PHYSICS 4C: QUIZ 3 SOLUTIONS

Problem 1

In equilibrium, the two spheres are at the same potential

$$V = \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}, \tag{1}$$

The total charge on the surfaces of the two spheres is Q, so:

$$Q_2 = Q - Q_1. \tag{2}$$

Plugging this into equation (1) and rearranging,

$$Q_1 = \frac{QR_1}{R_1 + R_2}. \tag{3}$$

Problem 2

No work done means the potential at points A and B is the same. The potentials are

$$V_A = k\left(\frac{30\mu\text{C}}{30\text{cm}} - \frac{10\mu\text{C}}{20\text{cm}}\right) = 0.5k\frac{\mu\text{C}}{\text{cm}} \tag{4}$$

$$V_B = k\left(\frac{30\mu\text{C}}{L\text{ cm}} - \frac{10\mu\text{C}}{(L+50)\text{cm}}\right) \tag{5}$$

Setting them equal to each other and canceling k and units,

$$\frac{30}{L} - \frac{10}{L+50} = 0.5 \tag{6}$$

$$L^2 + 10L - 3000 = 0 \tag{7}$$

$$L = -5 \pm 55 \tag{8}$$

The negative sign gives a solution to the right of q_2, so we choose the positive, $L = 50$cm.

Problem 3

The equivalent capacitance of C_2 and C_3 (series) is

$$C_a = \left(\frac{1}{2C} + \frac{1}{C}\right)^{-1} = \frac{2C}{3}. \tag{9}$$

The equivalent capacitance of C_a and C_4 (parallel) is

$$C_b = \frac{2C}{3} + 2C = \frac{8C}{3}. \tag{10}$$

The equivalent capacitance of the circuit is the equivalent capacitance of C_b and C_1 (series),

$$C_{circuit} = \left(\frac{3}{8C} + \frac{1}{C}\right)^{-1} = \frac{8C}{11}. \tag{11}$$

For the second part, we can write down the following equations for voltages, using the same equivalent capacitors as before:

$$V_1 + V_b = V, \text{and} \tag{12}$$

$$Q_1 = Q_b$$
$$C_1V_1 = C_bV_b$$
$$CV_1 = \frac{8CV_b}{3}. \tag{13}$$

Eliminating V_1 between equations (12) and (13) and solving for V_b,

$$V_b = \frac{3V}{11}. \tag{14}$$

Next, we find V_2 from V_b:

$$V_2 + V_3 = V_b = \frac{3V}{11} \tag{15}$$

$$Q_2 = Q_3$$
$$C_2V_2 = C_3V_3$$
$$2CV_2 = CV_3. \tag{16}$$

Eliminating V_3 between equations (15) and (16) and solving for V_2,

$$V_2 = \frac{V_b}{3} = \frac{V}{11} \tag{17}$$

$$Q_2 = C_2V_2 = \frac{2CV}{11}. \tag{18}$$

Problem 4

The two halves of the capacitor act as two capacitors in parallel, each of area $A/2$. Their capacitances add:

$$C = C_1 + C_2 = (K_1 + K_2)\frac{\epsilon_0 A}{2d}. \tag{19}$$