PHYSICS 4C: QUIZ 5 SOLUTIONS

Problem 1

The total voltage drop for any loop is 0, so

$$V = \frac{Q}{C} + V_R \tag{1}$$

for either capacitor and either resistor. Specifically, the voltage drop over R_1 at the instant when $Q_1 = 7\mu\text{C}$ is

$$V_{R_1} = V - \frac{Q_1}{C_1} = 12V - \frac{7\mu\text{C}}{1.8\mu\text{F}} = 8.11\text{V} \tag{2}$$

and the current through it is

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{8.11\text{V}}{1.8\text{k}\Omega} = 4.5\text{mA} \tag{3}$$

Alternative solution. The equivalent resistance is $R_{eq} = 1.35\text{k}\Omega$ (in parallel, inverses of resistances add). The equivalent capacitance is $C_{eq} = 7.2\mu\text{F}$ (in parallel, capacitances add). The voltage droop over the two capacitors is the same,

$$V_C = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_{eq}}{C_{eq}}, \tag{4}$$

so we can solve for the charge on the equivalent capacitor:

$$Q_{eq} = \frac{Q_1 C_{eq}}{C_1} = 28\mu\text{C} \tag{5}$$

Now we use the formula for charging the equivalent capacitor

$$Q_{eq} = C_{eq} V \left(1 - e^{-t/R_{eq}C_{eq}}\right), \tag{6}$$

frown which we can solve for time:

$$t = -R_{eq}C_{eq} \ln\left(1 - \frac{Q_{eq}V}{C_{eq}}\right) = 0.0038\text{s}. \tag{7}$$

Dividing both sides of equation (6) by C_{eq}, we can solve for the voltage drop across the capacitors:

$$V_C = V\left(1 - e^{-t/R_{eq}C_{eq}}\right), \tag{8}$$

from which we find the voltage drop across the resistors

$$V_R = V - V_C = Ve^{-t/R_{eq}C_{eq}} = 8.11\text{V} \tag{9}$$

and current through the first resistor

$$I_{R_1} = \frac{V_R}{R_1} = \frac{8.11\text{V}}{1.8\text{k}\Omega} = 4.5\text{mA} \tag{10}$$

Problem 2

Adding resistors in series for each branch and replacing them by equivalent resistances, we get I_1 flowing through $R_1 = 19\Omega$, I_2 flowing through $R_2 = 12\Omega$, and I_3 flowing through $R_3 = 35\Omega$. The current going into and out of a node is the same:

$$I_1 = I_2 + I_3 \tag{11}$$

The voltage drop is 0 over any closed loop, so for the top loop,

$$6\text{V} - (19\Omega)I_1 + 12\text{V} - (12\Omega)I_2 = 0 \tag{12}$$

and for the outer loop,

$$6\text{V} - (19\Omega)I_1 + 12\text{V} - (35\Omega)I_3 = 0 \tag{13}$$

Solving these three equations for the three unknowns, we get

$$I_1 = 0.64\text{A}, I_2 = 0.48\text{A}, I_3 = 0.16\text{A} \tag{14}$$

Problem 3

$$F = ma = \frac{mv^2}{R} = 3.1 \times 10^{-4}\text{N} \tag{15}$$

$$B = \frac{F}{qv} = 0.023\text{T} \tag{16}$$

$$f = \frac{1}{T} = \frac{v}{2\pi R} = 3 \times 10^3\text{Hz} \tag{17}$$

Problem 4

$$F = I\vec{l} \times \vec{B} \tag{18}$$

When the wire is along x-axis,

$$F_1 = Il\hat{\imath} \times \vec{B} = 5\left(B_y\hat{k} - B_z\hat{\jmath}\right) = -2.5\hat{\jmath} \tag{19}$$

When the wire is along y-axis,

$$F_2 = Il\hat{\jmath} \times \vec{B} = 5\left(B_z\hat{\imath} - B_x\hat{k}\right) = 2.5\hat{\imath} - 5\hat{k} \tag{20}$$

Setting vector components in the same direction equal to each other,

$$B_x = 1\text{T}, B_y = 0, B_z = 0.5\text{T} \tag{21}$$