PHYSICS 2D — QUIZ 5 — FALL QUARTER 2009
PROF. HIRSCH — NOVEMBER 20th, 2009

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma\Delta t' \equiv \gamma\,\Delta t_p$; $L = L_p/\gamma$; $c = 3\times10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \to -v$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c}\,/\,\sqrt{1 - v/c}$ (approaching)

Momentum: $\vec{p} = \gamma m\vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\,MeV/c^2$ Proton: $m_p = 938.26\,MeV/c^2$ Neutron: $m_n = 939.55\,MeV/c^2$

Atomic mass unit: $1\,u = 931.5\,MeV/c^2$; electron volt: $1\text{eV} = 1.6\times10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda,T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda)\times\overline{E}(\lambda,T) = \dfrac{8\pi}{\lambda^4}\times\dfrac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\;eV\,A$; $k_B = (1/11{,}600)eV/K$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation: $n\lambda = 2d\sin\vartheta$

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_ec}(1 - \cos\theta)$; $\dfrac{h}{m_ec} = 0.0243A$; Coulomb constant: $ke^2 = 14.4\;eV\,A$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$; Drag force: $D = 6\pi a\eta v$

Rutherford scattering: $\Delta n = \dfrac{C}{\sin^4(\phi/2)}$; $\hbar c = 1{,}973\;eV\,A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097\times10^7\;m^{-1} = \dfrac{1}{911.3A}$

Electrostatic force, energy: $F = \dfrac{kq_1q_2}{r^2}$; $U = \dfrac{kq_1q_2}{r}$. Centripetal force: $F_c = \dfrac{mv^2}{r}$

Bohr atom: $E_n = -\dfrac{ke^2Z}{2r_n} = -\dfrac{Z^2E_0}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = 13.6eV$; $K = \dfrac{m_ev^2}{2}$; $U = -\dfrac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_eke^2} = 0.529A$; $L = m_evr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j\cos(k_jx - \omega_jt)$, or $y(x,t) = \int dk\,a(k)\,e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; Heisenberg: $\Delta x\Delta p \sim \hbar$; $\Delta t\Delta E \sim \hbar$

Schrodinger equation: $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\dfrac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 eV A^2$ (electron)

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$; $\Delta n = \pm 1$

Expectation value of$[Q]$: $< Q >= \int \psi^*(x)[Q]\psi(x)\, dx$; Momentum operator: $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[Q]\ \Psi = q\ \Psi$ (q is a constant) ; uncertainty: $\Delta Q = \sqrt{< Q^2 > - < Q >^2}$

Step potential: reflection coef: $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Justify all your answers to all problems

Problem 1 (10 points)

An electron is described by the wavefunction
$\psi(x) = 0$ for $x < 0$; $\psi(x) = Cxe^{-x/2}$ for $x \geq 0$
with C a constant.
(a) Find C from normalization
(b) Find $<x>$
(c) Find $<p>$, $<p^2>$ and Δp, with p the momentum operator.

Hint: $\int_0^\infty dx\, x^n e^{-\lambda x} = \frac{n!}{\lambda^{n+1}}$

Problem 2 (10 points)
Consider an electron described by the wavefunction:
$\psi(x) = 0$ for $x < 0$; $\psi(x) = Cxe^{-3x}$ for $x \geq 0$
Consider the operator
$$Q = \frac{1}{x} - \frac{\partial}{\partial x}$$
(a) Show that the wavefunction ψ is an eigenfunction of this operator, and find the eigenvalue.
(b) Find $<Q^2>$ and the uncertainty ΔQ.

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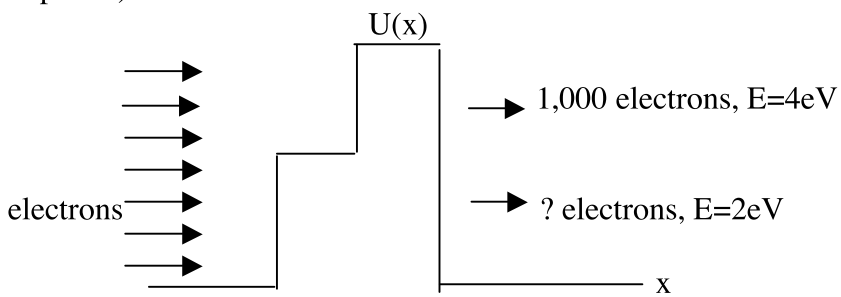
<u>Problem 3</u> (10 points)



The potential in the figure is given by:
U(x)=0 for x<0
U(x)=3eV for 0<x<1A
U(x)=5eV for 1A<x<2A
U(x)=0 for x>2A.
Electrons are incident from the left, uniformly distributed in energy (i.e. the same number of electrons are incident for any energy).
If 1,000 electrons are found on the right side of the barrier with energy 4eV, how many are found on the right side with energy 2eV?

Justify all your answers to all problems