HW2 Scattering theory

1) The scattering problem of particle with mass m, in the centeral force field $V(r) = \frac{\alpha}{r^2}$, where $\alpha > 0$.

a) Usi partial wave method to find δ_ℓ each partial wave channel ℓ. Hint: the radial equation can be solved exactly by using spherical Bessel function of order ν, where ν satisfies:

$$\ell(\ell+1) + \frac{2m\alpha}{\hbar^2} = \nu(\nu+1) \quad \text{for each channel of } \ell.$$

b) under the condition $\frac{m\alpha}{\hbar^2} << \frac{1}{8}$, find the approximate formulae for δ_ℓ, scattering amplitude $f(\theta)$, and differential cross section $\sigma(\theta, \varphi)$.

c) Use Born approximation to calculate $f(\theta)$ and $\sigma(\theta, \varphi)$ and compare with b).

You might need to use the formula

$$\sum_\ell P_\ell(\cos\theta) = \frac{1}{\sin\theta/2}$$

2) Scattering on a spherical δ-potential shell.

A particle with mass m is scattered by a repulsive δ-shell

$$\frac{2m}{\hbar^2} V(r) = \gamma\, \delta(r-R), \text{ where } \gamma > 0 .$$

a) Set up an equation that determines the s-wave phase shift δ_0 as a function of k which satisfies $E = \frac{\hbar^2 k^2}{2m}$.

b) Assume if $\gamma \gg \frac{1}{R}$, and $\gamma \gg k$. Show if $\tan kR$ is not close to zero, δ_0 resembles the hard-sphere result given in the notes. Also show if $\tan kR$ is close (but not exactly equal to) zero, resonance occurs, i.e. $\operatorname{ctg} \delta_0$ goes through zero from the positive side as k increases.

c) Determine approximatly the positions of resonances of incoming energe $E = \frac{\hbar^2 k^2}{2m}$ keeps terms of order $1/\gamma$.

Compare them with the bound state energies for a particle confined inside a spherical wall of the same radius of

$$\begin{cases} V = 0 & r < R \\ V = \infty & r > R . \end{cases}$$

d) Obtain an approximate expression for the resonace width Γ defined by $$\Gamma = \frac{-2}{d(\mathrm{ctg}\,\delta_0/dE)\Big|_{E=E_{\mathrm{resonace}}}}.$$

Notice that resonance is sharp as γ goes large.