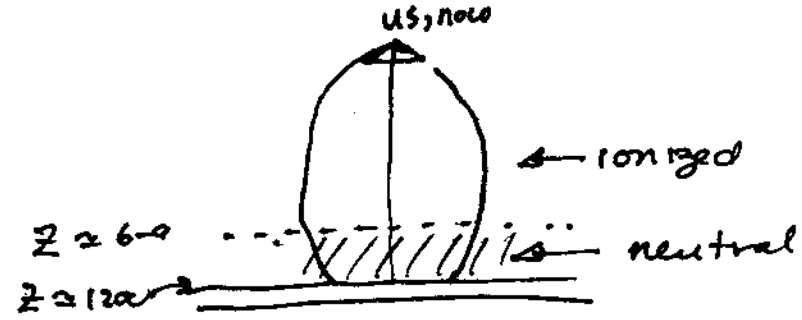
Conclude:

(1) $z \leq 5.7$: $\tau(z) \propto (1+z)^{4.3}$: close to predicted $(1+z)^{4.5}$ assuming ~ const ionization rate

(2) $z > 5.7$ $\tau(z) \propto (1+z)^{11}$: significantly above extrapolation of const. ionizatn. rate.
- Inference The decreased significantly with redshift at $z > 6$

Implication: We may be probing epochs at which rise of ionization began or at which the Universe became re-ionized.

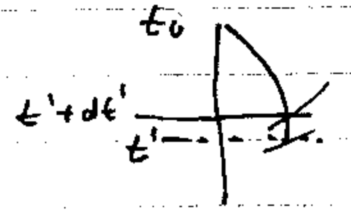
schematic:



what does the ionizing: newly formed galaxies? newly formed quasars?

Let's return to question of background radiation!

$$I_{\nu_0}(t_0) = \int_{t_{min}}^{t_0} \frac{j_{\nu_0(1+z')} \exp\left[-\tau_{\nu_0(1+z')}(t')\right] c\,dt'}{(1+z')^3}$$

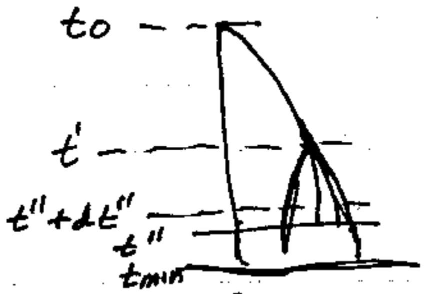


$$\tau_{\nu_0(1+z')} = \int_{t'}^{t_0} k_{\nu_0(1+z'')}\, c\,dt''$$

opacity between $t' \to t_0$

Simple model: Consider background radiation at t_0 due to galaxies. Let's ignore optical depth

Aside



What about backgrounds at t': Observer detects $\nu_{obs} = \nu_0$

$$I_{\nu_0}(t') = \int_{t_{min}}^{t'} \frac{j_{\nu_0(1+z_{rel})} \exp\left[-\tau_{\nu_0(1+z_{rel})}(t'')\right] c\,dt''}{1+z_{rel}(t'')}$$

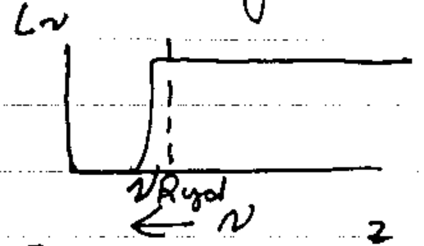
Recall $\quad 1+z_{rel}(t'') \equiv \dfrac{1+z(t'')}{1+z(t')}$

$$\tau_{\nu_0(1+z_{rel})}(t'') = \int_{t''}^{t'} k_{\nu_0(1+z_{rel}(t'''))}\, c\,dt'''$$

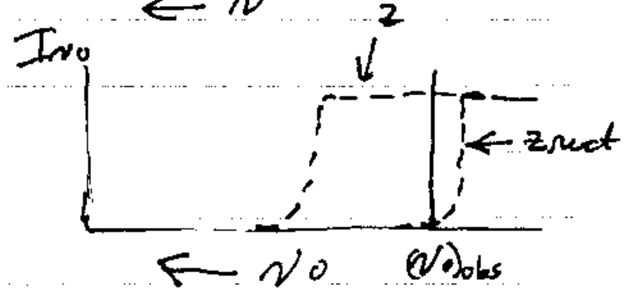
$$k_{\nu_0(1+z_{rel})} = n_0(z(t''))\,\sigma_{\nu_0(1+z_{rel}(t'''))}$$

Background radiation due to galaxies

Intrinsic Spectrum



Redshifted



So for a given $(\nu_0)_{obs}$, when ~~[illegible]~~ $1+z > \frac{\nu_{Ryd}}{\nu_{obs}}$ Lyman limit is shifted into $(\nu_0)_{obs}$ and no flux is observed. Thus practical value of

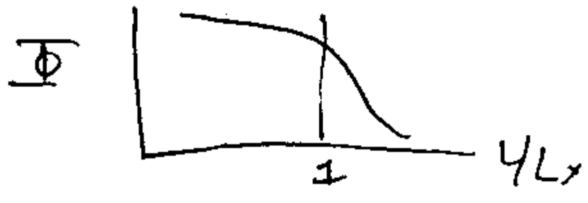
$$1+z_{max} = \frac{\nu_{Ryd}}{\nu_0} = \frac{\lambda_0}{\lambda_{Ryd}} \quad ; \lambda_{Ryd} = 912\,\text{Å}$$

Volume emissivity:

$$j_\nu(z) = \begin{cases} \frac{(1+z)^3}{4\pi} \int \Phi\left(\frac{L}{L_*}\right) L_\nu(z)\, d\left(\frac{L}{L_*}\right) & ; \nu \le \nu_{Ryd} \\ 0 & ; \nu \ge \nu_{Ryd} \end{cases}$$

Schechter function:

$$\Phi\left(\frac{L}{L_*}\right) = \Phi_*\left(\frac{L}{L_*}\right)^\alpha \exp\left(-\frac{L}{L_*}\right)$$



In general $\alpha = \sim 1$ for nearby galaxies

$$I_{\nu_0}(H_0) = \frac{L_{\nu_*} \Phi_* c}{4\pi \; H_0} \int_0^{z_{out}} \frac{dz}{(1+z)\, H(z)/H_0}$$

Numbers: ~~Einstein-deSitter~~: $H(z) = H_0 (1+z)^{3/2}$

In that case: $I_{\nu_0} = \frac{L_{\nu_*} \Phi_*}{4\pi} \times \frac{c}{H_0} \times \left[\frac{2}{3}\right]\left[1 - \frac{1}{(1+z_{out})^{3/2}}\right]$

$M_* = -19.67 + 5\log h$. $h = 0.7$, $M_* = -20.4$

Recall $m_{AB} = -2.5\log S_\nu^{(d)} - 48.6$

$$M_{AB} = -2.5 \log S_\nu(10pc) - 48.6$$

$$M_{AB} = -2.5 \log\left(\frac{L_\nu}{4\pi (10pc)^2}\right) - 48.6$$

$$\Rightarrow L_\nu = 4\pi \times (10 \times 3.08\times10^{18})^2 \times 10^{-.4(M_{AB}+48.6)}$$

If $M_{AB}^* = -20.4 \Rightarrow L_\nu^* = 6.5\times10^{28}$ erg/(s·Hz

$$I_{\nu_0} = \frac{6.5\times10^{28} \times 5.6\times10^{-3}}{4\pi \; (3.08\times10^{24})^3} \times \frac{3000}{.7} \times 3.08\times10^{24} \left(\frac{2}{3}\right)$$

$I_{\nu_0} \simeq 9\times10^{-21}$ erg cm^{-2}s^{-1} Hz^{-1}

if $\nu_0 = 4550$ Å (Blue)

$I \simeq \nu_0 I_{\nu_0} \simeq 5.8\times10^{-6}$

$u = \frac{4\pi I}{c} = 2.436\times10^{-15}$ erg/cm^3

$u_{CMB} = 4.17\times10^{-13}$ erg/cm^3 $\left(\begin{array}{c} 0.0068 \\ \text{CMB} \end{array}\right)$