4

4-14 (a) $r_n = \frac{n^2\hbar^2}{m_e k e^2}$; where $n = 1,\ 2,\ 3,\ \ldots$

$$r_n = n^2 \frac{\left(1.055 \times 10^{-34}\ \text{Js}\right)^2}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(9.0 \times 19^{9}\ \text{Nm}^2/\text{C}^2\right)\left(1.6 \times 10^{-19}\ \text{C}\right)^2} = \left(0.052\,9\ \text{nm}\right)n^2$$

For $n = 1$: $r_n = 0.052\,9$ nm

For $n = 2$: $r_n = 0.212\,1$ nm

For $n = 3$: $r_n = 0.477\,2$ nm

(b) From Equation 4.26, $v = \left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$v_1 = \left[\frac{\left(8.99 \times 10^{9}\ \text{Nm}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(0.052\,9 \times 10^{-9}\ \text{m}\right)}\right]^{1/2} = 2.19 \times 10^{6}\ \text{m/s}$$

$v_2 = 1.09 \times 10^{6}\ \text{m/s}$

$v_3 = 7.28 \times 10^{5}\ \text{m/s}$

(c) As $c = 3.0 \times 10^{8}\ \text{m/s}$, $v << c$ and no relativistic correction is necessary.

4-17 $r = \frac{n^2\hbar^2}{Zm_e ke^2} = \left(\frac{n^2}{Z}\right)\left(\frac{\hbar^2}{m_e ke^2}\right)$; $n = 1$

$$r = \frac{1}{Z}\left[\frac{\left(1.055 \times 10^{-34}\ \text{Js}\right)^2}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(9 \times 10^{9}\ \text{Nm}^2/\text{C}^2\right)\left(1.6 \times 10^{-19}\ \text{C}\right)^2}\right] = \frac{5.30 \times 10^{-11}\ \text{m}}{Z}$$

(a) For He^{+}, $Z = 2$, $r = \frac{5.30 \times 10^{-11}\ \text{m}}{2} = 2.65 \times 10^{-11}\ \text{m} = 0.026\,5$ nm

(b) For Li^{2+}, $Z = 3$, $r = \frac{5.30 \times 10^{-11}\ \text{m}}{3} = 1.77 \times 10^{-11}\ \text{m} = 0.017\,7$ nm

(c) For Be^{3+}, $Z = 4$, $r = \frac{5.30 \times 10^{-11}\ \text{m}}{4} = 1.32 \times 10^{-11}\ \text{m} = 0.013\,2$ nm

4-22 $E = K + U = \frac{mv^2}{2} - \frac{ke^2}{r}$. But $\frac{mv^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r}$. Thus $E = \left(\frac{1}{2}\right)\left(\frac{-ke^2}{r}\right) = \frac{U}{2}$, so $U = 2E = 2(-13.6\ \text{eV}) = -27.2\ \text{eV}$ and $K = E - U = -13.6\ \text{eV} - (-27.2\ \text{eV}) = 13.6\ \text{eV}$.

4-23 (a) $r_1 = \left(0.052\,9\ \text{nm}\right)n^2 = 0.052\,9\ \text{nm}$ (when $n = 1$)

(b) $$m_e v = m_e\left(\frac{ke^2}{m_e r}\right)^{1/2}$$

$$m_e = \left[\frac{(9.1\times10^{-31}\ \text{kg})(9\times10^{9}\ \text{Nm}^2/\text{C}^2)}{5.29\times10^{-11}\ \text{m}}\right]^{1/2}\times(1.6\times10^{-19}\ \text{C})$$

$$M_e v = 1.99\times10^{-24}\ \text{kg m/s}$$

(c) $L = m_e vr = (1.99\times10^{-24}\ \text{kg m/s})(5.29\times10^{-11}\ \text{m}),\ L = 1.05\times10^{-34}(\text{kg m}^2/\text{s}) = \hbar$

(d) $K = |E| = 13.6\ \text{eV}$

(e) $U = -2K = -27.2\ \text{eV}$

(f) $E = K + U = -13.6\ \text{eV}$

4-25 (a) $\Delta E = hf = (13.6\ \text{eV})\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$ or $f = (13.6\ \text{eV})\left(\frac{\frac{1}{9}-\frac{1}{16}}{4.14\times10^{-15}\ \text{eV s}}\right) = 1.60\times10^{14}\ \text{Hz}$

(b) $T = \frac{2\pi r_n}{v}$ so $f_{\text{rev}} = \frac{1}{T} = \frac{v}{2\pi r_n}$. Using $v = \left(\frac{ke^2}{m_e r_n}\right)^{1/2}$, $f_{\text{rev}} = \left(\frac{ke^2}{mr_n}\right)^{1/2}$. For $n = 3$, $r_3 = (3)^2 a_0$ and

$$f_{\text{rev}} = \frac{(8.99\times10^{9}\ \text{Nm}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{\dfrac{[(9.11\times10^{-31}\ \text{kg})(9)(5.29\times10^{-11}\ \text{m})]^{1/2}}{(2)(3.14)(9)(5.29\times10^{-11}\ \text{m})}}$$

$$f_{\text{rev}} = 2.44\times10^{14}\ \text{Hz}\ (n=3)$$

$$f_{\text{rev}} = 1.03\times10^{14}\ \text{Hz}\ (n=4)$$

Thus the photon frequency is about halfway between the two frequencies of the revolution.

4-32 (a)

$$\mu_{1\text{H}} = \frac{m_e M}{m_e + M} = \frac{(9.109\,390\times10^{-31}\ \text{kg})(1.672\,63\times10^{-27}\ \text{kg})}{(9.109\,390\times10^{-31}\ \text{kg}) + (1.672\,63\times10^{-27}\ \text{kg})}$$

$$= \frac{(9.109\,390\times10^{-31}\ \text{kg})(1.672\,63\times10^{-27}\ \text{kg})}{(0.000\,910\,939\,0\times10^{-27}\ \text{kg}) + (1.672\,63\times10^{-27}\ \text{kg})} = 9.104\,431\times10^{-31}\ \text{kg}$$

$$\frac{1}{\lambda} = \frac{\mu}{m_e}(k)\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) = \left(\frac{9.104\,431\,6\times10^{-31}\ \text{kg}}{9.109\,390\times10^{-31}\ \text{kg}}\right)(1.097\,315\,3\times10^{7}\ \text{m}^{-1})\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\lambda_{1\text{H}} = 656.469\,1\ \text{nm}$$

(b) Similarly, we find $\lambda_{2\text{H}} = 656.292\,5\ \text{nm}$.

(c) $\lambda_{3\text{H}} = 656.232\,5\ \text{nm}$

4-33 $r_n = \frac{n^2 a_0}{Z} = \frac{n^2\hbar^2}{mke^2 Z} = (1)^2 \frac{\left(1.05\times 10^{-34}\text{ Js}\right)^2}{(207m_e)\left(8.99\times 10^9\text{ Nm}^2/\text{C}\right)\left(1.60\times 10^{-19}\text{ C}\right)^2(82)} = 3.1\times 10^{-15}\text{ m}$.

Note that this means the muon grazes the nuclear surface, and so experiments with muonic atoms give information about the nuclear charge distribution.

$$E_n = -\frac{\left[\left(ke^2Z^2\right)/\left(2a_0n^2\right)\right]/ke^2Z^2}{2\hbar^2n^2/mke^2} = \frac{mk^2e^4Z^2}{2\hbar^2n^2}$$

Using $m = 207m_e$, $n = 1$, and $Z = 82$ yields $E_1 = 18.9\text{ MeV}$. (Using the reduced mass makes no difference in the answers to three significant figures.)

5

5-2 The issue is: Can we use the simpler classical expression $p = (2mK)^{1/2}$ instead of the exact relativistic expression $p = \frac{K\left(1+\frac{2mc^2}{K}\right)^{1/2}}{c}$? As the relativistic expression reduces to $p = (2mK)^{1/2}$ for $K << 2mc^2$, we can use the classical expression whenever $K << 1\text{ MeV}$ because mc^2 for the electron is 0.511 MeV.

(a) Here $50\text{ eV} << 1\text{ MeV}$, so $p = (2mK)^{1/2}$

$$\lambda = \frac{h}{p} = \frac{h}{\left[(2)\left(\frac{0.511\text{ MeV}}{c^2}\right)(50\text{ eV})\right]^{1/2}} = \frac{hc}{\left[(2)(0.511\text{ MeV})(50\text{ eV})\right]^{1/2}}$$

$$= \frac{1\,240\text{ eV nm}}{\left[(2)\left(0.511\times 10^6\right)(50)(\text{eV})^2\right]^{1/2}} = 0.173\text{ nm}$$

(b) As $50\text{ eV} << 1\text{ MeV}$, $p = (2mK)^{1/2}$

$$\lambda = \frac{hc}{\left[(2)\left(\frac{0.511\text{ MeV}}{c^2}\right)\left(50\times 10^3\text{ eV}\right)\right]^{1/2}} = 5.49\times 10^{-3}\text{ nm}.$$

As this is clearly a worse approximation than in (a) to be on the safe side use the relativistic expression for p: $p = K\frac{\left(1+\frac{2mc^2}{K}\right)^{1/2}}{c}$ so

$$\lambda = \frac{h}{p} = \frac{hc}{\left(K^2+2Kmc^2\right)^{1/2}} = \frac{1\,240\text{ eV nm}}{\left[\left(50\times 10^3\right)^2 + (2)\left(50\times 10^3\right)\left(0.511\times 10^6\text{ eV}\right)\right]^{1/2}}$$

$$= 5.36\times 10^{-3}\text{ nm} = 0.005\,36\text{ nm}$$

5-3 $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{Js}}{74\ \text{kg}}(5\ \text{m/s}) = 1.79 \times 10^{-36}\ \text{m}$

5-7 A 10 MeV proton has $K = 10\ \text{MeV} << 2mc^2 = 1\,877\ \text{MeV}$ so we can use the classical expression $p = (2mK)^{1/2}$. (See Problem 5-2)

$$\lambda = \frac{h}{p} = \frac{hc}{[(2)(938.3\ \text{MeV})(10\ \text{MeV})]^{1/2}} = \frac{1\,240\ \text{MeV fm}}{\left[(2)(938.3)(10)(\text{MeV})^2\right]^{1/2}} = 9.05\ \text{fm} = 9.05 \times 10^{-15}\ \text{m}$$

5-8 $\lambda = \frac{h}{p} = \frac{h}{(2mK)^{1/2}} = \frac{h}{(2meV)^{1/2}} = \left[\frac{h}{(2me)^{1/2}}\right]V^{-1/2}$

$$\lambda = \left[\frac{6.626 \times 10^{-34}\ \text{Js}}{\left(2 \times 9.105 \times 10^{-31}\ \text{kg} \times 1.602 \times 10^{-19}\ \text{C}\right)^{1/2}}\right]V^{-1/2}$$

$$\lambda = \left[\frac{1.226 \times 10^{-9}\ \text{kg}^{1/2}\text{m}^2}{sC^{1/2}}\right]V^{-1/2}$$

5-10 As $\lambda = 2a_0 = 2(0.052\,9)\ \text{nm} = 0.105\,8\ \text{nm}$ the energy of the electron is nonrelativistic, so we can use

$$p = \frac{h}{\lambda} \text{ with } K = \frac{p^2}{2m};$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)^2}{2\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(1.058 \times 10^{-10}\ \text{m}\right)^2} = 21.5 \times 10^{-18}\ \text{J} = 134\ \text{eV}$$

This is about ten times as large as the ground-state energy of hydrogen, which is 13.6 eV.

5-11 (a) In this problem, the electron must be treated relativistically because we must use relativity when $pc \approx mc^2$. (See problem 5-5). the momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{10^{-14}\ \text{m}} = 6.626 \times 10^{-20}\ \text{kg} \cdot \text{m/s}$$

and $pc = 124\ \text{MeV} >> mc^2 = 0.511\ \text{MeV}$. The energy of the electron is

$$E = \left(p^2c^2 + m^2c^4\right)^{1/2}$$
$$= \left[\left(6.626 \times 10^{-20}\ \text{kg} \cdot \text{m/s}\right)^2\left(3 \times 10^8\ \text{m/s}\right)^2 + \left(0.511 \times 10^6\ \text{eV}\right)^2\left(1.602 \times 10^{-19}\ \text{J/eV}\right)^2\right]^{1/2}$$
$$= 1.99 \times 10^{-11}\ \text{J} = 1.24 \times 10^8\ \text{eV}$$

so that $K = E - mc^2 \approx 124\ \text{MeV}$.

(b) The kinetic energy is too large to expect that the electron could be confined to a region the size of the nucleus.