4-1 F corresponds to the charge passed to deposit one mole of monovalent element at a cathode. As one mole contains Avogadro's number of atoms,

$e = \frac{96\ 500\ \text{C}}{6.02 \quad 10^{23}} = 1.60 \quad 10^{\#19}\ \text{C}$.

4-2 (a) Total charge passed $= i * t = (1.00\ \text{A})(3\,600\ \text{s}) = 3\ 600\ \text{C}$. This is

$$\frac{3\ 600\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 2.25 \times 10^{22}\ \text{electrons.}$$

As the valence of the copper ion is two, two electrons are required to deposit each ion as a neutral atom on the cathode.

$$\text{The number of Cu atoms} = \frac{\text{number of electrons}}{2} = 1.125 \times 10^{22}\ \text{Cu atoms.}$$

(b) So the weight (mass) of a Cu atom is: $\frac{1.185\ \text{g}}{1.125 \times 10^{22}\ \text{atoms}} = 1.05 \times 10^{-22}\ \text{g}$.

(c) $m = q\frac{\text{molar weight}}{96\ 500}(2)$ or

$$\text{molar weight} = m(96\ 500)\frac{2}{q} = (1.185\ \text{g})(96\ 500\ \text{C})\frac{2}{3\ 600\ \text{C}} = 63.53\ \text{g}.$$

4-3 Thomson's device will work for positive and negative particles, so we may apply $\frac{q}{m} \approx \frac{V\theta}{B^2 ld}$.

(a) $\frac{q}{m} \approx \frac{V\theta}{B^2 ld} = (2\ 000\ \text{V})\frac{0.20\ \text{radians}}{(4.57 \times 10^{-2}\ \text{T})^2 (0.10\ \text{m})(0.02\ \text{m})} = 9.58 \times 10^7\ \text{C/kg}$

(b) As the particle is attracted by the negative plate, it carries a positive charge and is a proton. $\left(\frac{q}{m_p} = \frac{1.60 \times 10^{-19}\ \text{C}}{1.67 \times 10^{-27}\ \text{kg}} = 9.58 \times 10^7\ \text{C/kg}\right)$

(c) $v_x = \frac{E}{B} = \frac{V}{\text{d}B} = \frac{2\ 000\ \text{V}}{0.02\ \text{m}}(4.57 \times 10^{-2}\ \text{T}) = 2.19 \times 10^6\ \text{m/s}$

(d) As $v_x \sim 0.01c$ there is no need for relativistic mechanics.

4-6 The velocity of fall $v = \frac{\Delta y}{\Delta t} = \frac{0.004\ \text{m}}{15.9\ \text{s}} = 2.52 \times 10^{-4}\ \text{m/s}$.

(a) The radius, a, is given by

$$a = \left(\frac{9\eta v}{2\rho g}\right)^{1/2} = \left(\frac{9(1.81 \times 10^{-5}\ \text{kg/m s})(2.52 \times 10^{-4}\ \text{m/s})}{2(800\ \text{kg/m}^3)(9.81\ \text{m/s}^2)}\right)^{1/2} = 1.62 \times 10^{-6}\ \text{m}.$$

The mass, m, is given by

$$m = \rho V = \rho\left(\frac{4}{3}\right)\pi a^3 = (1.33)\left(800\ \text{kg}/\text{m}^3\right)(\pi)\left(1.62\times10^{-6}\ \text{m}\right)^3 = 1.42\times10^{-14}\ \text{kg}.$$

(b) $q_1 = \dfrac{\frac{mg}{E}(v + v_1')}{v}$ where v is the velocity of fall and v' is the velocity of rise. The electric field is given by $E = \dfrac{V}{d} = \dfrac{4\,000\ \text{V}}{0.020\,0\ \text{m}} = 200\,000\ \text{V}/\text{m}$, $v = 2.52\times10^{-4}\ \text{m}/\text{s}$, $v_1' = \dfrac{0.004\ \text{m}}{36.0\ \text{s}} = 1.11\times10^{-4}\ \text{m}/\text{s}$, $v_2' = 2.31\times10^{-4}\ \text{m}/\text{s}$, $v_3' = 1.67\times10^{-4}\ \text{m}/\text{s}$, $v_4' = 3.51\times10^{-4}\ \text{m}/\text{s}$, $v_5' = 5.31\times10^{-4}\ \text{m}/\text{s}$

$$q_1 = \left(\frac{mg}{E}\right)\left(\frac{v + v_1'}{v}\right) = \frac{\left(6.97\times10^{-19}\ \text{C}\right)(2.52 + 1.11)}{2.52} = 10.0\times10^{-19}\ \text{C}$$

$$q_2 = \left(\frac{mg}{E}\right)\left(\frac{v + v_2'}{v}\right) = 13.4\times10^{-19}\ \text{C}$$

$$q_3 = 11.6\times10^{-19}\ \text{C}$$

$$q_4 = 16.7\times10^{-19}\ \text{C}$$

$$q_5 = 21.6\times10^{-19}\ \text{C}$$

(c) By inspection we choose integers that will yield an elementary charge between 1.5 and 2.0×10^{-19} C

$$\frac{q_1}{6} = 1.67\times10^{-19}\ \text{C}$$

$$\frac{q_2}{8} = 1.68\times10^{-19}\ \text{C}$$

$$\frac{q_3}{7} = 1.66\times10^{-19}\ \text{C}$$

$$\frac{q_4}{10} = 1.67\times10^{-19}\ \text{C}$$

$$\frac{q_5}{13} = 1.66\times10^{-19}\ \text{C}$$

The amount of charge gained or lost:

$$q_2 - q_1 = 3.4\times10^{-19}\ \text{C}$$

$$q_3 - q_1 = 1.6\times10^{-19}\ \text{C}$$

$$q_2 - q_3 = 1.8\times10^{-19}\ \text{C}$$

$$q_4 - q_1 = 6.7\times10^{-19}\ \text{C}$$

$$q_4 - q_3 = 5.1\times10^{-19}\ \text{C}$$

$$q_4 - q_2 = 3.4\times10^{-19}\ \text{C}$$

$$q_5 - q_1 = 11.6\times10^{-19}\ \text{C}$$

$$q_5 - q_4 = 4.9\times10^{-19}\ \text{C}$$

$$q_5 - q_3 = 10.0\times10^{-19}\ \text{C}$$

By inspection we again find integers that yield a value of e between 1.5 and 2.0×10^{-19} C

$$\frac{q_2-q_1}{2}=1.70\times 10^{-19}\text{C}$$
$$\frac{q_3-q_1}{1}=1.60\times 10^{-19}\text{C}$$
$$\frac{q_2-q_3}{1}=1.80\times 10^{-19}\text{C}$$
$$\frac{q_4-q_1}{4}=1.68\times 10^{-19}\text{C}$$
$$\frac{q_4-q_3}{3}=1.70\times 10^{-19}\text{C}$$
$$\frac{q_4-q_2}{2}=1.65\times 10^{-19}\text{C}$$
$$\frac{q_5-q_1}{7}=1.66\times 10^{-19}\text{C}$$
$$\frac{q_5-q_4}{3}=1.63\times 10^{-19}\text{C}$$
$$\frac{q_5-q_3}{6}=1.67\times 10^{-19}\text{C}$$

Average of all values $=1.67\times 10^{-19}\text{ C}$.

4-8 (a) From Equation 4.16 we have $\Delta n \propto \left(\frac{\sin\phi}{2}\right)^{-4}$ or $\Delta n_2=\Delta n_1\frac{\left(\frac{\sin\phi_1}{2}\right)^4}{\left(\frac{\sin\phi_2}{2}\right)^4}$. Thus the number of α's scattered at 40 degrees is given by

$$\Delta n_2=(100\text{ cpm})\frac{\left(\sin\frac{20}{2}\right)^4}{\left(\sin\frac{40}{2}\right)^4}=(100\text{ cpm})\left(\frac{\sin 10}{\sin 20}\right)^4=6.64\text{ cpm}.$$

Similarly

$$\Delta n\text{ at 60 degrees}=1.45\text{ cpm}$$
$$\Delta n\text{ at 80 degrees}=0.533\text{ cpm}$$
$$\Delta n\text{ at 100 degrees}=0.264\text{ cpm}$$

(b) From 4.16 doubling $\left(\frac{1}{2}\right)m_\alpha v_\alpha^2$ reduces Δn by a factor of 4. Thus Δn at 20 degrees $=\left(\frac{1}{4}\right)(100\text{ cpm})=25\text{ cpm}$.

(c) From 4.16 we find $\frac{\Delta n_{\text{Cu}}}{\Delta n_{\text{Au}}}=\frac{Z_{\text{Cu}}^2 N_{\text{Cu}}}{Z_{\text{Au}}^2 N_{\text{Au}}}$, $Z_{\text{Cu}}=29$, $Z_{\text{Au}}=79$.

$$N_{\text{Cu}} = \text{number of Cu nuclei per unit area}$$
$$= \text{number of Cu nuclei per unit volume * foil thickness}$$
$$= \left[\left(8.9\ \text{g/cm}^3\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{63.54\ \text{g}}\right)\right]t = 8.43\times10^{22}t$$
$$N_{\text{Au}} = \left[\left(19.3\ \text{g/cm}^3\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{197.0\ \text{g}}\right)\right]t = 5.90\times10^{22}t$$

So $\Delta n_{\text{Cu}} = \Delta n_{\text{Au}}(29)^2\dfrac{8.43\times10^{22}}{(79)^2}\left(5.90\times10^2\right) = (100)\left(\dfrac{29}{79}\right)^2\left(\dfrac{8.43}{5.90}\right) = 19.3\ \text{cpm}$.

4-9 The initial energy of the system of α plus copper nucleus is 13.9 MeV and is just the kinetic energy of the α when the α is far from the nucleus. The final energy of the system may be evaluated at the point of closest approach when the kinetic energy is zero and the potential energy is $k(2e)\dfrac{Ze}{r}$ where r is approximately equal to the nuclear radius of copper. Invoking conservation of energy $E_i = E_f$, $K_\alpha = (k)\dfrac{2Ze^2}{r}$ or

$$r = (k)\frac{2Ze^2}{K_\alpha} = \frac{(2)(29)\left(1.60\times10^{-19}\right)^2\left(8.99\times10^9\right)}{\left(13.9\times10^6\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 6.00\times10^{-15}\ \text{m}.$$

4-11 $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_{\text{f}}^2} - \dfrac{1}{n_{\text{i}}^2}\right)$. For the Balmer series, $n_{\text{f}} = 2$; $n_{\text{i}} = 3, 4, 5, \ldots$. The first three lines in the series have wavelengths given by $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$ where $R = 1.097\ 37\times10^7\ \text{m}^{-1}$.

$$\text{1st line: } \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \left(\frac{5}{36}\right)R;\ \lambda = \frac{36}{5R} = 656.112\ \text{nm}$$
$$\text{2nd line: } \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \left(\frac{3}{16}\right)R;\ \lambda = \frac{16}{3R} = 486.009\ \text{nm}$$
$$\text{3rd line: } \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{25}\right) = \left(\frac{21}{100}\right)R;\ \lambda = \frac{100}{21R} = 433.937\ \text{nm}$$

4-12 $\dfrac{1}{\lambda} = R\left(\dfrac{1-1}{n^2}\right)$ where $n = 2, 3, 4, \ldots$ and $R = 1.097\ 373\,2\times10^7\ \text{m}^{-1}$;

$$\text{For } n = 2:\ \lambda = R^{-1}\left(1 - \frac{1}{2^2}\right)^{-1} = 1.215\ 02\times10^{-7}\ \text{m} = 121.502\ \text{nm (UV)}$$
$$\text{For } n = 3:\ \lambda = R^{-1}\left(1 - \frac{1}{3^2}\right)^{-1} = 1.025\,17\times10^{-7}\ \text{m} = 102.517\ \text{nm (UV)}$$
$$\text{For } n = 4:\ \lambda = R^{-1}\left(1 - \frac{1}{4^2}\right)^{-1} = 1.972\ 018\times10^{-7}\ \text{m} = 97.201\ 8\ \text{nm (UV)}$$

4-13 (a) $\lambda = 102.6\text{ nm}\ ;\ \frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right) \Rightarrow n = \frac{R}{\left(R - \frac{1}{\lambda}\right)^{1/2}} = \frac{R}{\left(R - \frac{1}{102.6\times 10^{-9}\text{ m}}\right)^{1/2}} = 2.99 \approx 3$

(b) This wavelength cannot belong to either series. Both the Paschen and Brackett series lie in the IR region, whereas the wavelength of 102.6 nm lies in the UV region.

4-15 (a) The energy levels of a hydrogen-like ion whose charge number is 2 is given by

$E_n = (-13.6\text{ eV})\frac{Z^2}{n^2} = \frac{-54.4\text{ eV}}{n^2}$ for $Z = 2$. (He^+)

0

$E_3 = -6.04\text{ eV}$

$E_2 = -13.6\text{ eV}$

So $E_1 = -54.4\text{ eV}$
$E_2 = -13.6\text{ eV}$
$E_3 = -6.04\text{ eV}$, etc.

$E_1 = -54.4\text{ eV}$

(b) For He^+, $Z = 2$ so we see that the ionization energy (the energy required to take the electron from the state $n = 1$ to the state $n = \infty$ is

$E = (-13.6\text{ eV})\frac{2^2}{1^2} = \frac{-54.4\text{ eV}}{n^2}$ for $Z = 2$. (He^+)

4-16 For Li^{2+}, $Z = 3$ from Equation 4.36

0

$E_3 = -13.6\text{ eV}$

$$E_n = -\frac{13.6 Z^2}{n^2\text{ eV}} = -\frac{122.4}{n^2\text{ eV}}$$

$E_2 = -30.6\text{ eV}$

So $E_1 = -122.4\text{ eV}$
$E_2 = -30.6\text{ eV}$
$E_3 = -13.6\text{ eV}$, etc.

$E_1 = -122.4\text{ eV}$

4-18 (a) $\Delta E = (13.6\text{ eV})\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1\text{ eV}$

(b) Either $\Delta E = 12.1$ eV or $\Delta E = (13.6 \text{ eV})\left(\frac{1}{1} - \frac{1}{2^2}\right) = 10.2$ eV and

$\Delta E = (13.6 \text{ eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89$ eV.

4-20 $\Delta E = (-13.6 \text{ eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

(a) $\Delta E = (-13.6 \text{ eV})\left(\frac{1}{25} - \frac{1}{16}\right) = 0.306 \text{ eV}$

(b) $\Delta E = (-13.6 \text{ eV})\left(\frac{1}{36} - \frac{1}{25}\right) = 0.166 \text{ eV}$

4-21 (a) For the Paschen series; $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$; the maximum wavelength corresponds to $n_i = 4$, $\frac{1}{\lambda_{\max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$; $\lambda_{\max} = 1\,874.606$ nm. For minimum wavelength, $n_i \to \infty$, $\frac{1}{\lambda_{\min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$; $\lambda_{\min} = \frac{9}{R} = 820.140$ nm.

(b)

$$\frac{hc}{\lambda_{\min}} = \frac{\left(\frac{hc}{1\,874.606 \text{ nm}}\right)}{1.6 \times 10^{-19} \text{ J/eV}} = 0.662\,7 \text{ nm}$$

$$\frac{hc}{\lambda_{\min}} = \frac{\left(\frac{hc}{820.140 \text{ nm}}\right)}{1.6 \times 10^{-19} \text{ J/eV}} = 1.515 \text{ nm}$$

4-27 (a) $\lambda = \frac{C_2 n^2}{n^2 - 2^2}$ so $\frac{1}{\lambda} = \left(\frac{1}{C_2}\right)\frac{n^2 - 2^2}{n^2} = \left(\frac{1}{C_2}\right)\left(\frac{1 - 2^2}{n^2}\right)$ or

$\frac{1}{\lambda} = \left(\frac{2^2}{C_2}\right)\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ where $R = \frac{2^2}{C_2}$.

(b) $R = \frac{2^2}{36\,545.6 \times 10^{-8} \text{ cm}} = 109\,720 \text{ cm}^{-1}$