

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation :

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

$$y' = y, \quad z' = z$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$y = y', \quad z = z'$$

$$t' = \gamma(t - vx/c^2)$$

$$t = \gamma(t' + vx'/c^2)$$

Velocity transformation :

$$u_x' = \frac{u_x - v}{1 - u_x v / c^2}$$

$$u_x = \frac{u_x' + v}{1 + u_x' v / c^2}$$

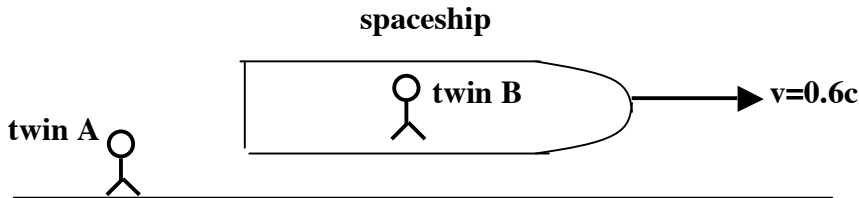
$$u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$$

$$u_y = \frac{u_y'}{\gamma(1 + u_x' v / c^2)}$$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Justify all your answers to all problems

Problem 1 (10 points)

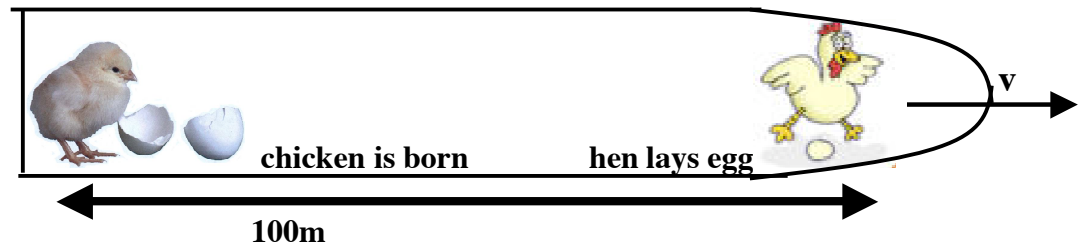


When twins A and B reach their 20th birthday, twin B departs on a spaceship going at speed 0.6c. When the twins reach their 21st birthday according each to their own clocks, they light up candles to celebrate (twin A on the earth, twin B on the spaceship).

- How old is twin B when she lights up her candle, as shown by clocks on the earth's reference frame?
- How old is twin A when the light from twin B's candle first reaches him?
- How old is twin B when the light from twin A's candle first reaches her?

Note: assume the effects of accelerating the spaceship from rest to speed 0.6c can be neglected, so you can deal with inertial frames only.

Problem 2 (10 points)



ground

A hen lays an egg at the front end of a 100 m long spaceship. $0.1\mu\text{s}$ later (according to clocks on the spaceship), a chicken is born at the back end of this spaceship ($1\mu\text{s}=10^{-6}\text{s}$). The spaceship is moving at speed v with respect to the ground.

- Find how fast the spaceship has to be moving so that for an observer on the ground the chicken is born simultaneously with the hen laying the egg. Give your answer in terms of c , i.e. find v/c .
- Find the spacetime interval for these events in the spaceship reference frame. Is it timelike or spacelike?
- Find the spacetime interval for these events in the ground reference frame.

Hint: define carefully your event coordinates (x_1', t_1') and (x_2', t_2') in the spaceship reference frame, then use the Lorentz transformation to find the coordinates of these events in the ground coordinates.

Problem 3 (10 points)



Two identical spaceships are moving in opposite directions, each of them moves at speed $0.8c$ with respect to the ground. The length of the spaceships measured in their own reference frame (proper length) is 200m.

- What is the speed of one ship relative to the other ship?
- What is the length of one ship as measured from the ground?
- What is the length of one ship as measured from the other ship?