Homework for the week of December 1st. 10th week of classes.
Ch. 31: 4, 6, 18, 20

4. The current in the wires is the rate at which charge is accumulating on the plates and also is the displacement current in the capacitor. Because the location in question is outside the capacitor, use the expression for the magnetic field of a long wire.

$$B=\frac{\mu_0 I}{2\pi R}=\left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R}=\frac{\left(10^{-7}\,\mathrm{T\cdot m/A}\right)2\left(38.0\times10^{-3}\,\mathrm{A}\right)}{(0.100\,\mathrm{m})}=\boxed{7.60\times10^{-8}\,\mathrm{T}}$$

After the capacitor is fully charged, all currents will be zero, so the magnetic field will be $\boxed{\text{zero.}}$

6. (*a*) The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate,
and so the conduction current is the same value as the displacement current. Thus the maximum conduction current is $\boxed{35\mu\mathrm{A}}$.

(*b*) The charge on the pages is given by $Q=CV=C\mathrm{e}_0\cos\omega t$. The current is the derivative of this.

$$I=\frac{dQ}{dt}=-\omega C\mathrm{e}_0\sin\omega t\ \ ;\ I_{\max}=\omega C\mathrm{e}_0\ \rightarrow$$

$$\mathrm{e}_0=\frac{I_{\max}}{\omega \mathrm{C}}=\frac{I_{\max}d}{2\pi f\varepsilon_0 A}=\frac{\left(35\times10^{-6}\mathrm{A}\right)\left(1.6\times10^{-3}\mathrm{m}\right)}{2\pi(76.0\,\mathrm{Hz})\left(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)\pi(0.025\mathrm{m})^2}$$

$$=6749\,\mathrm{V}\approx\boxed{6700\,\mathrm{V}}$$

(*c*) From Eq. 31-3, $I_\mathrm{D}=\varepsilon_0\dfrac{d\Phi_E}{dt}$.

$$I_\mathrm{D}=\varepsilon_0\frac{d\Phi_E}{dt}\ \rightarrow\ \left(\frac{d\Phi_E}{dt}\right)_{\max}=\frac{(I_\mathrm{D})_{\max}}{\varepsilon_0}=\frac{35\times10^{-6}\,\mathrm{A}}{8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}}=\boxed{4.0\times10^{6}\,\mathrm{V\cdot m/s}}$$

18. The length of the pulse is $\Delta d=c\Delta t$. Use this to find the number of wavelengths in a pulse.

$$N=\frac{(c\Delta t)}{\lambda}=\frac{\left(3.00\times10^{8}\,\mathrm{m/s}\right)\left(38\times10^{-12}\mathrm{s}\right)}{\left(1062\times10^{-9}\mathrm{m}\right)}=10734\approx\boxed{11{,}000\text{ wavelengths}}$$

If the pulse is to be only one wavelength long, then its time duration is the period of the wave, which is the reciprocal of the wavelength.

$$T=\frac{1}{f}=\frac{\lambda}{c}=\frac{\left(1062\times10^{-9}\mathrm{m}\right)}{\left(3.00\times10^{8}\,\mathrm{m/s}\right)}=\boxed{3.54\times10^{-15}\mathrm{s}}$$

20. (*a*) The general form of a plane wave is given in Eq. 31-7. For this wave, $E_x=E_0\sin(kz-\omega t)$.

$$\lambda=\frac{2\pi}{k}=\frac{2\pi}{0.077\mathrm{m}^{-1}}=81.60\,\mathrm{m}\approx\boxed{82\,\mathrm{m}}$$

$$f=\frac{\omega}{2\pi}=\frac{2.3\times10^{7}\,\mathrm{rad/s}}{2\pi}=3.661\times10^{6}\,\mathrm{Hz}\approx\boxed{3.7\,\mathrm{MHz}}$$

Note that $\lambda f = (81.60\,\mathrm{m})(3.661\times10^{6}\,\mathrm{Hz}) = 2.987\times10^{8}\,\mathrm{m/s} \approx c$.

(*b*) The magnitude of the magnetic field is given by $B_0 = E_0/c$. The wave is traveling in the $\hat{\mathbf{k}}$ direction, and so the magnetic field must be in the $\hat{\mathbf{j}}$ direction, since the direction of travel is given by the direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$.

$$B_0 = \frac{E_0}{c} = \frac{225\,\mathrm{V/m}}{3.00\times10^{8}\,\mathrm{m/s}} = 7.50\times10^{-7}\,\mathrm{T} \rightarrow$$

$$\vec{\mathbf{B}} = \boxed{\hat{\mathbf{j}}\left(7.50\times10^{-7}\,\mathrm{T}\right)\sin\left[\left(0.077\mathrm{m}^{-1}\right)z - \left(2.3\times10^{7}\,\mathrm{rad/s}\right)t\right]}$$