Physics 215C – Problem Set #1
due Monday April 20

1. Derive the Feynman rules for the trilinear and quadrilinear gauge boson vertices in Yang-Mills theory.

2. Consider a theory of massless non-Abelian gauge fields coupled to a multiplet of Dirac fields transforming according to some irreducible representation of the gauge group. Compute the amplitude to $O(g^2)$ for gauge boson-fermion elastic scattering in Feynman-'t Hooft gauge. (There are three relevant graphs.) Show that when ϵ, the polarization vector of the incoming gauge boson, is set equal to k, the gauge boson momentum, the amplitude vanishes, and thus, that this result doesn't depend on the choice $\xi = 1$. Note that for a massless vector boson, the momentum k and polarization vector ϵ satisfy $k^2 = 0$, $\epsilon \cdot k = 0$ and $\epsilon^2 = -1$.

The result of this problem generalizes the Ward identity for the Abelian case. In the non-Abelian case, the trilinear gauge boson vertex is required for exact cancellation to occur. In addition, the trilinear gauge boson coupling g must be equal to the fermion-gauge boson coupling constant. This Ward identity is associated with the fact that there exists a Lorentz transformation which leaves k unchanged but transforms ϵ to $\epsilon + ak$ for any real k. Thus there is no Lorentz-invariant way of stating whether the polarization vector assigned to a given helicity state is ϵ or $\epsilon + ak$.