PHYSICS 227: Cosmology

Instructor: Dr. A. M. Wolfe (phone: 47435)
Office Hours: Fri. 10-12

Please do not hand in mathematica output for your results. Rather take your mathematica output and plot them as separate graphs with detailed written explanations explaining what you have done.

Homework no. 3
Due: Tues. May 12
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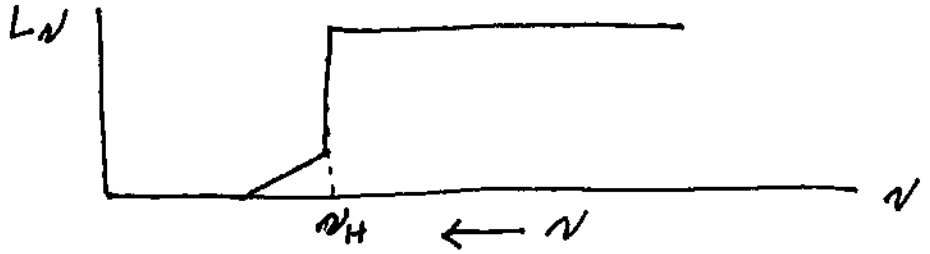
The problem is to determine whether ionizing background radiation emitted by galaxies is sufficient to ionize the Universe. To address this question, assume the following cosmological parameters: Ω_m=0.3, Ω_Λ=0.7, $\Omega_b h^2$=0.02, and h=0.7, and then proceed as follows:

(a) First, derive the neutral fraction of hydrogen as a function of redshift between z=3 and z=6. Do this by making the following assumptions: (i) assume the gas is in ionization equilibrium; (ii) read off the ionization rate, $\Gamma(z)$, from the following figure, where $\Gamma_{-12}=\Gamma(z)/10^{-12}$ s^{-1}. (iii) assume the coefficient for recombination of e^- onto H^+ is for gas at temperature, $T=10^4$K, i.e., $\alpha=4.3\times10^{-13}$ cm^{3}s^{-1}.(iv) Does the intergalactic gas become neutral by redshifts as large as z=6?

(b) Now compute the ionizing background radiation intensity $I_\nu(z)$ irradiating intergalactic gas for the same redshift interval. Assume the Universe is optically thin to this radiation. Assume it is emitted by galaxies drawn from a Schechter function with the following parameters: $\Phi_*=1.2\times10^{-3}$ Mpc^{-3}, $\alpha=-1.7$., and $M_*=-20.5$. Assume that only a small fraction say 3 % of the ionizing flux escapes at $\nu \geq \nu_H$ (where ν_H is the Rydberg frequency for hydrogen) and that the spectrum of the galaxies at $\nu \geq \nu_H$ is given by $L_\nu \propto \nu^{-2}$ (see graph).



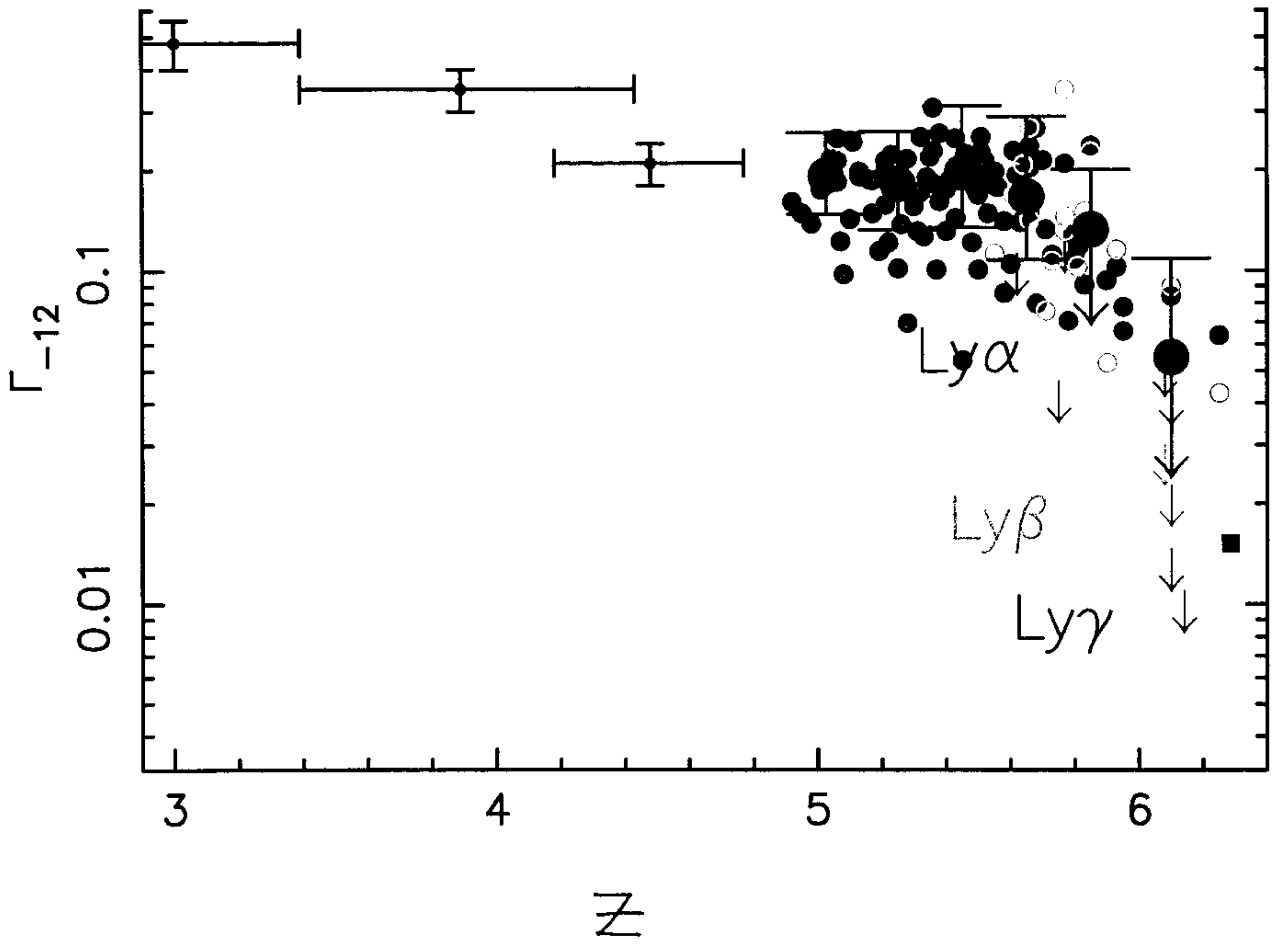
(i) Plot your the resulting intensity at $\nu = \nu_H$, ie., I_{ν_H}, for the redshift interval z = 3 to 6. (ii) Is the resulting background intensity of ionizing radiation sufficiently large to account for the ionization fractions computed in (a)? Hint: derive the relationship between ionization rate and I_{ν_H} from the following equation:

$$\Gamma = \int_{\nu_H}^{\infty} \frac{4\pi I_\nu \sigma_\nu}{h\nu} d\nu$$

where the photo-ionization cross section at frequency $\nu \geq \nu_H$ is given by $\sigma_\nu = 6.3\times10^{-18}(\nu_H/\nu)^3$ cm^2.

(c) Now make a more realistic computation by accounting for the optical depth of the intergalactic gas to photoionizing radiation. First compute this optical depth for z=3 to 6 from the results in part (a).

(d) Recompute the ionizing intensity for z= 3 to 6 and at frequency ν_H. You may do this by direct numerical integration, or by assuming the universe is optically thin between z and some larger redshift where the optical depth to photoionization at $\nu = \nu_H$ equals 1.



Γ_{-12}
0.1
0.01
3
4
5
6
z
Lyα
Lyβ
Lyγ

Appendix: Background intensity at redshift z and frequency ν is given by

$$I_\nu(t(z)) = \int_{t_{min}}^{t(z)} \frac{j_{\nu(1+z_{rel})}(t')\exp(-\tau_{\nu(1+z_{rel})}(t'))}{(1+z_{rel})^3} c dt'$$

where

$$\tau_{\nu(1+z_{rel})}(t') = \int_{t'}^{t(z)} k_{\nu(1+z_{rel})}(t'') c dt''$$

and

$$1 + z_{rel}(t') = \frac{1+z(t')}{1+z}$$