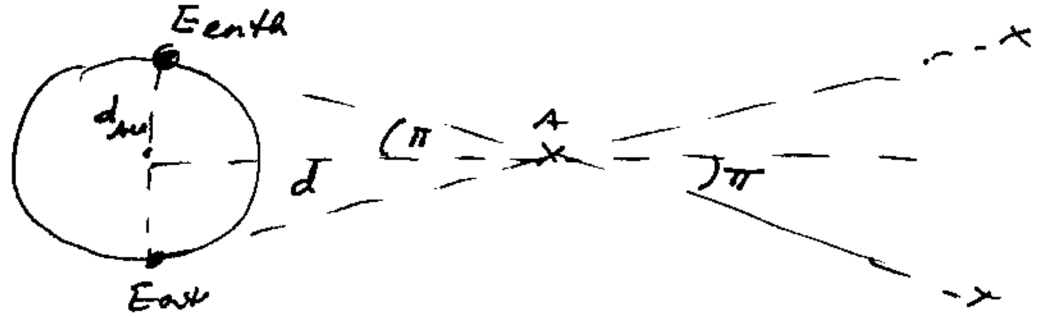
Empirical Basis for Hubble Law:

The most challenging task for establishing the Hubble law is to make estimates of galaxy distances, then compare them with recessional velocities inferred from spectral shifts to see (a) whether $u = H_0 d$ Holds and (b) Determine distance d from u when d is very large & calibrators too faint!

Parallax:

For nearby stars trigonometric parallax's provide unambiguous distances:



$$\tan(\pi) = \frac{d_{AU}}{d} \quad : d_{AU} = 1.5\times10^{13}\,\mathrm{cm}$$

In all cases $d \gg d_{AU} \Rightarrow \pi = d_{AU}/d$

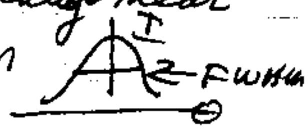
pc defined as distance at which $\pi = 1''$

But $\exists$ $\frac{180}{\pi}\times 60\times60 = 2.063\times10^{5}$ $''$/radian

$$\therefore\ d = \frac{2.063\times10^{5}\, d_{AU}}{\pi('')} = \frac{3.08\times10^{18}\,\mathrm{cm}}{\pi('')}$$

Largest parallaxes: $\sim 1'' \Rightarrow d \gg d_{AU}$

Difficult measurements to do from ground because atmospheric twinkling effects typically smear stars light distribution out to FWHM



$\simeq 1''$. Heroic efforts led to brightness centroid determinations to $\pi \simeq .03''$ or $d_\pi \simeq 30$ pc or so

Hipparchos satellite has now determined parallax angles for some 50,000 stars with median 1σ errors $\sigma_\pi \simeq 1$ mas and $V \simeq 12.5$. Best cases $\sigma_\pi \simeq 0.3$ mas $\Rightarrow$ $3\sigma_\pi \simeq 1\times10^3$ pc.
Eventually we will get π out to LMC $3\sigma_\pi \simeq 0.01$ mas

<u>Cepheid Variable Stars</u>:

But for me, the main achievement of Hipparchos satellite was to measure parallax angles for over 200 Cepheid variable stars, most of which can be used to calibrate period luminosity relationship for the <u>Cepheid Variables</u>

Importance of these stars:

(1) Easily recognized variables (pulsating envelopes)

$5 \leq P \leq 50$ days

(2) Excellent correlation between Luminosity and Period

$$M_V = -2.76 \log (P/1\,\text{day}) - 1.468$$

∴ Range in <u>absolute</u> magnitudes of Cepheids:

$$-6.5 \leq M_V \leq -3.7$$

Recall: $M_V - (M_V)_\odot = -2.5 \log \left(\frac{L_V}{(L_V)_\odot}\right)$

$$\Rightarrow \quad L_V = (L_V)_\odot \times 10^{-0.4[M_V - (M_V)_\odot]}$$

Since $(M_V)_\odot = +4.8$, luminosity of most luminous Cepheids will be: $L_V = (L_V)_\odot \times 10^{-0.4(-6.5-4.8)} \simeq 10^{4.5} L_\odot$

The high luminosities of these stars enables them to be observed in other galaxies.

Apparent magnitude: $\begin{cases} F(d) = \frac{L}{4\pi d^2} \; : \; F(d=10) = \frac{L}{4\pi (10)^2} \\ m - m(10) = -2.5 \log \frac{F(d)}{F(10)} = \\ \quad = -2.5 \log \left(\frac{10\,\text{pc}}{d}\right)^2 = +5 \log \frac{d}{1\,\text{kpc}} \end{cases}$

∴ For $M_V = -6$

d (pc)	$\underbrace{m - M}_{\text{distance modulus}}$	object	m	Comments
5×10^3	18.5	LMC	12.5	accurate measures
7×10^6	29	NGC4258	23	maser distances
1.5×10^7	31	Virgo Cluster	24.8	limit

<u>Physics</u>: instability strip in which gas becomes opaque at max. compression: $\kappa \propto \rho / T^{3.5}$ (partially ionized gas does not heat up during compression)

<u>Secondary distance Indicators</u>: Now we need indicators at d where $u \gg \sigma$

Figure Captions

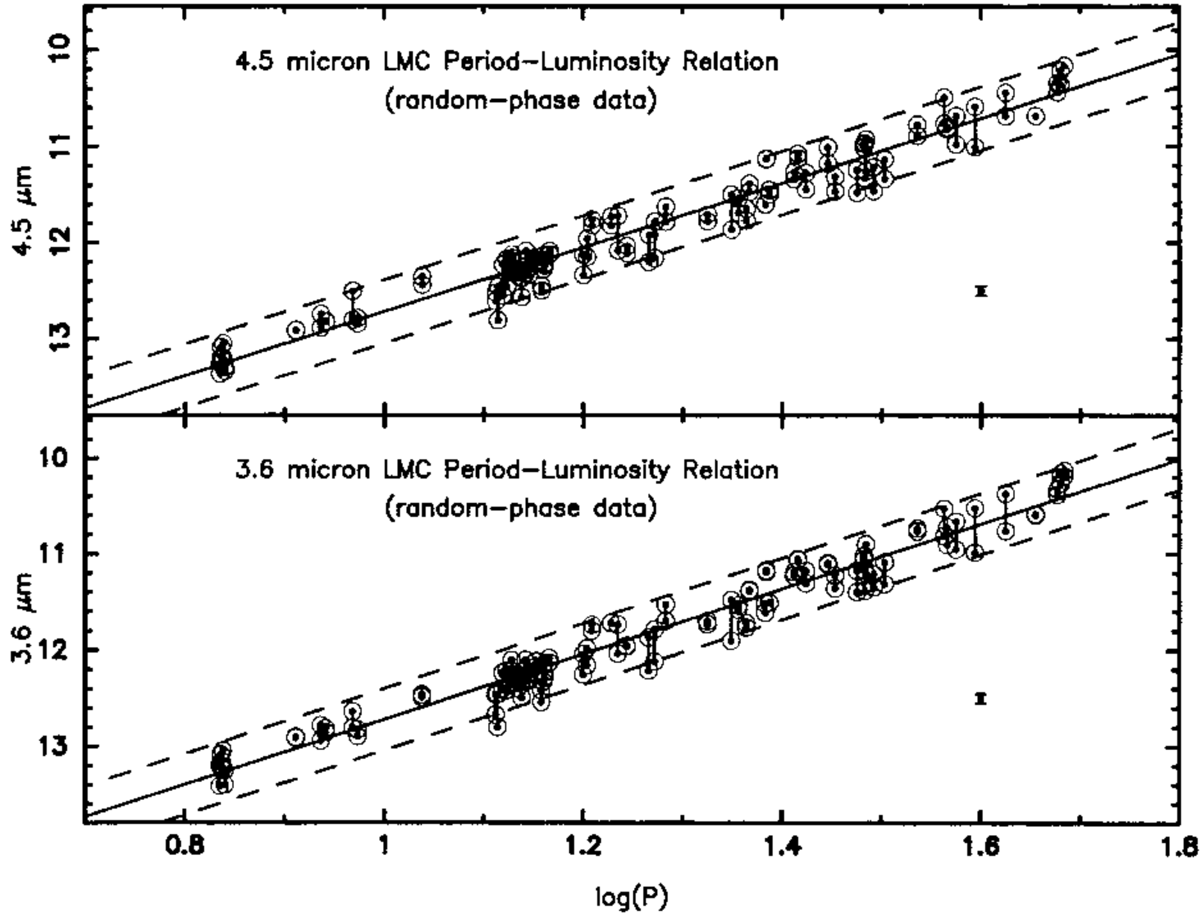
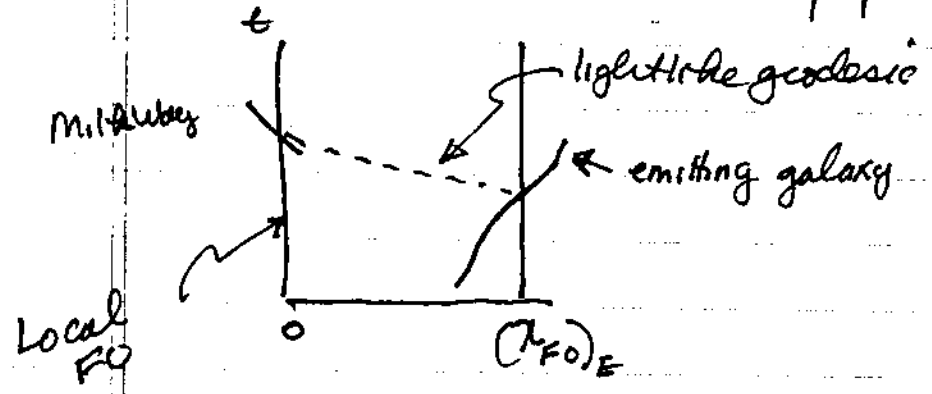


Fig. 1 – Random-phase (3.6 and 4.5μm) IRAC Period-Luminosity relations for LMC Cepheids plotted over the log(P) range 0.7 to 1.8. Observations of the same star seen at different epochs are joined by solid vertical lines. The solid line is a weighted least-squares fit to the data. The broken lines represent $\pm 2\sigma$ (typically ± 0.33 mag) bounds on the instability strip taken

It is only when $u \gg \sigma \simeq 300\ \mathrm{km\,s^{-1}}$ that approximation that we are measuring redshifts etc. of a fundamental observer becomes an excellent approximation.



Observer (us) in Milky Way receives light signal from distant emitting galaxy. That galaxy has velocity V_g wrst local FO. We have velocity V_{mw} wrst our local FO.

Suppose ν_{obs} is photon frequency that we measure and ν_E is frequency of photon emitted by distant galaxy. Then we measure redshift

$$1 + z_{meas} = \frac{\nu_E}{\nu_{obs}}$$

Let $(\nu_{FO})_g$ = be frequency measured by FO with $\chi = (\chi_{FO})_E$

$(\nu_{FO})_{mw}$ = be frequency measured by FO with $\chi = 0$

Then $$1 + z_{meas} = \frac{\nu_E}{(\nu_{FO})_g} \times \frac{(\nu_{FO})_g}{(\nu_{FO})_{mw}} \times \frac{(\nu_{FO})_{mw}}{\nu_{obs}}$$

$$1 + z_{meas} = \left(1 + \left(\frac{V_g}{c}\right)\right)(1+z)\left(1 + \frac{V_{mw}}{c}\right)$$

Therefore $1+z_{meas} \simeq 1+z$; i.e., z_{meas} approaches cosmological redshift provided:

(a) $V_g/c << 1$

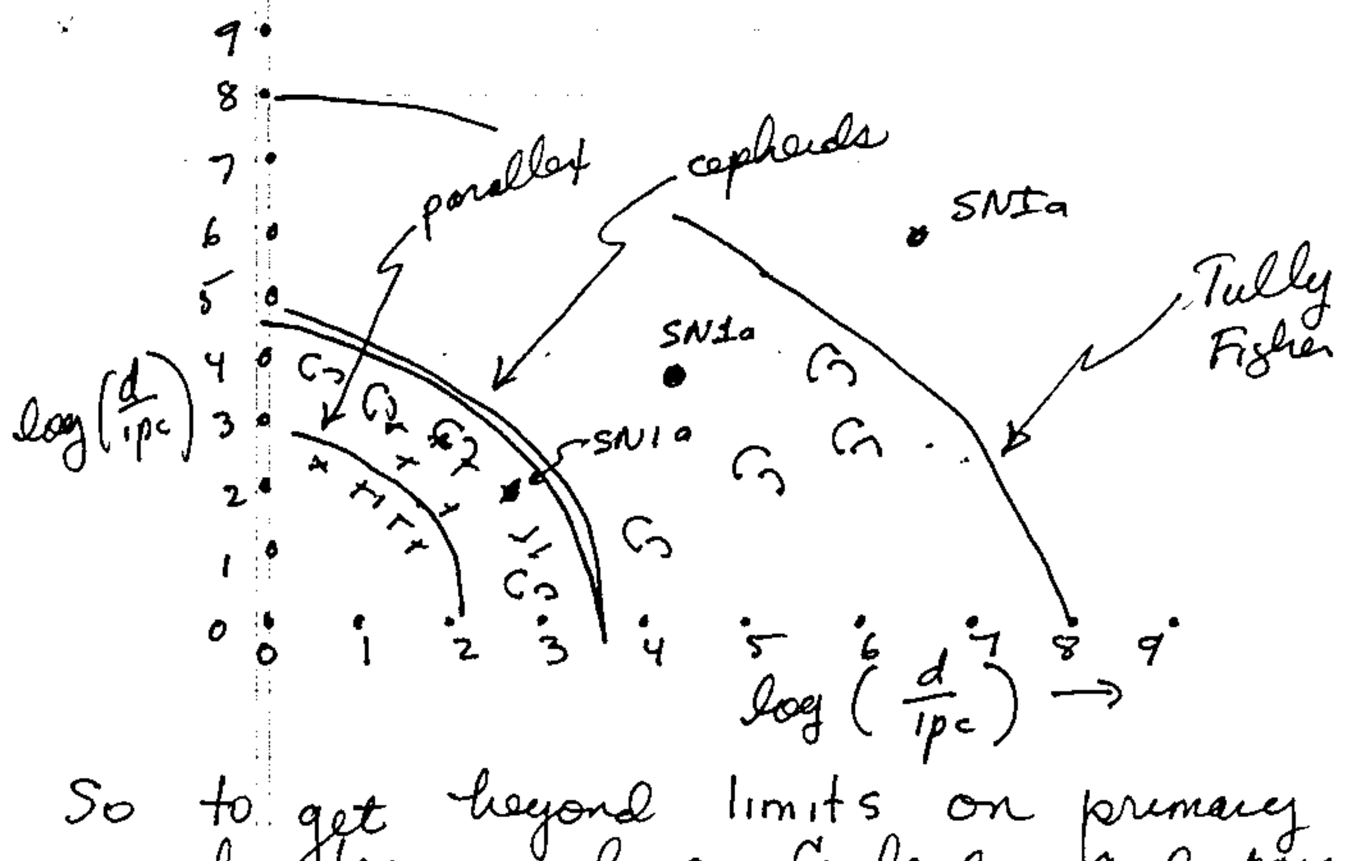
(b) $z >> V_g/c$

To be conservative, let $V_g \simeq 10^3$ km s^{-1}. Therefore, observed redshift will be dominated by cosmic expansion when

$$z >> \frac{10^3}{3\times10^5} = .003$$

or $$d >> \frac{V_g}{H_0} = \frac{10^3}{100h} >> \frac{10}{h}\ \mathrm{Mpc}$$

More like 30 - 50 Mpc is minimum!



So to get beyond limits on primary indicators such as Cepheids and parallax (and others not mentioned here) we need secondary indicators which

(a) are numerous enough so ~~they~~ exist in sufficiently high densities that a few are detected within realm of cepheids; i.e., the can be calibrated with Cepheid distances.

(b) But also luminous and numerous enough to

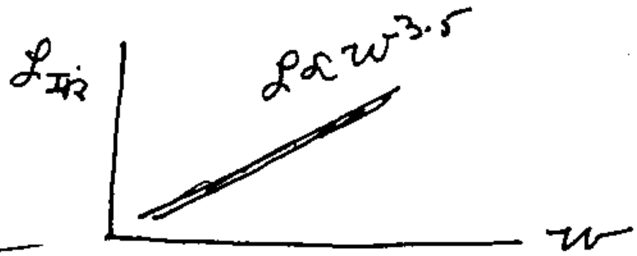
(a) be in "pure Hubble flow"

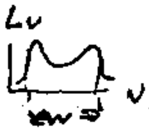
(b) remove statistical uncertainties.

Tully Fisher Technique

IR luminosity of spiral galaxies is tightly correlated with width of 21cm line emitted by interstellar HI

nearby galaxies





(a) Galaxies with distances calibrated by Cepheid variables

(b) out to galaxies with $v \simeq 10{,}000$ km/s (i.e., $z \simeq 0.03$)

Thus by measuring W we infer $\mathcal{L}$ and by measuring F_{IR} we get D from

$$F_{IR} = \frac{\mathcal{L}}{4\pi D^2}$$

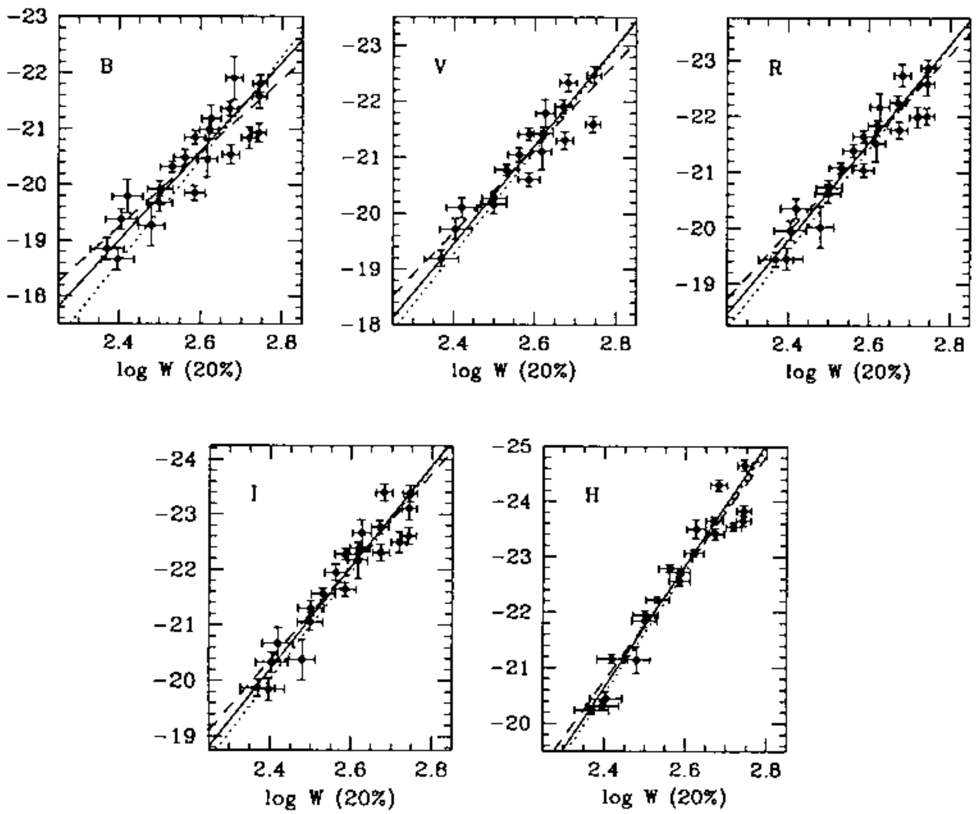


FIG. 1.—$BVRIH_{-0.5}$ Tully-Fisher relations for spiral galaxies with Cepheid distances, using 20% line width. Solid lines represent the bivariate fits, while the dotted and dashed lines represent inverse and direct fits, respectively.

Physically, we don't fully understand this "law" since

(a) $\mathcal{L}_{IR}$ comes mainly from older stars.

(b) But rotational speeds in spirals are controlled by centrifugal equilibrium between Gravitational pull exerted by dark matter and centripetal acceleration of "test particles" HI atoms.

$M = \frac{r^2 V}{G}$

In simple models $r \propto V$

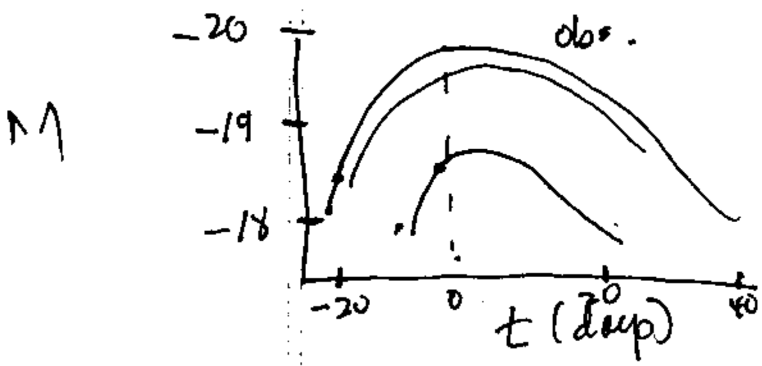
$\Rightarrow M \propto V^3$

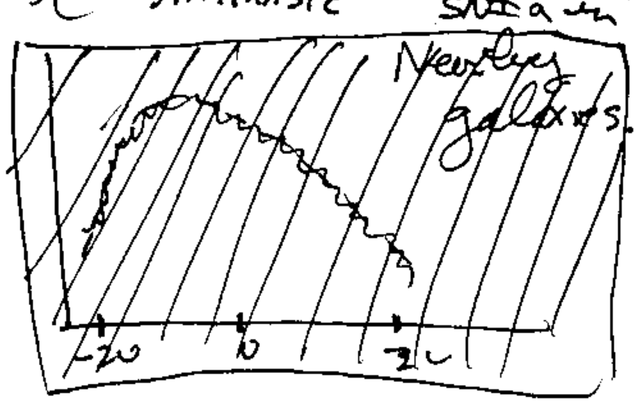
In any case $H_0 = 71 \pm 3 \pm 7$ km s^{-1} Mpc by this technique

SNIa : To go beyond ~ 100 Mpc or $z = 0.03$ we need more luminous calibrator since TF not accurate at such large redshifts.

- SNIa : Empirical fact: Tight correlation between time scale for SNIa to decline a measured time interval after peak brightness and $\mathcal{L}_{peak}$. More luminous SNIa take longer to decline than less SNIa. After correcting for time dilation, i.e.,

$$(\Delta t)_{obs} = (1+z)(\Delta t)_{intrinsic}$$





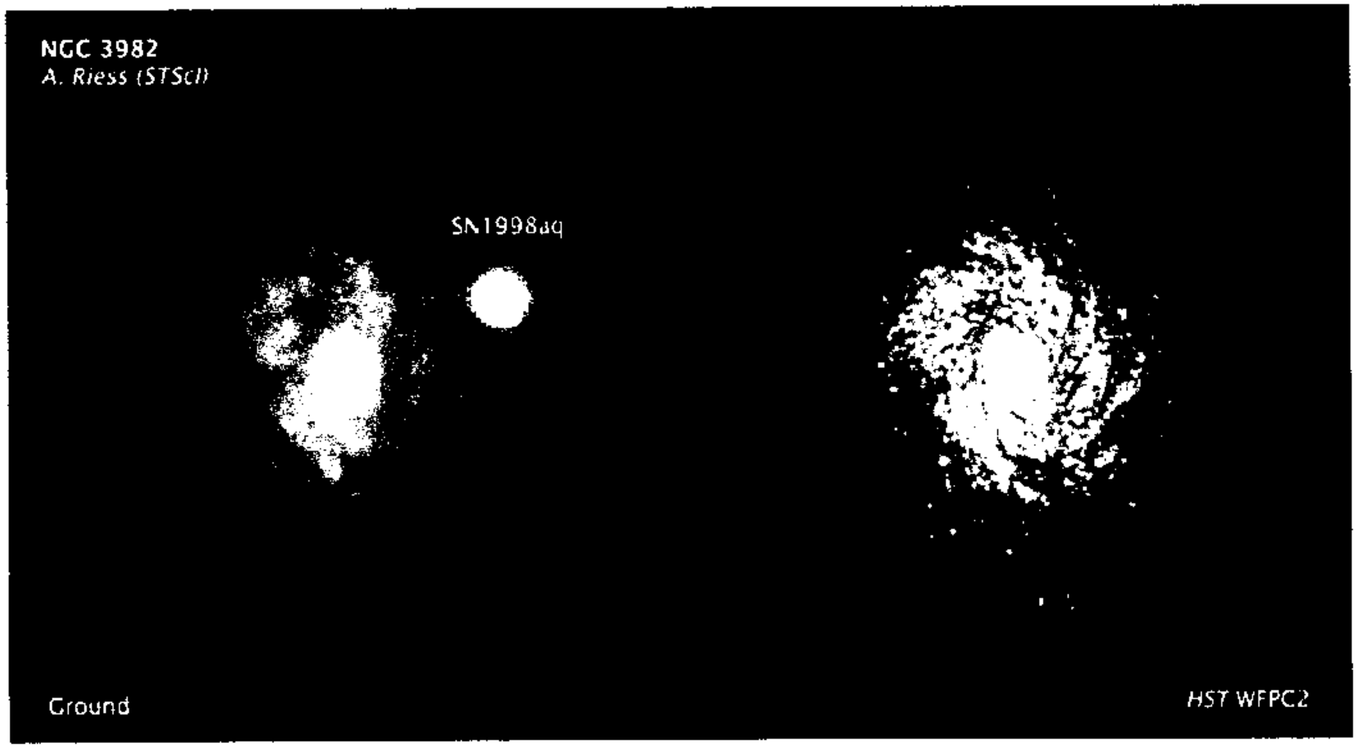
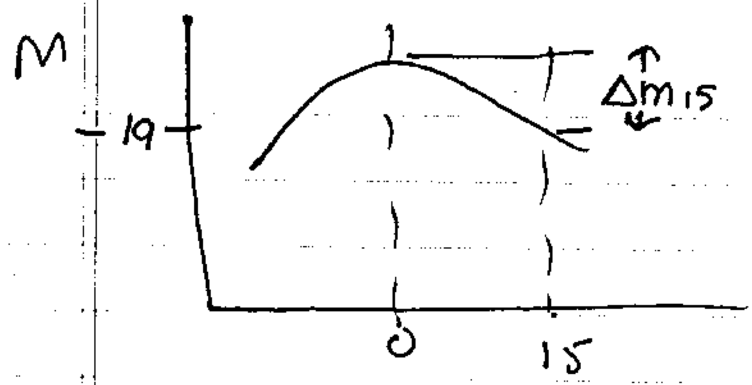
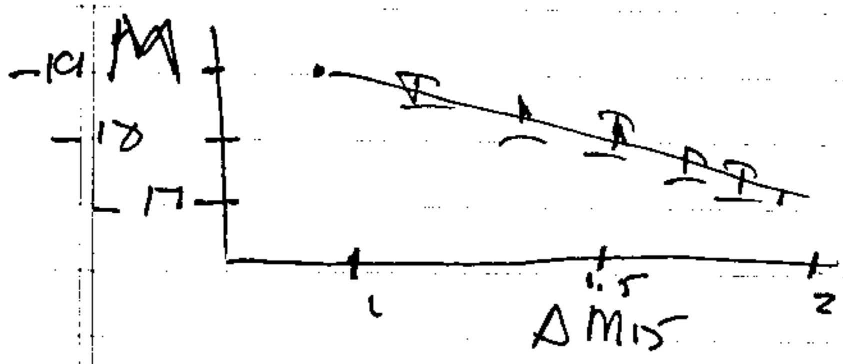


FIG. 3.—Color images of a $3' \times 3'$ region of the field of NGC 3982 and the SN Ia 1998aq. The left panel was produced from *B*-, *V*-, and *I*-band images obtained at the FLWO 1.2 m telescope in 1998 when SN 1998aq was near maximum brightness. The right panel is the same region produced from a stack of F435W, F555W, and F814W images obtained with WFPC2 on *HST* in 2001.

cos09-31



Measure Δm in 15 days



Find that more luminous SN exhibit smaller Δm than less luminous

Hubble diagram constructed by obtaining luminosities or absolute magnitudes for many SN Ia and from measured fluxes get distances. Redshifts best determined from parent galaxies.

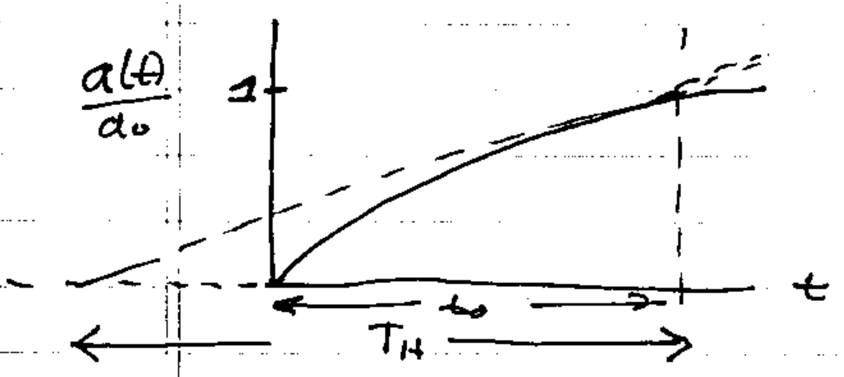
$$\boxed{H_0 = 73 \pm 4 \pm 5 \text{ km s}^{-1}\text{ Mpc}^{-1}}$$

~~Back to Theory: Dynamics of Expansion~~

Self-Consistency of expansion hypothesis

(1) Age of the Universe: are there independent tests that tell us the Universe is expanding? Can we relate predictions of expansion to other phenomena? What are these tests?

As universe expands $a(t)$ increases from $a=0$ to $a(t_0)$. We define current epoch as time at which $(\dot{a}/a)_{t_0} = H_0$.



(i) Naive model in which $\dot{a}$ = const (no matter, no stress energy). $\dot{a} = \frac{a_0}{T_H}$

$T_H = H_0^{-1}$

(ii) By contrast, decelerating model in which $\ddot{a} < 0$, age will be less than T_H

So, T_H is a rough order-of-magnitude estimate. (We will do more precise estimates).

$$H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \Rightarrow H_0 = (100\times10^5/3.08\times10^{24})h^{+1}\,s^{-1}$$
$$H_0 = 3.25\times10^{-18}h^{+1}\,s^{-1} \Rightarrow$$

$$T_H = 3.08\times10^{17}h^{-1}\,s = 10.3\,h^{-1}\,\mathrm{Gyr}$$
So for $h = 0.7$ $T_H = 14.7$ Gyr!

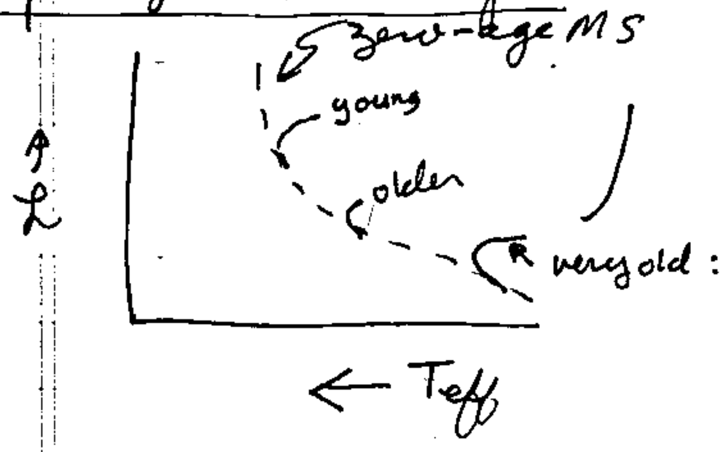
Compare:

Age of Solar System: Radioactive decay ages from meteorites $U^{235} \rightarrow Pb^{207}$
$U^{238} \rightarrow Pb^{206}$

$t = 4.6 \pm 0.1$ Gyr!

Globular Clusters: Oldest stellar systems in the Milky Way Galaxy

Hertzsprung Russel diagram:



$\mathcal{L} \propto M^{3.5}$: $M_{fuel} = \epsilon X M$: $\epsilon \simeq \frac{B_{He}}{4m_p} \sim 6\times10^{18}$ erg/g

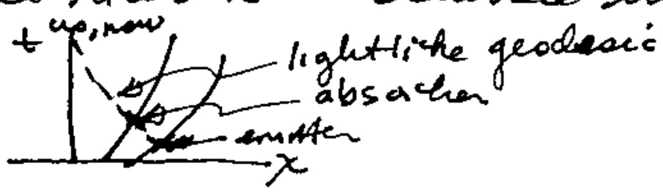
$$t_{MS} \propto \frac{M_{fuel}}{\mathcal{L}} \propto \frac{1}{M^{2.5}}$$

$t_{GC} \simeq 13 \pm 2$ Gyr

Other Tests

(1) Redshift independent of frequency $1+z = \frac{a(t_0)}{a(t_{em})}$ holds for photons at all emitted frequencies.

(2) Best evidence: my own work. Found that 21cm absorption predicted by optical redshift occurred where predicted



$1+z_{obs} = \frac{(\nu_E)_{opt}}{(\nu_{obs})_{opt}}$: Predicted 21cm abs. line

$$(\nu_{obs})_{21} = \frac{(\nu_E)_{21}}{1+z_{opt}} = \frac{(\nu_E)_{21}}{(\nu_E)_{opt}} \times (\nu_{obs})_{opt}$$

$\Rightarrow$ Ratio of frequencies not altered by cosmic expansion since

$$\left(\frac{\nu_{21}}{\nu_{opt}}\right)_{obs} = \left(\frac{\nu_{21}}{\nu_{opt}}\right)_{intrinsic}$$

Furthermore, these frequency ratios depend on physical constants: $\alpha^2 g_p m_e/m_p$ - Demonstration that dimensionless physical constants haven't varied!

Expansion Dynamics

How does $a(t)$ behave as a function of time? To answer this question we must insert FRW metric into the Einstein field equations and then solve for $a(t)$. Also impose homogeneous & isotropic symmetry!

Einstein field eqs.

cosmological constant

$$R_{ab} - \frac{1}{2} g_{ab} R^c_c + \Lambda g_{ab} = -\kappa T_{ab}$$

$$\kappa = \frac{8\pi G}{c^4}$$

Terms: R_{ab} is the Ricci tensor

$$R_{ab} = \Gamma^c_{ac,b} - \Gamma^c_{ab,c} + \Gamma^c_{ad}\Gamma^d_{cb} - \Gamma^c_{ab}\Gamma^d_{cd}$$

$$\Gamma^c_{ab} = \frac{1}{2} g^{cd}\left[g_{da,b} + g_{db,a} - g_{ab,d}\right]$$