5-12 Using $p = \frac{h}{\lambda} = mv$, we find that $v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(1 \times 10^{-10}\ \text{m}\right)} = 7.27 \times 10^{6}\ \text{m/s}$.
From the principle of conservation of energy, we get

$$eV = \frac{mv^2}{2} = \frac{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(7.27 \times 10^{6}\ \text{m/s}\right)^2}{2} = 2.41 \times 10^{-17}\ \text{J} = 151\ \text{eV}.$$

Therefore $V = 151\ \text{V}$.

5-15 For a free, non-relativistic electron $E = \frac{m_e v_0^2}{2} = \frac{p^2}{2m_e}$. As the wavenumber and angular frequency of the electron's de Broglie wave are given by $p = \hbar k$ and $E = \hbar\omega$, substituting these results gives the dispersion relation $\omega = \frac{\hbar k^2}{2m_e}$. So $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m_e} = \frac{p}{m_e} = v_0$.

5-17 $E^2 = p^2c^2 + \left(m_e c^2\right)^2$

$E = \left[p^2c^2 + \left(m_e c^2\right)^2\right]^{1/2}$. As $E = \hbar\omega$ and $p = \hbar k$

$\hbar\omega = \left[\hbar^2 k^2 c^2 + \left(m_e c^2\right)^2\right]^{1/2}$ or

$$\omega(k) = \left[k^2c^2 + \frac{\left(m_e c^2\right)^2}{\hbar^2}\right]^{1/2}$$

$$v_p = \frac{\omega}{k} = \frac{\left[k^2c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}}{k} = \left[c^2 + \left(\frac{m_e c^2}{\hbar k}\right)^2\right]^{1/2}$$

$$v_g = \left.\frac{d\omega}{dk}\right|_{k_0} = \frac{1}{2}\left[k^2c^2 + \left(\frac{m_e c^2}{\hbar}\right)^2\right]^{-1/2} 2kc^2 = \frac{kc^2}{\left[k^2c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}}$$

$$v_p v_g = \left\{\frac{\left[k^2c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}}{k}\right\}\left\{\left[k^2c^2 + \left(m_e c^2/\hbar\right)^2\right]^{1/2}\right\} = c^2$$

Therefore, $v_g < c$ if $v_p > c$.

5-18 $\Delta x \Delta p \geq \frac{\hbar}{2}$ where $\Delta p = m\Delta v = \left(0.05\ \text{kg}\right)\left(10^{-3} \times 30\ \text{m/s}\right) = 1.5 \times 10^{-3}\ \text{kg}\cdot\text{m/s}$. Therefore,

$$\Delta x = \frac{\hbar}{2\Delta p} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(1.5 \times 10^{-3}\ \text{kg}\cdot\text{m/s}\right)} = 3.51 \times 10^{-32}\ \text{m}.$$

5-19 $K=\dfrac{mv^2}{2}=\dfrac{p^2}{2m}$:

$$\left(1\times10^{6}\ \text{eV}\right)\left(1.6\times10^{-19}\ \text{J/eV}\right)=\frac{p^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)}\Rightarrow p=2.312\times10^{-20}\ \text{kg}\cdot\text{m/s},$$

$\Delta p=0.05p=1.160\times10^{-21}\ \text{kg}\cdot\text{m/s}$ and $\Delta x\Delta p=\dfrac{h}{4\pi}$. Thus

$$\Delta x=\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.16\times10^{-21}\ \text{kg}\cdot\text{m/s}\right)(4\pi)}=4.56\times10^{-14}\ \text{m}.$$

Note that non-relativistic treatment has been used, which is justified because the kinetic energy is only $\dfrac{\left(1.6\times10^{-13}\right)\times100\%}{1.50\times10^{-10}}=0.11\%$

of the rest energy.

5-23 (a) $\Delta p\Delta x=m\Delta v\Delta x\geq\dfrac{\hbar}{2}$

$$\Delta v\geq\frac{h}{4\pi m\Delta x}=\frac{2\pi\,\text{J}\cdot\text{s}}{4\pi(2\ \text{kg})(1\ \text{m})}=0.25\ \text{m/s}$$

(b) The duck might move by $(0.25\ \text{m/s})(5\ \text{s})=1.25\ \text{m}$. With original position uncertainty of 1m, we can think of Δx growing to $1\ \text{m}+1.25\ \text{m}=2.25\ \text{m}$.

5-24 (a) $\Delta x\Delta p=\hbar$ so if $\Delta x=r$, $\Delta p\approx\dfrac{\hbar}{r}$

(b)

$$K=\frac{p^2}{2m_e}\approx\frac{(\Delta p)^2}{2m_e}=\frac{\hbar^2}{2m_er^2}$$

$$U=-\frac{ke^2}{r}$$

$$E=\frac{\hbar^2}{2m_er^2}-\frac{ke^2}{r}$$

(c) To minimize E take $\dfrac{dE}{dr}=-\dfrac{\hbar^2}{m_er^3}+\dfrac{ke^2}{r^2}=0\Rightarrow r=\dfrac{\hbar^2}{m_eke^2}=\text{Bohr radius}=a_0$. Then

$$E=\left(\frac{\hbar}{2m_e}\right)\left(\frac{m_eke^2}{\hbar^2}\right)^2-ke^2\left(\frac{m_eke^2}{\hbar^2}\right)=\frac{m_ek^2e^4}{2\hbar^2}=-13.6\ \text{eV}.$$

5-25 To find the energy width of the γ-ray use $\Delta E\Delta t\geq\dfrac{\hbar}{2}$ or

$$\Delta E\geq\frac{\hbar}{2\Delta t}\geq\frac{6.58\times10^{-16}\ \text{eV}\cdot\text{s}}{(2)\left(0.10\times10^{-9}\ \text{s}\right)}\geq3.29\times10^{-6}\ \text{eV}.$$

As the intrinsic energy width of $\sim \pm 3\times10^{-6}$ eV is so much less than the experimental resolution of ± 5 eV, the intrinsic width can't be measured using this method.

5-26 The full width at half-maximum (FWHM) is 110 MeV. So $\Delta E = 55$ MeV and using $\Delta E_{\text{min}}\Delta t_{\text{min}} = \frac{\hbar}{2}$,

$$\Delta t_{\text{min}} = \frac{\hbar}{2\Delta E} = \frac{6.58\times10^{-16}\ \text{eV}\cdot\text{s}}{2\left(55\times10^{6}\ \text{eV}\right)} \cong 6.0\times10^{-24}\ \text{s}$$

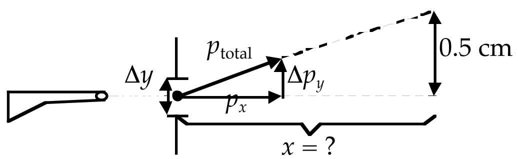
$$\tau = \text{lifetime} \sim 2\Delta t_{\text{min}} = 1.2\times10^{-23}\ \text{s}$$

5-27 For a single slit with width a, minima are given by $\sin\theta = \frac{n\lambda}{a}$ where $n = 1, 2, 3, \ldots$ and $\sin\theta \approx \tan\theta = \frac{x}{L}$, $\frac{x_1}{L} = \frac{\lambda}{a}$ and $\frac{x_2}{L} = \frac{2\lambda}{a} \Rightarrow \frac{x_2 - x_1}{L} = \frac{\lambda}{a}$ or

$$\lambda = \frac{a\Delta x}{L} = \frac{5\ \text{Å}\times 2.1\ \text{cm}}{20\ \text{cm}} = 0.525\ \text{Å}$$

$$E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{\left(1.24\times10^{4}\ \text{eV}\cdot\text{Å}\right)^2}{2\left(5.11\times10^{5}\ \text{eV}\right)\left(0.525\ \text{Å}\right)^2} = 546\ \text{eV}$$

5-31

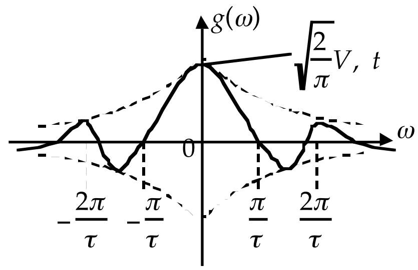


$\Delta y \Delta p_y \sim \hbar$ $\quad\quad$ $\Delta p_y = \frac{\hbar}{\Delta y}$. From the diagram, because the momentum triangle and space triangle are similar, $\frac{\Delta p_y}{p_x} = \frac{0.5\ \text{cm}}{x}$;

$$x = \frac{(0.5\ \text{cm})p_x}{\Delta p_y} = \frac{(0.5\ \text{cm})p_x\Delta y}{\hbar} = \frac{\left(0.5\times10^{-2}\ \text{m}\right)(0.001\ \text{kg})(100\ \text{m/s})\left(2\times10^{-3}\ \text{m}\right)}{1.05\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 9.5\times10^{27}\ \text{m}$$

Once again we see that the uncertainty relation has no observable consequences for macroscopic systems.

5-34 (a) $g(\omega) = (2\pi)^{-1/2}\int_{-\infty}^{\infty} V(t)(\cos\omega t - i\sin\omega t)dt$, $V(t)\sin\omega t$ is an odd function so this integral vanishes leaving $g(\omega) = 2(2\pi)^{-1/2}\int_0^{\tau} V_0\cos\omega t\,dt = \left(\frac{2}{\pi}\right)^{1/2} V_0 \frac{\sin\omega\tau}{\omega}$. A sketch of $g(\omega)$ is given below.



(b) As the major contribution to this pulse comes from ω's between $-\frac{\pi}{\tau}$ and $\frac{\pi}{\tau}$, let $\Delta\omega \approx \frac{\pi}{\tau}$ and since $\Delta t = \tau$.

$$\Delta\omega\Delta t = \left(\frac{\pi}{\tau}\right)\tau = \pi$$

(c) Substituting $\Delta t = 0.5\ \mu\text{s}$ in $\Delta\omega = \frac{\pi}{\Delta t}$ we find $\frac{\Delta 1}{2\Delta t} = \frac{1}{2\left(0.5\times 10^{-6}\ \text{s}\right)} = 1\times 10^{6}\ \text{Hz}$.
As the range is $2\Delta f$, the range is $2\times 10^{6}\ \text{Hz}$. For $\Delta t = 0.5\ \text{ns}$, the range is $2\Delta f = 2\times 10^{9}\ \text{Hz}$.

5-35 (a)

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} a(k)e^{ikx}\,dk = \frac{A}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-\alpha^2(k-k_0)^2}e^{ikx}dk = \frac{A}{\sqrt{2\pi}}e^{-\alpha^2 k_0^2}\int_{-\infty}^{+\infty} e^{-\alpha^2\left(k^2-\left(2k_0+ix/\alpha^2\right)k\right)}dk$$

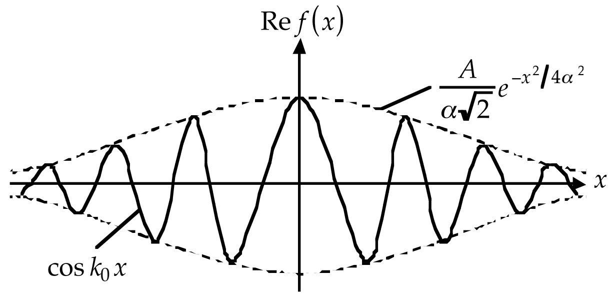
. Now complete the square in order to get the integral into the standard form $\int_{-\infty}^{+\infty} e^{-az^2}dz$:

$$e^{-\alpha^2\left(k^2-\left(2k_0+ix/\alpha^2\right)k\right)} = e^{+\alpha^2\left(k_0+ix/2\alpha^2\right)^2}e^{-\alpha^2\left(k-\left(k_0+ix/2\alpha^2\right)\right)^2}$$

$$f(x) = \frac{A}{\sqrt{2\pi}}e^{-\alpha^2 k_0^2}e^{\alpha^2\left(k_0+ix/2\alpha^2\right)^2}\int_{k=-\infty}^{+\infty} e^{-\alpha^2\left(k-\left(k_0+ix/2\alpha^2\right)\right)^2}dk$$

$$= \frac{A}{\sqrt{2\pi}}e^{-x^2/4\alpha^2}e^{ik_0x}\int_{z=-\infty}^{+\infty} e^{-\alpha^2 z^2}dz$$

where $z = k - \left(k_0 + \frac{ix}{2\alpha^2}\right)$. Since $\int_{z=-\infty}^{+\infty} e^{-\alpha^2 z^2}dz = \frac{\pi^{1/2}}{\alpha}$, $f(x) = \frac{A}{\alpha\sqrt{2}}e^{-x^2/4\alpha^2}e^{ik_0x}$. The real part of $f(x)$, $\operatorname{Re} f(x)$ is $\operatorname{Re} f(x) = \frac{A}{\alpha\sqrt{2}}e^{-x^2 4\alpha^2}\cos k_0x$ and is a gaussian envelope multiplying a harmonic wave with wave number k_0. A plot of $\operatorname{Re} f(x)$ is shown below:



Comparing $\frac{A}{\alpha\sqrt{2}}e^{-x^2 4\alpha^2}$ to $Ae^{-(x/2\Delta x)^2}$ implies $\Delta x = \alpha$.

(c) By same reasoning because $\alpha^2 = \frac{1}{4\Delta k^2}$, $\Delta k = \frac{1}{2\alpha}$. Finally $\Delta x \Delta k = \alpha\left(\frac{1}{2\alpha}\right) = \frac{1}{2}$.

5-36 $E = K = \frac{1}{2}mu^2 = hf$ and $\lambda = \frac{h}{mu}$. $v_{\text{phase}} = f\lambda = \frac{mu^2}{2h}\frac{h}{mu} = \frac{u}{2} = v_{\text{phase}}$. This is different from the speed u at which the particle transports mass, energy, and momentum.

6-2 (a) Normalization requires

$$1 = \int_{-\infty}^{\infty} |\psi|^2 dx = A^2 \int_{-\frac{L}{4}}^{\frac{L}{4}} \cos^2\left(\frac{2\pi x}{L}\right) dx = \left(\frac{A^2}{2}\right)\int_{-\frac{L}{4}}^{\frac{L}{4}}\left(1 + \cos\left(\frac{4\pi x}{L}\right)\right) dx$$

so $A = \frac{2}{\sqrt{L}}$.

(b) $$P = \int_0^{\frac{L}{8}} |\psi|^2 dx = A^2 \int_0^{\frac{L}{8}} \cos^2\left(\frac{2\pi x}{L}\right) dx = \left(\frac{4}{L}\right)\left(\frac{1}{2}\right)\int_0^{\frac{L}{8}}\left(1 + \cos\left(\frac{4\pi x}{L}\right) dx\right)$$

$$= \left(\frac{2}{L}\right)\left(\frac{L}{8}\right) + \left(\frac{2}{L}\right)\left(\frac{L}{4\pi}\right)\sin\left(\frac{4\pi x}{L}\right)\Bigg|_0^{\frac{L}{8}} = \frac{1}{4} + \frac{1}{2\pi} = 0.409$$

6-3 (a) $A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin\left(5 \times 10^{10} x\right)$ so $\left(\frac{2\pi}{\lambda}\right) = 5 \times 10^{10}\ \text{m}^{-1}$,

$\lambda = \frac{2\pi}{5 \times 10^{10}} = 1.26 \times 10^{-10}\ \text{m}$.

(b) $p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{Js}}{1.26 \times 10^{-10}\ \text{m}} = 5.26 \times 10^{-24}\ \text{kg m/s}$

(c) $K = \dfrac{p^2}{2m}$ $m = 9.11 \times 10^{-31}$ kg

$$K = \frac{\left(5.26 \times 10^{-24}\ \text{kg m/s}\right)^2}{\left(2 \times 9.11 \times 10^{-31}\ \text{kg}\right)} = 1.52 \times 10^{-17}\ \text{J}$$

$$K = \frac{1.52 \times 10^{-17}\ \text{J}}{1.6 \times 10^{-19}\ \text{J/eV}} = 95\ \text{eV}$$

6-4 The time development of Ψ is given by Equation 6.8 or

$$\Psi(x,\ t) = \int a(k) e^{i\{kx - \omega(k)t\}} dk = \left(\frac{C\alpha}{\sqrt{\pi}}\right)\int_{-\infty}^{\infty} e^{\{ikx - i\omega(k)t - \alpha^2 k^2\}} dk,$$

with $\omega(k) = \dfrac{\hbar k^2}{2m}$ for a free particle of mass m. As in Example 6.3, the integral may be reduced to a recognizable form by completing the square in the exponent. Since $\omega(k)t = \left(\dfrac{\hbar t}{2m}\right)k^2$, we group this term together with $\alpha^2 k^2$ by introducing $\beta^2 = \alpha^2 + \dfrac{i\hbar t}{2m}$ to get

$$ikx - \omega(k)t - \alpha^2 k^2 = -\left(\beta k - \frac{ix}{2\beta}\right)^2 - \frac{x^2}{4\beta^2}.$$

Then, changing variables to $z = \beta k - \dfrac{ix}{2\beta}$ gives

$$\Psi(x,\ t) = \left(\frac{C\alpha}{\beta\sqrt{\pi}}\right) e^{-x^2/4\beta^2} \int_{-\infty}^{\infty} e^{-z^2} = \left(\frac{C\alpha}{\beta}\right) e^{-x^2/4\beta^2}.$$

To interpret this result, we must recognize that β is complex and separate real and imaginary parts. Thus, $\left|\beta^2\right|^2 = \left|\alpha^2 + \dfrac{i\hbar t}{2m}\right|^2 = \alpha^4 + \left(\dfrac{\hbar t}{2m}\right)^2$ and the exponent for Ψ is

$$\frac{x^2}{4\beta^2} = \frac{x^2\left(\alpha^2 - \frac{i\hbar t}{2m}\right)}{4\left|\beta^2\right|^2} = \frac{x^2}{4\left[\alpha^2 + \left(\frac{\hbar t}{2m\alpha}\right)^2\right]} + (\text{imaginary terms})$$

then

$$\left|\Psi(x,\ t)\right| = \frac{C\alpha}{\left(\alpha^4 + \left(\frac{\hbar t}{2m}\right)^2\right)^{1/4}} e^{\left\{-x^2/\left[4\left\{\alpha^2 + (\hbar t/2m\alpha)^2\right\}\right]\right\}}.$$

We see that apart from a phase factor, $\Psi(x,\ t)$ is still a gaussian but with amplitude diminished by $\dfrac{\alpha}{\left(\alpha^4 + \left(\frac{\hbar t}{2m}\right)^2\right)^{1/4}}$ and a width $\Delta x(t) = \left(\alpha^2 + \left(\dfrac{\hbar t}{2m\alpha}\right)^2\right)^{1/2}$ where $\alpha = \Delta x(0)$ is the initial width.