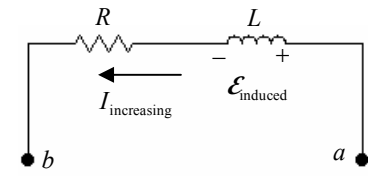


Homework for the week of November 24. 9th week of classes.
Ch. 30: 3, 9, 12, 21, 24, 27, 34, 37, 46, 53, 65

3. We find the mutual inductance of the inner loop. If we assume the outer solenoid is carrying current I_1 , then the magnetic field inside the outer solenoid is $B = \mu_0 \frac{N_1}{l} I_1$. The magnetic flux through each loop of the small coil is the magnetic field times the area perpendicular to the field. The mutual inductance is given by Eq. 30-1.

$$\Phi_{21} = BA_2 \sin \theta = \mu_0 \frac{N_1 I_1}{l} A_2 \sin \theta ; M = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2 \mu_0 \frac{N_1 I_1}{l} A_2 \sin \theta}{I_1} = \boxed{\frac{\mu_0 N_1 N_2 A_2 \sin \theta}{l}}$$

9. We draw the coil as two elements in series, and pure resistance and a pure inductance. There is a voltage drop due to the resistance of the coil, given by Ohm's law, and an induced emf due to the inductance of the coil, given by Eq. 30-5. Since the current is increasing, the inductance will create a potential difference to oppose the increasing current, and so there is a drop in the potential due to the inductance. The potential difference across the coil is the sum of the two potential drops.



$$V_{ab} = IR + L \frac{dI}{dt} = (3.00 \text{ A})(3.25 \Omega) + (0.44 \text{ H})(3.60 \text{ A/s}) = \boxed{11.3 \text{ V}}$$

12. The inductance of the solenoid is given by $L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi d^2}{4}$. The (constant) length of the wire is given by $l_{\text{wire}} = N \pi d_{\text{sol}}$, and so since $d_{\text{sol 2}} = 2.5 d_{\text{sol 1}}$, we also know that $N_1 = 2.5 N_2$. The fact that the wire is tightly wound gives $l_{\text{sol}} = N d_{\text{wire}}$. Find the ratio of the two inductances.

$$\frac{L_2}{L_1} = \frac{\frac{\mu_0 \pi N_2^2 d_{\text{sol 2}}^2}{4 l_{\text{sol 2}}}}{\frac{\mu_0 \pi N_1^2 d_{\text{sol 1}}^2}{4 l_{\text{sol 1}}}} = \frac{N_2^2 d_{\text{sol 2}}^2}{N_1^2 d_{\text{sol 1}}^2} = \frac{l_{\text{wire}}^2 / \pi^2}{l_{\text{sol 1}}^2} = \frac{l_{\text{sol 1}}}{l_{\text{sol 2}}} = \frac{N_1 d_{\text{wire}}}{N_2 d_{\text{wire}}} = \frac{N_1}{N_2} = \boxed{2.5}$$

21. We create an Amperian loop of radius r to calculate the magnetic field within the wire using Eq. 28-3. Since the resulting magnetic field only depends on radius, we use Eq. 30-7 for the energy density in the differential volume $dV = 2\pi r l dr$ and integrate from zero to the radius of the wire.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}} \rightarrow B(2\pi r) = \mu_0 \left(\frac{I}{\pi R^2} \right) (\pi r^2) \rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

$$\frac{U}{l} = \frac{1}{l} \int u_B dV = \int_0^R \frac{1}{2\mu_0} \left(\frac{\mu_0 I r}{2\pi R^2} \right)^2 2\pi r dr = \frac{\mu_0 I^2}{4\pi R^4} \int_0^R r^3 dr = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

24. (a) We set I equal to 75% of the maximum value in Eq. 30-9 and solve for the time constant.

$$I = 0.75 I_0 = I_0 (1 - e^{-t/\tau}) \rightarrow \tau = -\frac{t}{\ln(0.25)} = -\frac{(2.56 \text{ ms})}{\ln(0.25)} = 1.847 \text{ ms} \approx \boxed{1.85 \text{ ms}}$$

- (b) The resistance can be calculated from the time constant using Eq. 30-10.

$$R = \frac{L}{\tau} = \frac{31.0 \text{ mH}}{1.847 \text{ ms}} = \boxed{16.8 \Omega}$$

27. (a) We use Eq. 30-5 to determine the emf in the inductor as a function of time. Since the exponential term decreases in time, the maximum emf occurs when $t = 0$.

$$\mathbf{e} = -L \frac{dI}{dt} = -L \frac{d}{dt} [I_0 e^{-t/\tau}] = \frac{LI_0 R}{L} e^{-t/\tau} = V_0 e^{-t/\tau} \rightarrow \boxed{\mathbf{e}_{\max} = V_0}.$$

- (b) The current is the same just before and just after the switch moves from A to B. We use Ohm's

law for a steady state current to determine I_0 before the switch is thrown. After the switch is thrown, the same current flows through the inductor, and therefore that current will flow through the resistor R' . Using Kirchhoff's loop rule we calculate the emf in the inductor. This will be a maximum at $t = 0$.

$$I_0 = \frac{V_0}{R}, \quad \mathbf{e} - IR' = 0 \rightarrow \mathbf{e} = R' \frac{V_0}{R} e^{-t/\tau} \rightarrow \mathbf{e}_{\max} = \left(\frac{R'}{R}\right) V_0 = \left(\frac{55R}{R}\right) (120 \text{ V}) = \boxed{6.6 \text{ kV}}$$

34. (a) We calculate the resonant frequency using Eq. 30-14.

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = \frac{1}{2\pi} \sqrt{\frac{1}{(0.175 \text{ H})(425 \times 10^{-12} \text{ F})}} = 18,450 \text{ Hz} \approx \boxed{18.5 \text{ kHz}}$$

- (b) As shown in Eq. 30-15, we set the peak current equal to the maximum charge (from Eq. 24-1) multiplied by the angular frequency.

$$I = Q_0 \omega = CV(2\pi f) = (425 \times 10^{-12} \text{ F})(135 \text{ V})(2\pi)(18,450 \text{ Hz}) \\ = 6.653 \times 10^{-3} \text{ A} \approx \boxed{6.65 \text{ mA}}$$

- (c) We use Eq. 30-6 to calculate the maximum energy stored in the inductor.

$$U = \frac{1}{2} LI^2 = \frac{1}{2} (0.175 \text{ H})(6.653 \times 10^{-3} \text{ A})^2 = \boxed{3.87 \mu\text{J}}$$

- 37.** As in the derivation of 30-16, we set the total energy equal to the sum of the magnetic and electric energies, with the charge given by Eq. 30-19. We then solve for the time that the energy is 75% of the initial energy.

$$U = U_E + U_B = \frac{Q^2}{2C} + \frac{LI^2}{2} = \frac{Q_0^2}{2C} e^{-\frac{R}{L}t} \cos^2(\omega t + \phi) + \frac{Q_0^2}{2C} e^{-\frac{R}{L}t} \sin^2(\omega t + \phi) = \frac{Q_0^2}{2C} e^{-\frac{R}{L}t} \\ 0.75 \frac{Q_0^2}{2C} = \frac{Q_0^2}{2C} e^{-\frac{R}{L}t} \rightarrow t = -\frac{L}{R} \ln(0.75) = -\frac{L}{R} \ln(0.75) \approx \boxed{0.29 \frac{L}{R}}$$

46. (a) Since the resistor and capacitor are in parallel, they will have the same voltage drop across

them. We use Ohm's law to determine the current through the resistor and Eq. 30-25 to determine the current across the capacitor. The total current is the sum of the currents across each element.

$$I_R = \frac{V}{R} ; I_C = \frac{V}{X_C} = V(2\pi fC)$$

$$\frac{I_C}{I_R + I_C} = \frac{V(2\pi fC)}{V(2\pi fC) + V/R} = \frac{R(2\pi fC)}{R(2\pi fC) + 1} = \frac{(490 \Omega)2\pi(60 \text{ Hz})(0.35 \times 10^{-6} \text{ F})}{(490 \Omega)2\pi(60 \text{ Hz})(0.35 \times 10^{-6} \text{ F}) + 1} \\ = 0.0607 \approx \boxed{6.1\%}$$

(b) We repeat part (a) with a frequency of 60,000 Hz.

$$\frac{I_C}{I_R + I_C} = \frac{(490 \Omega)2\pi(60,000 \text{ Hz})(0.35 \times 10^{-6} \text{ F})}{(490 \Omega)2\pi(60,000 \text{ Hz})(0.35 \times 10^{-6} \text{ F}) + 1} = 0.9847 \approx \boxed{98\%}$$

53. We use the rms voltage across the resistor to determine the rms current through the circuit. Then, using the rms current and the rms voltage across the capacitor in Eq. 30-25 we determine the frequency.

$$I_{\text{rms}} = \frac{V_{R,\text{rms}}}{R} \quad V_{C,\text{rms}} = \frac{I_{\text{rms}}}{2\pi fC}$$

$$f = \frac{I_{\text{rms}}}{2\pi C V_{C,\text{rms}}} = \frac{V_{R,\text{rms}}}{2\pi C R V_{C,\text{rms}}} = \frac{(3.0 \text{ V})}{2\pi(1.0 \times 10^{-6} \text{ C})(750 \Omega)(2.7 \text{ V})} = \boxed{240 \text{ Hz}}$$

Since the voltages in the resistor and capacitor are not in phase, the rms voltage across the power source will not be the sum of their rms voltages.

65. (a) The peak voltage across the capacitor is the peak current multiplied by the capacitive reactance. We calculate the current in the circuit by dividing the source voltage by the impedance, where at resonance the impedance is equal to the resistance.

$$V_{C0} = X_C I_0 = \frac{1}{2\pi f_0 C} \frac{V_0}{R} = \frac{V_0}{2\pi(RC)} \frac{1}{f_0} = \boxed{\frac{V_0}{2\pi\tau} T_0}$$

(b) We set the amplification equal to 125 and solve for the resistance.

$$\beta = \frac{T_0}{2\pi\tau} = \frac{1}{2\pi f_0 RC} \rightarrow R = \frac{1}{2\pi f_0 \beta C} = \frac{1}{2\pi(5000 \text{ Hz})(125)(2.0 \times 10^{-9} \text{ F})} = \boxed{130 \Omega}$$