prove very useful in this case.

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Dynamical Role of Light Neutral Leptons in Cosmology

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Using the Vlasov equation, we show that massive galactic halos cannot be composed of stable neutral leptons of mass $\lesssim 1$ MeV. Since most of the mass in clusters of galaxies probably consists of stripped halos, we conclude that the "missing mass" in clusters does not consist of leptons of mass $\lesssim 1$ MeV (e.g., muon or electron neutrinos). Lee and Weinberg's hypothetical heavy leptons (mass ≈ 1 GeV) are not ruled out by this argument.

Observations of rich clusters of galaxies[1] show that most of their mass is in a form which emits very little light: the mass-to-blue-light ratio $M/L_B \approx 200(H/50)$ in solar units,[1] where H(km s^{-1} Mpc^{-1}) is the Hubble constant. This is the classical "missing-mass" problem, although really it is the light and not the mass which is missing. The problem persists in smaller systems. Groups of galaxies have $M/L_B \approx 140(H/50)$.[2] The Local Group has $M/L_B \approx 60(H/50)$ if $q_0 = 0$,[3] binary galaxies have $M/L_B \approx 65(H/50)$,[4] and even in our own Galaxy and $M31$ most of the mass probably lies at $r \gtrsim 10$ kpc, outside the visible regions.[5-7] We conclude that even for systems with radii as small as ~ 20 kpc, most of the mass lies in dark material unlike the stellar population with $M/L_B \approx 7(H/50)$,[8] which dominates the mass in the central regions of galaxies. It seems likely that the dark mass in binaries, groups, and clusters is material from the halos of galaxies that has been stripped off by tidal interactions.[9,10] More detailed and quantitative comparisons[11] based on expected values of M/L_B from stellar content yield the same conclusion; viz., that the ratio of "missing" to visible mass is similar for all systems large enough that separation would not occur by dissipative processes in the time available—and thus that the dark material is likely to be the same everywhere. These arguments and Occam's razor lead us to the assumption that the composition of the dark material is the same in galactic halos, binary galaxies, groups, and clusters.

In some gauge theories the electron and muon neutrinos have a small, nonzero rest mass,[12] and it has been proposed that the dark material consists of such particles.[13] Shortly after the big bang the neutrinos are relativistic and in thermal equilibrium, and so their momentum distribution is

$$n_{\nu_e}(\vec{p})\,d\vec{p} = n_{\bar{\nu}_e}(\vec{p})\,d\vec{p} = n_{\nu_\mu}(\vec{p})\,d\vec{p} = n_{\bar{\nu}_\mu}(\vec{p})\,d\vec{p} = \frac{g_\nu}{h^3}\,d\vec{p}\left\{\exp\left[\frac{p}{kT_\nu(z)}\right]+1\right\}^{-1}. \qquad (1)$$

In this equation $c=1$, $T_\nu(z)=T_\gamma(z)$ is the common temperature of the neutrinos and the radiation, and g_ν is the number of allowed helicity states. For neutrinos of nonzero rest mass two helicities are allowed, but the cross section for production of positive-helicity neutrinos is down by $\sim (m/kT_\nu)^2$ from the cross section for negative-helicity neutrinos, and as a result, the former decouple at a very high temperature, where the evolution of the universe is not well understood. Thus it is not clear whether both helicity states should be included in g_ν, and we only know that $1 \leq g_\nu \leq 2$. Both helicity states are decoupled by $kT_\nu \approx 1$ MeV,[14] and, since the electron and muon neutrinos are known to be less massive than 1 MeV, they are still relativistic when they decouple. Thereafter their momenta red shift like $1+z$, so that Eq. (1) continues to hold with $T_\nu(z) \propto 1+z$. But $T_\gamma(z)$ is also $\propto 1+z$ except for a jump of $(\frac{11}{4})^{1/3}$ at e^+e^- recombination,[14] and so we have

$$T_\nu(0)=(\tfrac{4}{11})^{1/3}T_\gamma(0)=1.9\text{ K}. \tag{2}$$

From Eqs. (1) and (2) the present mass density in neutrinos is

$$\rho_\nu=[(m_{\nu_e}+m_{\bar\nu_e})g_{\nu_e}+(m_{\nu_\mu}+m_{\bar\nu_\mu})g_{\nu_\mu}]\,6\pi\zeta(3)\left(\frac{kT_\nu(0)}{h}\right)^3=(1.9\times 10^{-31}\text{ g cm}^{-3})\left(\frac{m_{\nu_e}g_{\nu_e}+m_{\nu_\mu}g_{\nu_\mu}}{1\text{ eV}}\right). \tag{3}$$

In terms of the critical density $\rho_c=3H^2/8\pi G$,

$$\Omega_\nu=\frac{\rho_\nu}{\rho_c}=0.04\left(\frac{50}{H}\right)^2\frac{m_{\nu_e}g_{\nu_e}+m_{\nu_\mu}g_{\nu_\mu}}{1\text{ eV}}. \tag{4}$$

Neutrinos with a cosmologically interesting mass will have very small velocities: For $m \gtrsim 1$ eV, $v \approx kT_\nu(0)/m \lesssim 50$ km s^{-1}. Because this velocity is smaller than the typical velocity dispersion within galaxies or clusters, and because galaxies and clusters collapse in a dynamical time, the neutrinos will participate in the galaxy clustering. The ratio of the density distributed like the galaxies to ρ_c is called Ω^*. The best observational estimate of Ω^* is roughly 0.05.[15] Since $\Omega_\nu < \Omega^*$, we have

$$m_{\nu_e}g_{\nu_e}+m_{\nu_\mu}g_{\nu_\mu} \lesssim (1.2\text{ eV})(H/50)^2. \tag{5}$$

This mass is well below the current experimental upper limits on m_{ν_μ} and m_{ν_e} (cf. Ref. 13). This is the chain of argument which led Cowsik and McClelland[13] to the suggestion that massive neutrinos could supply the missing mass. If we look further, however, we can find a flaw. Because the neutrinos are noninteracting, the density of a fluid element in phase space is conserved (the Vlasov equation). Thus, the maximum fine-grained phase-space density is conserved. The maximum coarse-grained phase-space density is less well constrained: We know only that it must decrease with time, because the fluid can become "frothy" in phase space,[16] i.e., the regions of maximum fine-grained density may be mixed in with lower-density regions.

While neutrino background is homogeneous the maximum phase-space density is $2g_\nu h^{-3}$ from the four kinds of neutrinos, assuming $g_{\nu_e}=g_{\nu_\mu}\equiv g_\nu$ [cf. Eq. (1)]. We assume that the central regions of the bound systems formed by the neutrinos resemble isothermal gas spheres; then their velocity distribution is Maxwellian and the maximum phase-space density is $\rho_0 m_\nu^{-4}(2\pi\sigma^2)^{-3/2}$. Here ρ_0 is the central density and σ is the one-dimensional velocity dispersion, and we assume that $m_{\nu_e}=m_{\nu_\mu}\equiv m_\nu$. The core radius is defined by $r_c^2=9\sigma^2/4\pi G\rho_0$, and the requirement that the maximum phase-space density has decreased becomes

$$m_\nu^4>\frac{9h^3}{4(2\pi)^{5/2}g_\nu G\sigma r_c^2}, \quad m_\nu>(101\text{ eV})\left(\frac{100\text{ km s}^{-1}}{\sigma}\right)^{1/4}\left(\frac{1\text{ kpc}}{r_c}\right)^{1/4}g_\nu^{-1/4}. \tag{6}$$

In comparison, the Pauli principle requires only that the present maximum phase-space density is $<4g_\nu h^{-3}$, giving a limit on m_ν which is similar to Eq. (6) but less severe by a factor $2^{1/4}$. Note, however, that the principle that the maximum phase-space density decreases is quite distinct from the Pauli principle. For example, our argument would also work for any hypothetical noninteracting Maxwell-Boltzmann particles. It does not work for bosons because their equilibrium phase-space density does not have a maximum.

For rich clusters of galaxies[1] $r_c \simeq 0.25(50/H)$ Mpc, $\sigma \simeq 10^3$ km s^{-1}. Thus, $m_\nu \gtrsim (3.6\text{ eV})(H/50)^{1/2}g_\nu^{-1/4}$, roughly consistent with the limit (5). Binary galaxies, however, have $r_c \lesssim 100$ $(50/H)$ kpc (the typical separation of the components) and $\sigma \approx 100$ km s^{-1} (Ref. 4), and inequalities (5) and (6) yield $m_\nu \lesssim (0.6\text{ eV})(H/50)^2 g_\nu^{-1}$ and $m_\nu \gtrsim (10\text{ eV})(H/50)^{1/2} g_\nu^{-1/4}$, which are inconsistent. The inconsistency is worse still for galactic halos, where $r_c \lesssim 20$ kpc, $\sigma \approx 150$ km s^{-1}, and $m_\nu \gtrsim (20\text{ eV}) g_\nu^{-1/4}$. We conclude that massive muon or electron neutrinos cannot supply the dark material in galactic halos or binary galaxies or, by extension, in clusters of galaxies.

The contradiction cannot be removed by assuming that the luminous parts of the galaxies condense first and suck the neutrinos into their potensial well, because we have not assumed that the neutrinos supply all the gravitational potential.

Notice that this argument is not based on the Jeans mass. In fact, the present Jeans mass for the neutrinos is

$$M_J \approx \rho[\pi v_s^2/G\rho]^{3/2} \approx 3\times 10^{13} M_\odot [m_\nu/1\text{ eV}]^{-7/2} g_\nu^{-1/2}, \qquad (7)$$

where we have replaced the "sound speed" v_s by $[kT_\nu(0)/m_\nu]^{1/2}$. Thus systems of galactic or larger mass could form if $m_\nu \lesssim 4$ eV; the phase-space argument shows, however, that they would be larger than the systems we observe.

Are there any loopholes? The most uncertain parameter is Ω^*. If we regard these arguments as setting a lower limit on Ω^*, then the most conservative limit is obtained when $g_\nu = 1$ and one species of neutrinos is much more massive than the other (say, $m_{\nu_\mu} \gg m_{\nu_e}$). Then the values $r_c \lesssim 20$ kpc, $\sigma \approx 150$ km s^{-1} appropriate for galactic halos require $\Omega^* \gtrsim 1.0(50/H)^2$, $m_\nu \gtrsim 24$ eV, if massive neutrinos are to make up the halos. This result is well outside the acceptable limits to Ω^* and H (cf. Ref. 15). A very simple argument is worth noting at this point. Particles as heavy as 24 eV behave completely classically at the phase-space densities in the great clusters and would cluster on these scales precisely as the galaxies do.[17] If Ω^* were as large as unity in such particles, the ratio of unseen to visible matter in the clusters would be larger than it is, corresponding to mass-to-light ratios in excess of 1500—a value never approached in observation.

In a cosmology with nonzero muon or electron lepton number the problem only gets worse. The reason is that the maximum phase-space density of neutrinos plus antineutrinos is independent of chemical potential μ_ν, while the spatial density of neutrinos plus antineutrinos increases if μ_ν is nonzero. Thus inequality (6) is unchanged but inequality (5) requires smaller neutrino masses. (If m_ν is very small or zero, the neutrinos do not cluster with the galaxies; in this case, if $\mu_\nu/kT_\nu \gg 1$, neutrinos could close the universe, but they could not supply the missing mass in clusters. Cosmological helium production is also impossible in such models.[14])

Finally, we consider the possibility that there are other stable neutral leptons besides the electron and muon neutrino. If the lepton mass is $\lesssim 1$ MeV all of our arguments are still valid [if there are *more* than two species with comparable masses, the inconsistency of inequalities (5) and (6) is still worse]. If their mass is $\gtrsim 1$ MeV, the assumption that the leptons are relativistic when they decouple is no longer valid; in this case Lee and Weinberg[18] have shown that leptons with mass $\gtrsim 1$ GeV can contribute $\Omega_\nu \approx \Omega^*$. Such heavy leptons, although reprehensible on etymological grounds, are not ruled out by our phase-phase-density arguments (see also Ref. 17).

In summary, we know that there is some dark material which dominates the dynamics of binary galaxies, and makes up most of the mass in the outer halos of isolated galaxies. We have shown that this material cannot be muon or electron neutrinos of nonzero rest mass, or any stable neutral lepton less massive than 1 MeV. Several arguments suggest that the dark material is universal, so that the same restrictions apply to the unknown component which provides most of the mass in groups and clusters of galaxies.

We feel that the most likely remaining possibilities are small black holes or very low-mass stars, or perhaps some much heavier stable neutral particles.

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ERRATA

FIELD THEORY FOR THE STATISTICS OF BRANCHED POLYMERS, GELATION, AND VULCANIZATION. T. C. Lubensky and Joel Isaacson [Phys. Rev. Lett. 41, 829 (1978)].

This paper contains two errors. The first is the statement that dilute branched polymers with and without loops are in the same universality class. The fixed point for dilute branched polymers discussed in Eq. (10) and following applies only to branched polymers without loops. There is a new fixed point in $8-\epsilon'$ dimensions describing branched polymers with loops. The second error is in the value for ν [Eq. (10d)] for the loopless fixed point. The fixed-point values for x, y, and z and the value for η are, however, correct.

Both errors resulted from using the relation $G_\parallel = G_\perp$ [Eq. (6)] when $m=0$ in all calculations. Though this relation is true, the quantity $\tau_\perp = m^{-1}(G_\parallel - G_\perp)$ enters all recursion relations in a nontrivial way. The new fixed point in $8-\epsilon'$ dimensions for polymers with loops has $K_8 w^2 \tau_\perp = \frac{2}{9}\epsilon'$ and exponents $\eta = \frac{1}{9}\epsilon'$ and $\nu^{-1} = 2 - \frac{5}{9}\epsilon'$. The two exponents associated with r and $\tau_\perp$ at the loopless fixed point in $6-\epsilon$ dimensions are $\lambda_\pm = 2 - \frac{20}{3}y \pm \frac{1}{2}(96y^2 - 8z^2)^{1/2}$ ($\lambda_+ = 2 - 0.196\epsilon$ and $\lambda_- = 2 - 0.536\epsilon$). λ_+ is the inverse-correlation-length exponent and replaces Eq. (10d) for ν.

Details of these calculations will be published shortly.

CONFINING A TOKAMAK PLASMA WITH rf-DRIVEN CURRENTS. Nathaniel J. Fisch [Phys. Rev. Lett. 41, 873 (1978)].

Integrating over $v_\perp$ in Eq. (1) should give $\nu = 3\omega_p^4 \ln\Lambda / 4\pi n v_z^3$ in the normalizations following Eq. (2). Quantities which depend on the time normalization are affected. Of greater importance is that the numerical two-dimensional solution of Eq. (1) has recently been obtained.[1] Thus, for example, the ratio of the correct one-dimensional power dissipation to what is given in the Letter to the numerically determined power dissipation is $\frac{3}{2}$:1:0.6.

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