Homework for the week of October 13. 3rd week of classes.
Ch. 23: 3, 7, 15, 22, 26, 35, 51, 54, 61

3. The kinetic energy gained by the electron is the work done by the electric force. Use Eq. 23-2b to calculate the potential difference.

$$V_{\text{ba}} = -\frac{W_{\text{ba}}}{q} = -\frac{5.25\times10^{-16}\,\text{J}}{\left(-1.60\times10^{-19}\,\text{C}\right)} = \boxed{3280\,\text{V}}$$

The electron moves from low potential to high potential, so $\boxed{\text{plate B}}$ is at the higher potential.

$\boxed{7.}$ The maximum charge will produce an electric field that causes breakdown in the air. We use the same approach as in Examples 23-4 and 23-5.

$$V_{\text{surface}} = r_0 E_{\text{breakdown}} \text{ and } V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} \rightarrow$$

$$Q = 4\pi\varepsilon_0 r_0^2 E_{\text{breakdown}} = \left(\frac{1}{8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2}\right)(0.065\,\text{m})^2\left(3\times10^{6}\,\text{V/m}\right) = \boxed{1.4\times10^{-6}\,\text{C}}$$

15. (*a*) After the connection, the two spheres are at the $\boxed{\text{same potential}}$. If they were at different
potentials, then there would be a flow of charge in the wire until the potentials were equalized.

(*b*) We assume the spheres are so far apart that the charge on one sphere does not influence the charge on the other sphere. Another way to express this would be to say that the potential due to either of the spheres is zero at the location of the other sphere. The charge splits between the spheres so that their potentials (due to the charge on them only) are equal. The initial charge on sphere 1 is Q, and the final charge on sphere 1 is Q_1.

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1} \ ; \ V_2 = \frac{Q-Q_1}{4\pi\varepsilon_0 r_2} \ ; \ V_1 = V_2 \rightarrow \frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{Q-Q_1}{4\pi\varepsilon_0 r_2} \rightarrow Q_1 = Q\frac{r_1}{(r_1+r_2)}$$

$$\text{Charge transferred } Q - Q_1 = Q - Q\frac{r_1}{(r_1+r_2)} = \boxed{Q\frac{r_2}{(r_1+r_2)}}$$

22. Because of the spherical symmetry of the problem, the electric field in each region is the same as that of a point charge equal to the net enclosed charge.

(*a*) For $r > r_2$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r^2} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

For $r_1 < r < r_2$: $E = \boxed{0}$, because the electric field is 0 inside of conducting material.

For $0 < r < r_1$: $E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{1}{2}Q}{r^2} = \boxed{\frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}}$

(*b*) For $r > r_2$, the potential is that of a point charge at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r}}, \ r > r_2$$

(c) For $r_1 < r < r_2$, the potential is constant and equal to its value on the outer shell, because there is no electric field inside the conducting material.

$$V = V(r = r_2) = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r_2}}, \ r_1 < r < r_2$$

(d) For $0 < r < r_1$, we use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = E dr$.

$$V_r - V_{r_1} = -\int_{r_1}^{r} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} = -\int_{r_1}^{r} E\,dr = -\int_{r_1}^{r} \frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}dr = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$V_r = V_{r_1} + \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) = \boxed{\frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r_2} + \frac{1}{r}\right)}, \ 0 < r < r_1$$

(e) To plot, we first calculate $V_0 = V(r = r_2) = \dfrac{3Q}{8\pi\varepsilon_0 r_2}$ and $E_0 = E(r = r_2) = \dfrac{3Q}{8\pi\varepsilon_0 r_2^2}$.

Then we plot V/V_0 and E/E_0 as functions of r/r_2.

For $0 < r < r_1$:

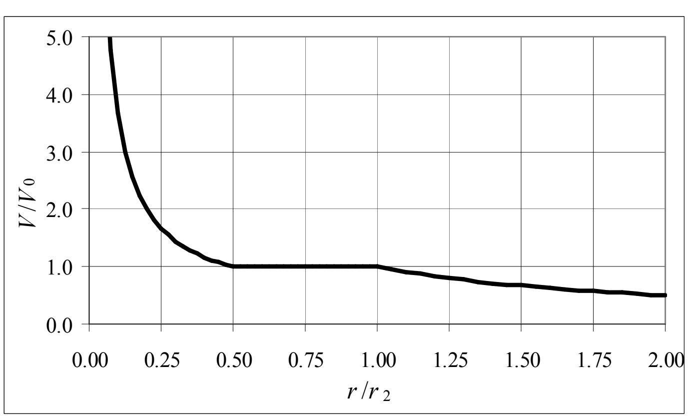
$$\frac{V}{V_0} = \frac{\frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r_2} + \frac{1}{r}\right)}{\frac{3Q}{8\pi\varepsilon_0 r_2}} = \tfrac{1}{3}\left[1 + (r/r_2)^{-1}\right] \ ; \ \frac{E}{E_0} = \frac{\frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2}}{\frac{3Q}{8\pi\varepsilon_0 r_2^2}} = \frac{1}{3}\frac{r_2^2}{r^2} = \tfrac{1}{3}(r/r_2)^{-2}$$

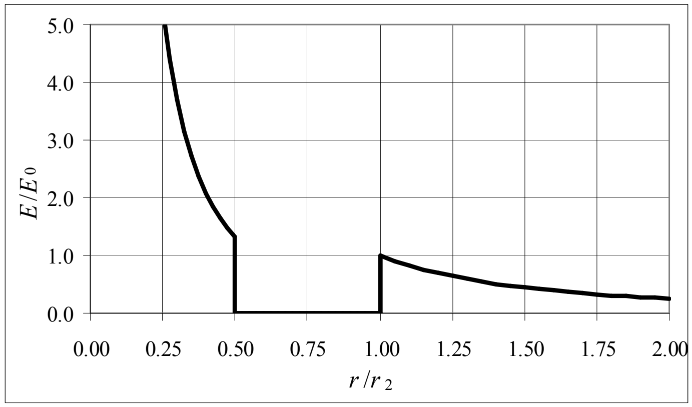
For $r_1 < r < r_2$:

$$\frac{V}{V_0} = \frac{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r_2}}{\frac{3Q}{8\pi\varepsilon_0 r_2}} = 1 \ ; \ \frac{E}{E_0} = \frac{0}{\frac{3Q}{8\pi\varepsilon_0 r_2^2}} = 0$$

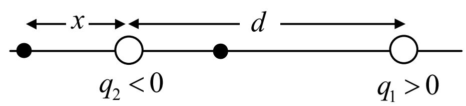
For $r > r_2$:

$$\frac{V}{V_0} = \frac{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r}}{\frac{3Q}{8\pi\varepsilon_0 r_2}} = \frac{r_2}{r} = (r/r_2)^{-1} \ ; \ \frac{E}{E_0} = \frac{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r^2}}{\frac{3Q}{8\pi\varepsilon_0 r_2^2}} = \frac{r_2^2}{r^2} = (r/r_2)^{-2}$$





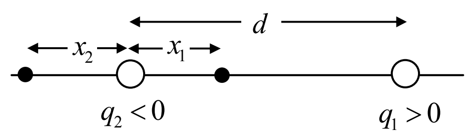
26. (*a*) Because of the inverse square nature of the electric field, any location where the field is zero must be closer to the weaker charge (q_2). Also, in between the two charges, the fields due to the two charges are parallel to each other (both to the left) and cannot cancel. Thus the only places where the field can be zero are closer to the weaker charge, but not between them. In the diagram, this is the point to the left of q_2. Take rightward as the positive direction.



$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q_2|}{x^2} - \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(d+x)^2} = 0 \rightarrow |q_2|(d+x)^2 = q_1 x^2 \rightarrow$$

$$x = \frac{\sqrt{|q_2|}}{\sqrt{q_1} - \sqrt{|q_2|}}d = \frac{\sqrt{2.0\times10^{-6}\,\text{C}}}{\sqrt{3.4\times10^{-6}\,\text{C}} - \sqrt{2.0\times10^{-6}\,\text{C}}}(5.0\,\text{cm}) = \boxed{16\,\text{cm left of } q_2}$$

(*b*) The potential due to the positive charge is positive everywhere, and the potential due to the negative charge is negative everywhere. Since the negative charge is smaller in magnitude than the positive charge, any point where the potential is zero must be closer to the negative charge. So consider locations between the charges (position x_1) and to the left of the negative charge (position x_2) as shown in the diagram.



$$V_{\text{location 1}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d-x_1)} + \frac{q_2}{x_1}\right] = 0 \rightarrow x_1 = \frac{q_2 d}{(q_2 - q_1)} = \frac{(-2.0\times10^{-6}\,\text{C})(5.0\,\text{cm})}{(-5.4\times10^{-6}\,\text{C})} = 1.852\,\text{cm}$$

$$V_{\text{location 2}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(d+x_2)} + \frac{q_2}{x_2}\right] = 0 \rightarrow$$

$$x_2 = -\frac{q_2 d}{(q_1 + q_2)} = -\frac{(-2.0\times10^{-6}\,\text{C})(5.0\,\text{cm})}{(1.4\times10^{-6}\,\text{C})} = 7.143\,\text{cm}$$

So the two locations where the potential is zero are 1.9 cm from the negative charge towards the positive charge, and 7.1 cm from the negative charge away from the positive charge.

35. We follow the development of Example 23-9, with Figure 23-15. The charge on a thin ring of radius R and thickness dR is $dq = \sigma dA = \sigma(2\pi R dR)$. Use Eq. 23-6b to find the potential of a continuous charge distribution.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{\sigma(2\pi R dR)}{\sqrt{x^2+R^2}} = \frac{\sigma}{2\varepsilon_0}\int_{R_1}^{R_2}\frac{R}{\sqrt{x^2+R^2}}dR = \frac{\sigma}{2\varepsilon_0}\left(x^2+R^2\right)^{1/2}\Big|_{R_1}^{R_2}$$

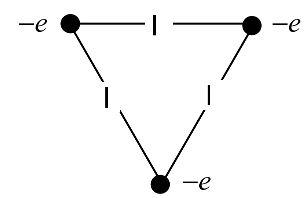
$$= \boxed{\frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R_2^2}-\sqrt{x^2+R_1^2}\right)}$$

51. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = -2.5y + 3.5yz \;;\; E_y = -\frac{\partial V}{\partial y} = -2y - 2.5x + 3.5xz \;;\; E_z = -\frac{\partial V}{\partial z} = 3.5xy$$

$$\vec{\mathbf{E}} = \boxed{(-2.5y+3.5yz)\hat{\mathbf{i}} + (-2y-2.5x+3.5xz)\hat{\mathbf{j}} + (3.5xy)\hat{\mathbf{k}}}$$

54. Let the side length of the equilateral triangle be L. Imagine bringing the electrons in from infinity one at a time. It takes no work to bring the first electron to its final location, because there are no other charges present. Thus $W_1 = 0$. The work done in bringing in the second electron to its final location is equal to the charge on the electron times the potential (due to the first electron) at the final location of the second electron.



Thus $W_2 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{l}\right) = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{L}$. The work done in bringing the third electron to its final

location is equal to the charge on the electron times the potential (due to the first two electrons). Thus $W_3 = (-e)\left(-\frac{1}{4\pi\varepsilon_0}\frac{e}{l} - \frac{1}{4\pi\varepsilon_0}\frac{e}{l}\right) = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{l}$. The total work done is the sum $W_1 + W_2 + W_3$.

$$W = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{l} + \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{l} = \frac{1}{4\pi\varepsilon_0}\frac{3e^2}{l} = \frac{3\left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\,\mathrm{C}\right)^2}{\left(1.0\times10^{-10}\,\mathrm{m}\right)}$$

$$= \boxed{6.9\times10^{-18}\,\mathrm{J}} = 6.9\times10^{-18}\,\mathrm{J}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right) = \boxed{43\,\mathrm{eV}}$$

$\boxed{61.}$ (*a*) The electron was accelerated through a potential difference of 1.33 kV (moving from low potential to high potential) in gaining 1.33 keV of kinetic energy. The proton is accelerated through the opposite potential difference as the electron, and has the exact opposite charge. Thus the proton gains the same kinetic energy, $\boxed{1.33\text{ keV}}$.

(*b*) Both the proton and the electron have the same KE. Use that to find the ratio of the speeds.

$$\tfrac{1}{2}m_p v_p^2 = \tfrac{1}{2}m_e v_e^2 \quad\rightarrow\quad \frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{\frac{1.67\times10^{-27}\,\mathrm{kg}}{9.11\times10^{-31}\,\mathrm{kg}}} = \boxed{42.8}$$

The lighter electron is moving about 43 times faster than the heavier proton.