<u>PHYSICS 210A, HOMEWORK ASSIGNMENT #2</u> April 23, 2009

1. In Problem 3.15, you examined a three-dimensional, extreme-relativistic gas (for which the single-particle energy $\varepsilon = pc$) in the framework of the canonical ensemble. Now, examine the same problem in the framework of the microcanonical ensemble.

To do so, you'll have to determine the volume of that particular region of the phase space which is defined by the condition

$$0 \leq \sum_{i=1}^{N} (p_i c) \leq E.$$

Hint: To determine this volume, follow the procedure of Appendix C; however, instead of using the integral C4, use the integral

$$\int_0^\infty e^{-r} r^2 \, dr = 2.$$

Check that, for the system under consideration, the present procedure yields the same density of states $g(E)$ that you obtained by taking the "inverse Laplace transform" of the partition function $Q_N(V, T)$.

2. You have already done Problem 3.29, which pertained to an inharmonic oscillator treated *classically*. Now do Problem 3.30, which pertains to a similar oscillator treated *quantum-mechanically*.

3. The rotational states of a diatomic molecule are characterized by energy eigenvalues

$$\varepsilon_l = l(l+1)\hbar^2/2I, \qquad l = 0, 1, 2, \ldots,$$

with multiplicity $g_l = 2l+1$; here, I is the moment of inertia of the molecule [for details, see pp. 144-146 of the text].

Set up the partition function of the problem and show that, at high temperatures, the specific heat per molecule turns out to be

$$C_{rot} = k\left[1 + \frac{1}{45}\left(\frac{\theta}{T}\right)^2 + \cdots\right],$$

where

$$\Theta = \hbar^2/2Ik.$$

Hint: write $l(l+1) = (l+\frac{1}{2})^2 - \frac{1}{4}$ and $2l+1 = 2(l+\frac{1}{2})$, and evaluate the summation over l (that appears in the expression for the partition function) by using the following version of the Euler-MacLaurin formula rather than the one employed in the text :

$$\sum_{l=0}^{\infty} f(l+\tfrac{1}{2}) = \int_0^{\infty} f(x)\,dx + \frac{1}{24} f'(0) - \frac{7}{5{,}760} f'''(0) + \cdots,$$

with

$$f(x) = x\, e^{-\left(\frac{\Theta}{T}\right)x^2}.$$

4. Consider a paramagnetic system consisting of N non-interacting magnetic dipoles, each with spin ½ and multiplicity *2*, in equilibrium at temperature T and subjected to an external magnetic field H. Calculate for this system the quantities $\overline{M}$, $\overline{M^2}$ and χ ----- and verify that that these quantities do conform to the *general* relationship

$$\overline{M^2} - \overline{M}^2 = kT\chi.$$

5. Solve Problem 3.36 of the text.
   Note that the factor $(\mu\mu')$ in the final result should be $(\mu\mu')^2$.

6. Solve Problems 4.10 and 4.11 of the text.
   Note that the parameter N_0 here plays the role of the <u>area</u> of the surface, which is similar to the <u>volume</u> of a box. So, you have to evaluate $Q_N(N_0, T)$ and $\mathcal{Q}(z, N_0, T)$.