

Homework for the week of October 20. 4th week of classes.

Ch. 24: 6, 15, 18, 27, 29, 31, 34, 44, 49, 58, 60

Ch. 25: 7, 8, 9

6. The total charge will be conserved, and the final potential difference across the capacitors will be the same.

$$Q_0 = Q_1 + Q_2 ; V_1 = V_2 \rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_0 - Q_1}{C_2} \rightarrow \boxed{Q_1 = Q_0 \frac{C_1}{C_1 + C_2}}$$

$$Q_2 = Q_0 - Q_1 = Q_0 - Q_0 \frac{C_1}{C_1 + C_2} = \boxed{Q_2 = Q_0 \left(\frac{C_2}{C_1 + C_2} \right)}$$

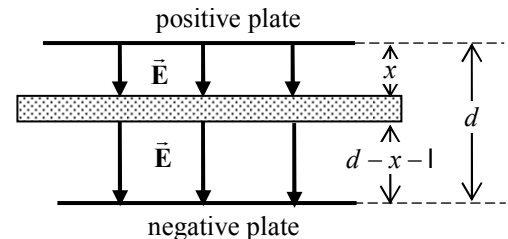
$$V_1 = V_2 = \frac{Q_1}{C_1} = \frac{Q_0 \frac{C_1}{C_1 + C_2}}{C_1} = \boxed{V = \frac{Q_0}{C_1 + C_2}}$$

15. We assume there is a uniform electric field between the capacitor plates, so that $V = Ed$, and then use Eqs. 24-1 and 24-2.

$$Q_{\max} = CV_{\max} = \epsilon_0 \frac{A}{d} (E_{\max} d) = \epsilon_0 A E_{\max} = (8.85 \times 10^{-12} \text{ F/m}) (6.8 \times 10^{-4} \text{ m}^2) (3.0 \times 10^6 \text{ V/m})$$

$$= \boxed{1.8 \times 10^{-8} \text{ C}}$$

18. (a) The uncharged plate will polarize so that negative charge will be drawn towards the positive capacitor plate, and positive charge will be drawn towards the negative capacitor plate. The same charge will be on each face of the plate as on the original capacitor plates. The same electric field will be in the gaps as before the plate was inserted. Use that electric field to determine the potential difference between the two original plates, and the new capacitance. Let x be the distance from one original plate to the nearest face of the sheet, and so $d - l - x$ is the distance from the other original plate to the other face of the sheet.



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0} ; V_1 = Ex = \frac{Qx}{A\epsilon_0} ; V_2 = E(d - l - x) = \frac{Q(d - l - x)}{A\epsilon_0}$$

$$\Delta V = V_1 + V_2 = \frac{Qx}{A\epsilon_0} + \frac{Q(d - l - x)}{A\epsilon_0} = \frac{Q(d - l)}{A\epsilon_0} = \frac{Q}{C} \rightarrow C = \boxed{\epsilon_0 \frac{A}{(d - l)}}$$

(b)

$$C_{\text{initial}} = \epsilon_0 \frac{A}{d} ; C_{\text{final}} = \epsilon_0 \frac{A}{(d - l)} ; \frac{C_{\text{final}}}{C_{\text{initial}}} = \frac{\epsilon_0 \frac{A}{(d - l)}}{\epsilon_0 \frac{A}{d}} = \frac{d}{d - l} = \frac{d}{d - 0.40d} = \frac{1}{0.60} = \boxed{1.7}$$

27. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\max} = C_1 + C_2 + C_3 = 3.6 \times 10^{-9} \text{ F} + 5.8 \times 10^{-9} \text{ F} + 1.00 \times 10^{-8} \text{ F} = \boxed{1.94 \times 10^{-8} \text{ F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\min} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = \left(\frac{1}{3.6 \times 10^{-9} \text{ F}} + \frac{1}{5.8 \times 10^{-9} \text{ F}} + \frac{1}{1.00 \times 10^{-8} \text{ F}} \right)^{-1} = \boxed{1.82 \times 10^{-9} \text{ F in series}}$$

29. (a) From the diagram, we see that C_1 and C_2 are in series. That combination is in parallel with C_3 ,

and then that combination is in series with C_4 . Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow C_{12} = \frac{1}{2} C ; C_{123} = C_{12} + C_3 = \frac{1}{2} C + C = \frac{3}{2} C ;$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{3C} = \frac{5}{3C} \rightarrow C_{1234} = \boxed{\frac{3}{5} C}$$

(b) The charge on the equivalent capacitor C_{1234} is given by $Q_{1234} = C_{1234} V = \frac{3}{5} CV$. This is the charge on both of the series components of C_{1234} .

$$Q_{123} = \frac{3}{5} CV = C_{123} V_{123} = \frac{3}{2} C V_{123} \rightarrow V_{123} = \frac{2}{5} V$$

$$Q_4 = \frac{3}{5} CV = C_4 V_4 \rightarrow V_4 = \frac{3}{5} V$$

The voltage across the equivalent capacitor C_{123} is the voltage across both of its parallel components. Note that the sum of the charges across the two parallel components of C_{123} is the same as the total charge on the two components, $\frac{3}{5} CV$.

$$V_{123} = \frac{2}{5} V = V_{12} ; Q_{12} = C_{12} V_{12} = \left(\frac{1}{2} C \right) \left(\frac{2}{5} V \right) = \frac{1}{5} CV$$

$$V_{123} = \frac{2}{5} V = V_3 ; Q_3 = C_3 V_3 = C \left(\frac{2}{5} V \right) = \frac{2}{5} CV$$

Finally, the charge on the equivalent capacitor C_{12} is the charge on both of the series components of C_{12} .

$$Q_{12} = \frac{1}{5} CV = Q_1 = C_1 V_1 \rightarrow V_1 = \frac{1}{5} V ; Q_{12} = \frac{1}{5} CV = Q_2 = C_2 V_2 \rightarrow V_2 = \frac{1}{5} V$$

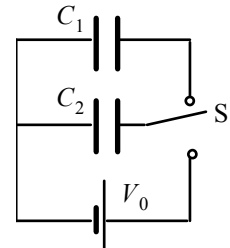
Here are all the results, gathered together.

$$\boxed{\begin{array}{l} Q_1 = Q_2 = \frac{1}{5} CV ; Q_3 = \frac{2}{5} CV ; Q_4 = \frac{3}{5} CV \\ V_1 = V_2 = \frac{1}{5} V ; V_3 = \frac{2}{5} V ; V_4 = \frac{3}{5} V \end{array}}$$

31. When the switch is down the initial charge on C_2 is calculated from Eq. 24-1.

$$Q_2 = C_2 V_0$$

When the switch is moved up, charge will flow from C_2 to C_1 until the voltage across the two capacitors is equal.



$$V = \frac{Q'_2}{C_2} = \frac{Q'_1}{C_1} \rightarrow Q'_2 = Q'_1 \frac{C_2}{C_1}$$

The sum of the charges on the two capacitors is equal to the initial charge on C_2 .

$$Q_2 = Q'_2 + Q'_1 = Q'_1 \frac{C_2}{C_1} + Q'_1 = Q'_1 \left(\frac{C_2 + C_1}{C_1} \right)$$

Inserting the initial charge in terms of the initial voltage gives the final charges.

$$Q'_1 \left(\frac{C_2 + C_1}{C_1} \right) = C_2 V_0 \rightarrow Q'_1 = \boxed{\frac{C_1 C_2 V_0}{C_2 + C_1}} ; Q'_2 = Q'_1 \frac{C_2}{C_1} = \boxed{\frac{C_2^2 V_0}{C_2 + C_1}}$$

34. We have $C_p = C_1 + C_2$ and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$. Solve for C_1 and C_2 in terms of C_p and C_s .

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{(C_p - C_1) + C_1}{C_1(C_p - C_1)} = \frac{C_p}{C_1(C_p - C_1)} \rightarrow$$

$$\frac{1}{C_s} = \frac{C_p}{C_1(C_p - C_1)} \rightarrow C_1^2 - C_p C_1 + C_p C_s = 0 \rightarrow$$

$$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_p C_s}}{2} = \frac{35.0 \mu\text{F} \pm \sqrt{(35.0 \mu\text{F})^2 - 4(35.0 \mu\text{F})(5.5 \mu\text{F})}}{2}$$

$$= 28.2 \mu\text{F}, 6.8 \mu\text{F}$$

$$C_2 = C_p - C_1 = 35.0 \mu\text{F} - 28.2 \mu\text{F} = 6.8 \mu\text{F} \text{ or } 35.0 \mu\text{F} - 6.8 \mu\text{F} = 28.2 \mu\text{F}$$

So the two values are $\boxed{28.2 \mu\text{F} \text{ and } 6.8 \mu\text{F}}$.

44. (a) The charge is constant, and the tripling of separation reduces the capacitance by a factor of 3.

$$\frac{U_2}{U_1} = \frac{\frac{Q^2}{2C_2}}{\frac{Q^2}{2C_1}} = \frac{C_1}{C_2} = \frac{\epsilon_0 \frac{A}{d}}{\epsilon_0 \frac{A}{3d}} = \boxed{3}$$

(b) The work done is the change in energy stored in the capacitor.

$$U_2 - U_1 = 3U_1 - U_1 = 2U_1 = 2 \frac{Q^2}{2C_1} = \frac{Q^2}{\epsilon_0 \frac{A}{d}} = \boxed{\frac{Q^2 d}{\epsilon_0 A}}$$

$\boxed{49}$. (a) With the plate inserted, the capacitance is that of two series capacitors of plate separations $d_1 = x$ and $d_2 = d - l - x$.

$$C_i = \left[\frac{x}{\epsilon_0 A} + \frac{d - x - l}{\epsilon_0 A} \right]^{-1} = \frac{\epsilon_0 A}{d - l}$$

With the plate removed the capacitance is obtained directly from Eq. 24-2.

$$C_f = \frac{\epsilon_0 A}{d}$$

Since the voltage remains constant the energy of the capacitor will be given by Eq. 24-5 written in terms of voltage and capacitance. The work will be the change in energy as the plate is removed.

$$W = U_f - U_i = \frac{1}{2}(C_f - C_i)V^2$$

$$= \frac{1}{2}\left(\frac{\epsilon_0 A}{d} - \frac{\epsilon_0 A}{d-1}\right)V^2 = \boxed{-\frac{\epsilon_0 A l V^2}{2d(d-1)}}$$

The net work done is negative. Although the person pulling the plate out must do work, charge is returned to the battery, resulting in a net negative work done.

(b) Since the charge now remains constant, the energy of the capacitor will be given by Eq. 24-5 written in terms of capacitance and charge.

$$W = \frac{Q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{Q^2}{2}\left(\frac{d}{\epsilon_0 A} - \frac{d-1}{\epsilon_0 A}\right) = \frac{Q^2 l}{2\epsilon_0 A}$$

The original charge is $Q = CV_0 = \frac{\epsilon_0 A}{d-1}V_0$ and so

$$W = \frac{\left(\frac{\epsilon_0 A}{d-1}V_0\right)^2 l}{2\epsilon_0 A} = \boxed{\frac{\epsilon_0 A V_0^2 l}{2(d-1)^2}}$$

58. The initial charge on the capacitor is $Q_{\text{initial}} = C_{\text{initial}}V$. When the mica is inserted, the capacitance changes to $C_{\text{final}} = KC_{\text{initial}}$, and the voltage is unchanged since the capacitor is connected to the same battery. The final charge on the capacitor is $Q_{\text{final}} = C_{\text{final}}V$.

$$\Delta Q = Q_{\text{final}} - Q_{\text{initial}} = C_{\text{final}}V - C_{\text{initial}}V = (K-1)C_{\text{initial}}V = (7-1)(3.5 \times 10^{-9}\text{F})(32\text{V})$$

$$= \boxed{6.7 \times 10^{-7}\text{C}}$$

60. The intermediate potential at the boundary of the two dielectrics can be treated as the “low” potential plate of one half and the “high” potential plate of the other half, so we treat it as two capacitors in series. Each series capacitor has half of the inter-plate distance of the original capacitor.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\frac{1}{2}d}{K_1\epsilon_0 A} + \frac{\frac{1}{2}d}{K_2\epsilon_0 A} = \frac{d}{2\epsilon_0 A} \frac{K_1 + K_2}{K_1 K_2} \rightarrow C = \boxed{\frac{2\epsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2}}$$

Ch. 25

7. Use Ohm’s Law, Eq. 25-2a, to find the current. Then use the definition of current, Eq. 25-1a, to calculate the number of electrons per minute.

$$I = \frac{V}{R} = \frac{\Delta Q}{\Delta t} = \frac{4.5\text{V}}{1.6\Omega} = \frac{2.8\text{C}}{\text{s}} \times \frac{1 \text{ electron}}{1.60 \times 10^{-19}\text{C}} \times \frac{60\text{s}}{1 \text{ min}} = \boxed{1.1 \times 10^{21} \frac{\text{electrons}}{\text{minute}}}$$

8. Find the potential difference from the resistance and the current.

$$R = (2.5 \times 10^{-5} \Omega/\text{m})(4.0 \times 10^{-2} \text{ m}) = 1.0 \times 10^{-6} \Omega$$

$$V = IR = (3100 \text{ A})(1.0 \times 10^{-6} \Omega) = \boxed{3.1 \times 10^{-3} \text{ V}}$$

9. (a) Use Eq. 25-2b to find the resistance.

$$R = \frac{V}{I} = \frac{12 \text{ V}}{0.60 \text{ A}} = \boxed{20 \Omega} \text{ (2 sig. fig.)}$$

- (b) An amount of charge ΔQ loses a potential energy of $(\Delta Q)V$ as it passes through the resistor.

The amount of charge is found from Eq. 25-1a.