1-5 This is a case of dilation. $T = \gamma T'$ in this problem with the proper time $T' = T_0$

$$T = \left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} T_0 \Rightarrow \frac{v}{c} = \left[1-\left(\frac{T_0}{T}\right)^2\right]^{1/2};$$

in this case $T = 2T_0$, $v = \left\{1-\left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1-\left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-6 This is a case of length contraction. $L = \frac{L'}{\gamma}$ in this problem the proper length $L' = L_0$,

$L = \left[1-\frac{v^2}{c^2}\right]^{-1/2} L_0 \Rightarrow v = c\left[1-\left(\frac{L}{L_0}\right)^2\right]^{1/2}$; in this case $L = \frac{L_0}{2}$,

$v = \left\{1-\left[\frac{L_0/2}{L_0}\right]^2\right\}^{1/2} = \left[1-\left(\frac{1}{4}\right)\right]^{1/2}$ therefore $v = 0.866c$.

1-7 The problem is solved by using time dilation. This is also a case of $v \ll c$ so the binomial expansion is used $\Delta t = \gamma\,\Delta t' \cong \left[1+\frac{v^2}{2c^2}\right]\Delta t'$, $\Delta t - \Delta t' = \frac{v^2\Delta t'}{2c^2}$; $v = \left[\frac{2c^2(\Delta t - \Delta t')}{\Delta t'}\right]^{1/2}$;

$\Delta t = (24\ \text{h/day})(3\,600\ \text{s/h}) = 86\,400\ \text{s}$; $\Delta t = \Delta t' - 1 = 86\,399\ \text{s}$;

$$v = \left[\frac{2(86\,400\ \text{s} - 86\,399\ \text{s})}{86\,399\ \text{s}}\right]^{1/2} = 0.004\,8c = 1.44\times10^{6}\ \text{m/s}.$$

1-8 $L = \frac{L'}{\gamma}$

$\frac{1}{\gamma} = \frac{L}{L'} = \left[1-\frac{v^2}{c^2}\right]^{1/2}$

$v = c\left[1-\left(\frac{L}{L'}\right)^2\right]^{1/2} = c\left[1-\left(\frac{75}{100}\right)^2\right]^{1/2} = 0.661c$

1-10 (a) $\tau = \gamma\tau'$ where $\beta = \frac{v}{c}$ and

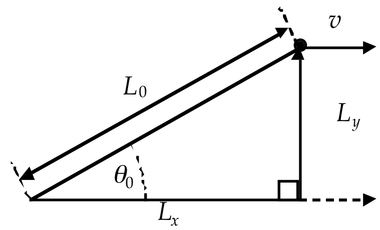
$$\gamma =' \left(1-\beta^2\right)^{-1/2} = \tau'\left(1-\frac{v^2}{c^2}\right)^{-1/2} = \left(2.6\times10^{-8}\ \text{s}\right)\left[1-(0.95)^2\right]^{-1/2} = 8.33\times10^{-8}\ \text{s}$$

(b) $d = v\tau = (0.95)\left(3\times10^{8}\right)\left(8.33\times10^{8}\ \text{s}\right) = 24\ \text{m}$

1-12 (a) 70 beats/min or $\Delta t' = \frac{1}{70}$ min

(b) $\Delta t = \gamma\ \Delta t' = \left[1-(0.9)^2\right]^{-1/2}\left(\frac{1}{70}\right)$ min $= 0.032\ 8$ min/beat or the number of beats per minute $\approx 30.5 \approx 31$.

1-14 (a) Only the $x$-component of $L_0$ contacts.




$$L_{x'} = L_0\cos\theta_0 \Rightarrow \frac{L_x\left[L_0\cos\theta_0\right]}{\gamma}$$

$$L_{y'} = L_0\sin\theta_0 \Rightarrow L_y = L_0\sin\theta_0$$

$$L = \left[(L_x)^2 + (L_y)^2\right]^{1/2} = \left[\left(\frac{L_0\cos\theta_0}{\gamma}\right)^2 + (L_0\sin\theta_0)^2\right]^{1/2}$$

$$= L_0\left[\cos^2\theta_0\left(1-\frac{v^2}{c^2}\right)+\sin^2\theta_0\right]^{1/2} = L_0\left[1-\frac{v^2}{c^2}\cos^2\theta_0\right]^{1/2}$$

(b) As seen by the stationary observer, $\tan\theta = \frac{L_y}{L_x} = \frac{L_0\sin\theta_0}{L_0\cos\theta_0/\gamma} = \gamma\tan\theta_0$.

1-16 For an observer approaching a light source, $\lambda_{\text{ob}} = \left[\frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}}\right]\lambda_{\text{source}}$. Setting $\beta = \frac{v}{c}$ and after some algebra we find,

$$\beta = \frac{\lambda_{\text{source}}^2 - \lambda_{\text{obs}}^2}{\lambda_{\text{source}}^2 + \lambda_{\text{obs}}^2} = \frac{(650\ \text{nm})^2-(550\ \text{nm})^2}{(650\ \text{nm})^2+(550\ \text{nm})^2} = 0.166$$

$$v = 0.166c = \left(4.98\times10^7\ \text{m/s}\right)\left(2.237\ \text{mi/h}\right)\left(\text{m/s}\right)^{-1} = 1.11\times10^8\ \text{mi/h}.$$

1-19 $u_{XA} = -u_{XB}$; $u'_{XA} = 0.7c = \frac{u_{XA}-u_{XB}}{1-u_{XA}u_{XB}/c^2}$; $0.70c = \frac{2u_{XA}}{1+(u_{XA}/c)^2}$ or $0.70u_{XA}^2 - 2cu_{XA} + 0.7c^2 = 0$. Solving this quadratic equation one finds $u_{XA} = 0.41c$ therefore $u_{XB} = -u_{XA} = -0.41c$.

1-21 $u'_X = \frac{u_X - v}{1-u_Xv/c^2} = \frac{0.50c-0.80c}{1-(0.50c)(0.80c)/c^2} = -0.50c$

1-23 (a) Let event 1 have coordinates $x_1 = y_1 = z_1 = t_1 = 0$ and event 2 have coordinates $x_2 = 100$ mm, $y_2 = z_2 = t_2 = 0$. In $S'$, $x'_1 = \gamma(x_1 - vt_1) = 0$, $y'_1 = y_1 = 0$, $z'_1 = z_1 = 0$, and $t'_1 = \gamma\left[t_1 - \left(\frac{v}{c^2}\right)x_1\right] = 0$, with $\gamma = \left[1-\frac{v^2}{c^2}\right]^{-1/2}$ and so $\gamma = \left[1-(0.70)^2\right]^{-1/2} = 1.40$. In system $S'$, $x'_2 = \gamma(x_2 - vt_2) = 140$ m, $y'_2 = z'_2 = 0$, and

$$t_2' = \gamma\left[t_2 - \left(\frac{v}{c^2}\right)x_2\right] = \frac{(1.4)(-0.70)(100\text{ m})}{3.00\times10^8\text{ m/s}} = -0.33\ \mu\text{s}.$$

1-31 In this case, the proper time is $T_0$ (the time measured by the students using a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$ where $\Delta t = T + t$. Here $T$ is the time she waits before sending a signal and $t$ is the time required for the signal to reach the students. Thus we have: $T + t = \gamma T_0$. To determine travel time $t$, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T+t)$. The time required for the signal to travel this distance is: $t = \frac{d}{c} = \frac{v}{c}(T+t)$. Solving for $t$ gives: $t = \left(\frac{v}{c}\right)T\left(1-\frac{v}{c}\right)^{-1}$. Substituting this into the above equation for $(T+t)$ yields: $T + \left(\frac{v}{c}\right)T\left(1-\frac{v}{c}\right)^{-1} = \gamma T_0$, or $T\left(1-\frac{v}{c}\right)^{-1} = \gamma T_0$. Using the expression for $\gamma$ this becomes: $T = \left(1-\frac{v}{c}\right)\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} T_0$, or

$$T = T_0\left(1-\frac{v}{c}\right)\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} = T_0\left[\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)^{-1}\right]^{1/2}.$$

1-33 (a) We in the spaceship do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-\left(\frac{v}{c}\right)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}.$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is $\frac{3.60\text{ ly}}{1.80c} = 2.00$ yr. We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for $\frac{3.60\text{ ly}}{0.200c} = 18.0$ yr and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

(b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

1-37 Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. We may take the $S$-frame coordinates of the events as ($x = 0$, $y = 0$, $z = 0$, $t = 0$) and ($x = 100$ m, $y = 0$, $z = 0$, $t = 0$). Then the coordinates in $S'$ are given by Equations 1.23 to 1.27. Event A is at ($x' = 0$, $y' = 0$, $z' = 0$, $t' = 0$). The time of event B is:

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1-0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\text{ m})\right) = 1.667\left(\frac{80\text{ m}}{3\times10^8\text{ m/s}}\right) = -4.44\times10^{-7}\text{ s}.$$

The time elapsing before A occurs is 444 ns.