PHYSICS 4C: QUIZ 2 SOLUTIONS

Problem 1

$E = 0$ inside a conductor, thus, all Gaussian surfaces inside the cube including those that enclose the cavity must enclose no total charge. Thus, charges inside the cube must distribute themselves in such way that they cancel the charge q in the center. This is accomplished by having a total charge of $-q$ uniformly (by symmetry) distributed on the inner surface of the conductor, resulting in surface charge density of

$$\sigma = \frac{-q}{A} = \frac{-q}{4\pi R^2}.$$

The total charge of the cube is Q with or without the center charge. With the center charge, there is a total of $-q$ on the inner surface, thus, rest of the cube must have charge $Q + q$. Again, since there is no electric field inside a conductor, this charge cannot be inside the cube and thus must be distributed on the outer surface. Since a cube has 6 identical sides, total charge per side is

$$Q_{side} = \frac{Q + q}{6}.$$

Problem 2

Position the plate in the xy plane such that the mid-plane is at $z = 0$ (the plate fills $-2.5\text{cm} < z < 2.5\text{cm}$). Draw a cylindrical Gaussian surface of base area A and height $2z$ such that its lower base is at $-z$ and its upper base is at z. By symmetry, electric field will point perpendicular to the plate and will be equal and opposite at the top and bottom bases of the Gaussian surface. There will be no flux through the rest of the cylinder, so only top and bottom will contribute. There are two regions that will have different solutions, outside and inside of the plate.

Outside ($|z| > 2.5\text{cm}$):

$$\oint E \cdot da = 2AE = \frac{Q_{encl}}{\epsilon_0} = \frac{\rho A \Delta z}{\epsilon_0},$$

where $\Delta z = 0.05\text{m}$ is the thickness of the plate. Solving for the electric field,

$$E_{out} = \frac{\rho \Delta z}{2\epsilon_0} = \frac{(10^{-5}\text{Cm}^{-3})(0.05\text{m})}{2(8.85 \times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2})} = 3 \times 10^4 \frac{\text{N}}{\text{C}},$$

pointing up above the plate and down below the plate.

Inside ($|z| < 2.5\text{cm}$):

$$\oint E \cdot da = 2AE = \frac{Q_{encl}}{\epsilon_0} = \frac{\rho A(2z)}{\epsilon_0},$$

which gives

$$E_{in} = \frac{\rho z}{\epsilon_0},$$

pointing up above the mid-plane and down below the mid-plane ($E = 0$ at $z = 0$). Note that this reduces to the previous equation at $z = \pm 0.025$m, as it should because electric field is continuous.

Problem 3

Divide the space into three regions: (1) the cavity, (2) the shell, and (3) space outside the shell.

Region 1 ($0 < r < R$): $E = 0$ since any Gaussian surface inside this region encloses no charge.

Region 2 ($R < r < 2R$): Spherical Gaussian surface of radius r will enclose the charge

$$Q_{encl} = \rho V = \rho(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R^3) = \frac{4\pi(r^3 - R^3)\rho}{3}.$$

Gauss's theorem gives

$$\oint E \cdot da = AE = 4\pi r^2 E = \frac{Q_{encl}}{\epsilon_0},$$

resulting in the electric field

$$E = \frac{Q_{encl}}{A\epsilon_0}\hat{r} = \frac{\frac{4}{3}\pi(r^3 - R^3)\rho}{4\pi r^2 \epsilon_0}\hat{r} = \frac{(r^3 - R^3)\rho}{3r^2\epsilon_0}\hat{r}.$$

It is instructive to note that at $r = R$, this equation reduces to $E = 0$, and at $r = 2R$, it reduces to the equation for Region 3, thus, the solution is continuous at both boundaries.

Region 3 ($r > 2R$):

$$Q_{encl} = \rho V = \rho(\frac{4}{3}\pi(2R)^3 - \frac{4}{3}\pi R^3) = \frac{4\pi(7R^3)\rho}{3}.$$

Using Gauss's theorem, we get electric field

$$E = \frac{Q_{encl}}{A\epsilon_0}\hat{r} = \frac{\frac{4}{3}\pi(7R^3)\rho}{4\pi r^2\epsilon_0}\hat{r} = \frac{7R^3\rho}{3r^2\epsilon_0}\hat{r}.$$