

Experiment 6: Diffraction and Interference with Coherent Light

Introduction

In this laboratory you will examine the diffraction and interference patterns caused by laser (coherent) light.

1 Physics

1.1 Coherent Light

A word is necessary about the unique character of laser radiation. All other sources of light produce what is called incoherent light, which means that a beam of such light is composed of a large number of short trains of waves randomly arranged in phase with respect to one another, and usually containing multiple different wavelengths. Indeed, ordinary white light contains all the wavelengths in the visible spectrum.

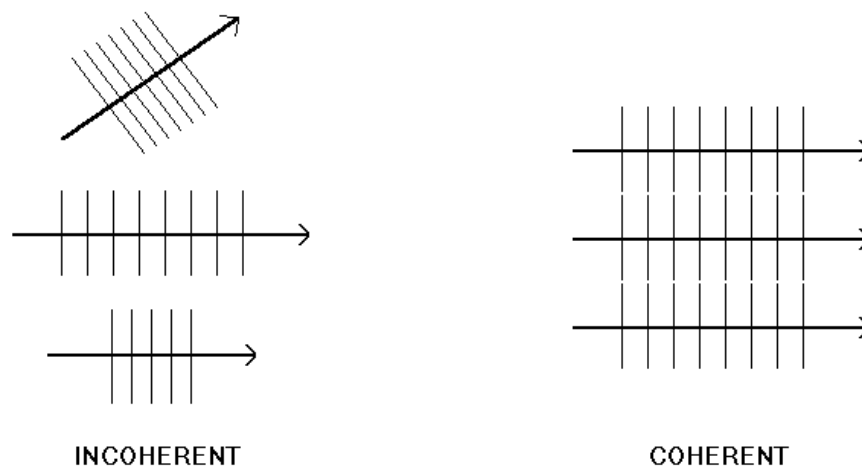


Figure 1 – *Packets of waves which are incoherent have various wavelengths, directions and phases with respect to each other. The coherent packets contain light traveling in the same direction with the same phase and the same wavelength.*

The mathematical derivation of the formula for diffraction from a slit assumes that a uniform straight wave front of light exists all across the slit, and radiates Huygens' wavelets. For incoherent light, a regular diffraction pattern will not be created. It will be smeared out, showing none of the sharp structure we will derive. Laser light, however, is coherent, which is to say that the beam is a single train of waves that extend uniformly over the whole cross-section. Furthermore, it is monochromatic, meaning that only a single, constant wavelength is present. Thus, in laser light, the idealized diffraction effects discussed below should actually occur.

1.2 Huygens' Principle

When a train of waves strikes an obstruction, the pattern of transmitted energy produces a characteristic interference or diffraction pattern beyond the object. While we are going to discuss the problem specifically for electromagnetic radiation, it is much more general than that; the analysis we shall do here applies, almost without any change, to sound waves and even ocean waves.

The discussion can be stated quite well by considering the behavior of ocean waves in a familiar situation. Suppose you are in an airplane watching the waves approach a breakwater that has a gap in it. The waves that come into the gap can certainly get through. What happens to them then? Do they continue on in the manner of Figure 2a? It is our experience that they do not. Rather, we know that they spread radially away from the gap, as in Figure 2b.

Similarly, one knows that an object such as a board set into the path of a wave train in water will not cast a long "shadow" as in Figure 3a, but will cause only a momentary break in the otherwise straight waves (Figure 3b).

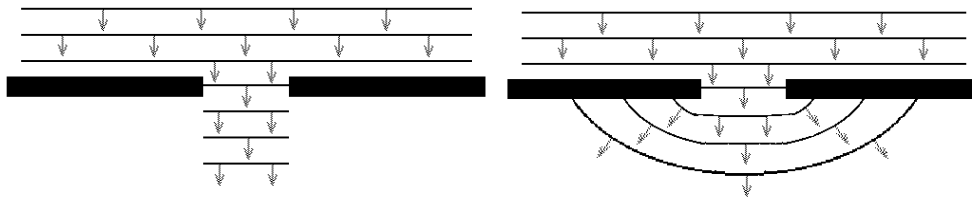


Figure 2 – Waves impinge on a hole. False version is (a – left): correct one is (b – right).

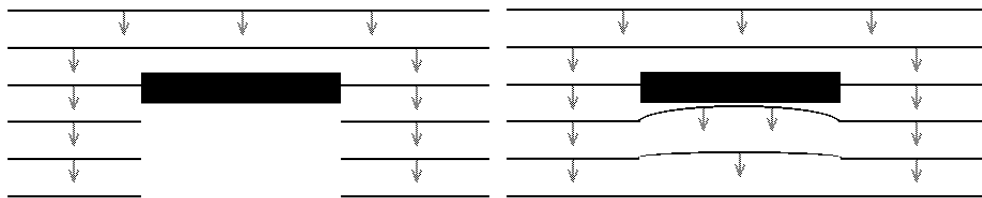


Figure 3 – Waves impinge on an obstacle. Once again, the correct version is (b – right).

The behavior of waves in these circumstances is described accurately by a very simple but profound principle set forth by Christian Huygens in the 17th century. The picture shows this principle, which is **every point on a wave front (i.e., a surface of equal phase) acts as a point source of spherical waves**. Thus, when only part of the wave front is allowed to pass a barrier and the barrier is smaller than or comparable to the wavelength, we can approximate the passing wave as several spherical waves emanating from each point in that opening.

1.3 Diffraction

Recall Huygens' principle states that every point on a wave front (i.e., a surface of equal phase) acts as a point source of spherical waves. To discover what results this may have, let us return to our two-dimensional case of surface waves in water. The analogy to Huygens' principle here would be that each

point along a wave is a source of circular waves. Let us draw our open breakwater again, and the wave which is just coming through it:

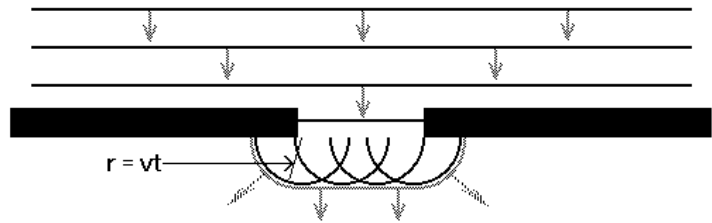


Figure 4 – Huygens' wavelet construction.

If we wish to predict the form of this wave a time t later, we simply draw a great number of circles of radius $r = vt$ and having centers all along the original wave. Their envelope will indeed be the wave shape at the new time. (For 3 dimensions, we would superimpose spherical waves originating from every point on some initial wave front.) This clearly describes the behavior shown in Figure 2(b) and 3(b).

We will next consider the problem of these "diffracted" waves in a little more detail. We ask the following question: if we are at a distance from an aperture (the opening in our breakwater) which is large compared to the aperture width, how does the amplitude of the waves vary with the angle between the observer and the aperture? We assume that the plane (or straight) waves are incident upon the opening, and parallel to it (see Figure 5).

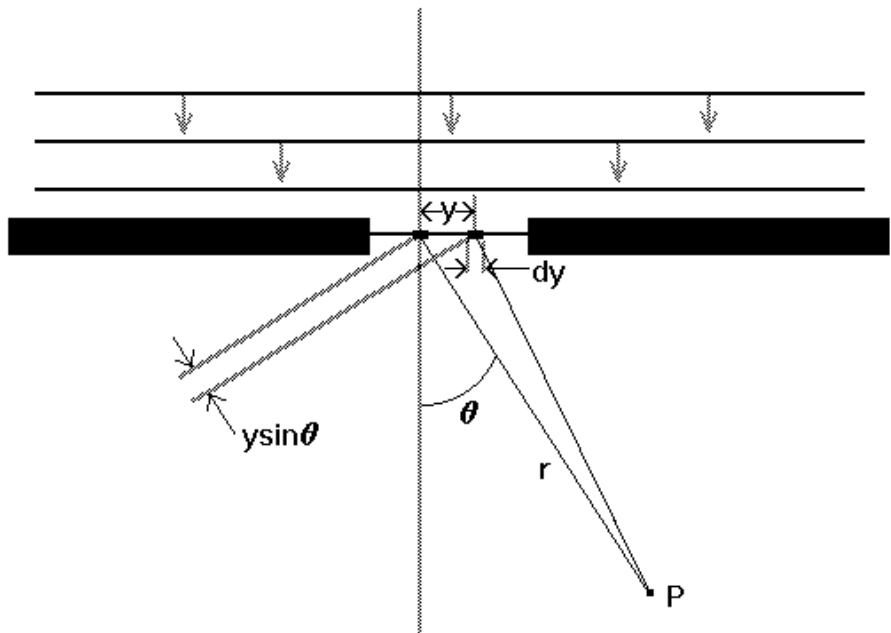


Figure 5 – Setup for integration of amplitudes from a single slit, out to point P .

The problem will be to calculate the wave amplitude at a point P, which is at an angle θ from the normal to the aperture and at a distance r from it.

Our reasoning here is qualitatively quite simple. We invoke Huygens' principle again, and assume that across the aperture there are, in a straight line, a multitude of little generators of spherical (or circular) waves, all oscillating in perfect synchronism together.

The amplitude of the wave at point P will be the sum of all the contributions from these little sources. The only complicating fact here is that point P is farther from the sources at the left of the aperture than from those at the right, and consequently, the various contributions will not all have the same phase when they arrive - the waves will cancel each other out in varying degrees depending on the angle at which the observation is made. (Since we have assumed that $r \gg a$, i.e., that P is quite distant from the aperture, we are safe in the assumption that the waves coming to P from all equal increments dy of the opening have the same maximum amplitude, and that they only differ in phase.)

$$dA_p = A_0 \cos(kr - \omega t) dy \quad (1)$$

Sources off the aperture center are closer to or farther from P by distances $y \sin \theta$ (Fig. 4), and so, the general increment is

$$dA_p = A_0 \cos[k(r - y \sin \theta) - \omega t] dy \quad (2)$$

and we simply integrate this over the width of the opening to obtain A_p :

$$A_p(\theta) = A_0 \int_{-a/2}^{a/2} \cos[k(r - y \sin \theta) - \omega t] dy \quad (3)$$

The result, which you could evaluate for yourself, is:

$$A_p(\theta) = A_0 a \frac{\sin\left(\frac{ka}{2} \sin \theta\right)}{\frac{ka}{2} \sin \theta} \cos(kr - \omega t) \quad (4)$$

The constant A_0 contains the dependence of the maximum amplitude upon the distance r . This will not interest us since our main concern is with the angular dependence. In most instances, the diffracted wave intensity is of more interest than the amplitude itself, since our means of detection of most wave phenomena are sensitive only to intensity. Since intensity is proportional to the square of the wave amplitude, the angular intensity dependence of the diffracted waves has the form

$$I = I_0 \left(\frac{\sin \beta}{\beta}\right)^2, \text{ where } \beta = \frac{ka}{2} \sin \theta \quad (5)$$

The radiation pattern then will look something like Figure 6.

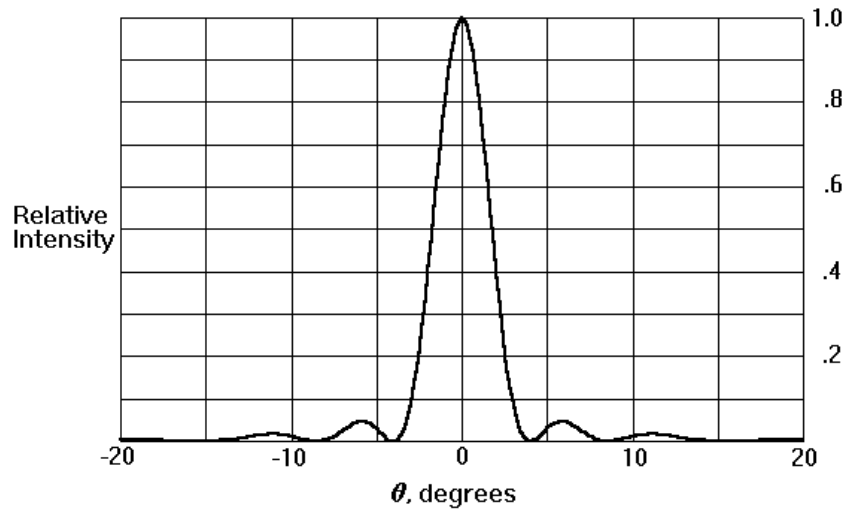


Figure 6 – Diffraction pattern for waves passing through narrow slit of width a .

This pattern contains a quite surprising result. It shows us that while the wave pattern does indeed spread out, as we guessed from a qualitative application of Huygens' principle, there are particular angles where the intensity goes exactly to zero. We could certainly not have anticipated such a result from the general "common-sense" arguments that preceded our calculation. We see that the "null angle" is related in a simple way to the ratio between the wavelength λ and a , the aperture width:

$$I = 0 \text{ for } \frac{ka}{2} \sin \theta = \pi, 2\pi, \dots \quad (6)$$

or, since $k = \frac{2\pi}{\lambda}$,

$$I = 0 \text{ for } \sin \theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \dots \quad (7)$$

In this experiment, you will determine λ for an electromagnetic wave by measuring the angle for the first minimum for a given aperture width. The other minimum values are typically around such low intensity of light that the measurement is not very reliable. See Figure 6 to examine the rapid drop off in intensity.

1.4 Babinet's Principle

A theorem called Babinet's Principle, which is not too difficult to derive, may be stated as follows:

Except for the intensity of the central spot, the diffraction pattern produced by an opaque object in a wave train (or beam) is the same as that produced by an aperture of the same size and shape in an otherwise opaque screen.

Thus, according to Babinet's Principle, cases A and B in Figure 7 will produce basically the same diffraction pattern. The only difference between them is that the beam passing around the obstruction will generally leave a bright extra spot at the center of the screen (in case B, the central spot from the diffraction pattern may be too dim to see).

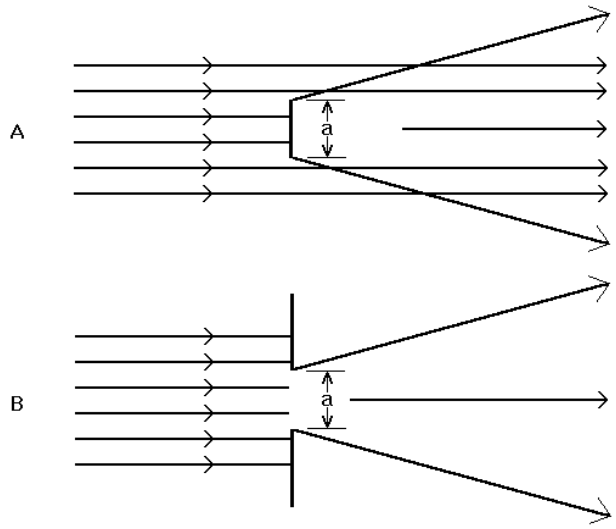


Figure 7 – Equivalence of diffraction pattern for a disk (a) and a hole (b), except for the overall forward direction beam intensity.

1.5 Interference

Interference phenomena in wave mechanics has already been encountered in what we have just described, since the existence of a null in the diffraction pattern is due to a perfect, canceling interference between waves from different parts of the aperture. However, this particular term is usually reserved for interference between waves from discrete, well-separated sources. The most commonly used and studied case is an array of linear or point sources of waves, such as slits in an opaque screen (light) or dipole antennas (radio waves).

Consider the array of narrow slits in a screen in Figure 8. We can suppose that electromagnetic plane waves are incident on the screen (as they are in this experiment), and we will simplify the problem by making $a \ll \lambda$, so each slit radiates isotropically (uniformly in all directions). This is an approximation, but it will produce results which are sufficiently accurate.

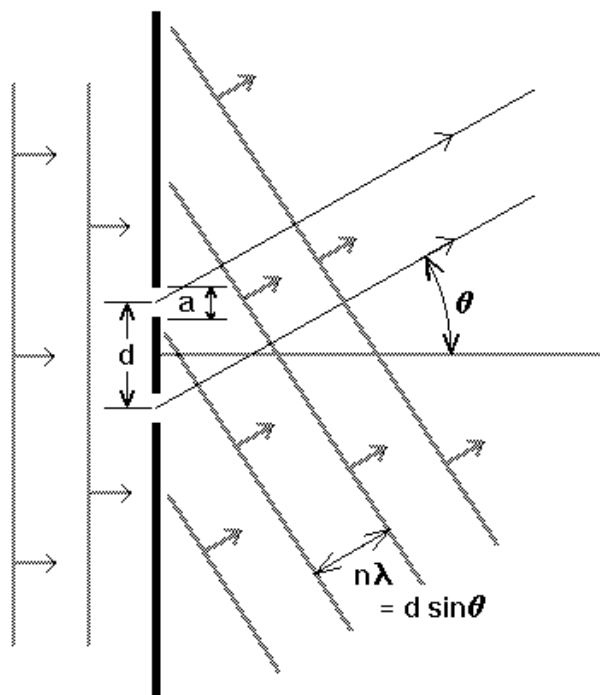


Figure 8 – *The two-slit interference geometry.*

For points of observation very far from the screen, strong intensity maxima will occur when all the slit contributions arrive in phase; but this clearly requires that the differences in distance must be an integral number of wavelengths, or that

$$d \sin \theta = n \lambda \text{ } ^1 \tag{8}$$

The greater the number of slits, the sharper and more closely defined in angle the maxima are. Optical spectrographs employ gratings having as many as 10^5 apertures. However, even two slits allow a determination of wavelength to accuracy somewhat better than can be accomplished by a measurement of the single-slit diffraction pattern.

2 The Small Angle Approximation

In this experiment your deflection from the central peak will be very small. It is appropriate for you to make small angle approximations based on this. Your results would not differ noticeably if you were to use the sine function so it will simplify your calculations to make the following approximations.

¹ Note that n is the number of wavelengths between any two peaks. When you are taking measurements between two peaks this n will be the number of peaks from the first to the second. **n is NOT the number of slits.**

The Figure below depicts your experimental setup although u will be much smaller in comparison to L in the experiment. From basic rules of trigonometry we recall that:

$$\sin \theta = \frac{u}{\sqrt{L^2 + u^2}} \quad (9)$$

However, $L \gg u$ in this laboratory so we may approximate in the following fashion:

$$\sin \theta \approx \frac{u}{L} \quad (10)$$

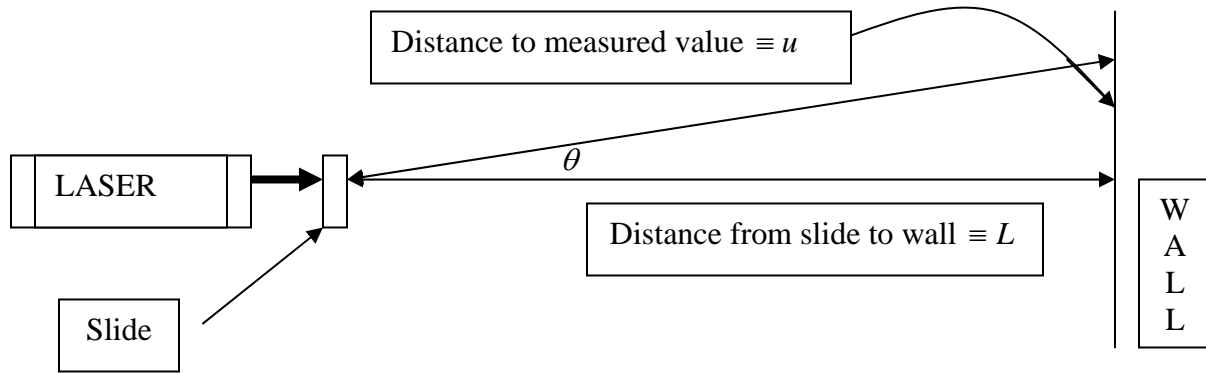


Figure 9 – *The small angle approximation is valid when $L \gg u$.*

3 The Experiment

3.1 Safety

You will study here the diffraction of light emitted by a Helium-Neon laser. The power of the beam from this unit is about one milliwatt (10^{-3} watt):

- You can damage your eyes by looking directly into the beam or a beam reflected by a mirror-like surface. Exercise reasonable caution! DO NOT look directly into the beam or a beam reflected by a mirror-like surface.
- When handling such objects as flat glass slides, or even film slides, be careful that they remain approximately perpendicular to the beam when in the beam, so you do not receive accidental eye exposure from reflections.

3.2 The Equipment

- A He-Ne laser;
- An "optical bench", which is a metal rail upon which the holders for various pieces of equipment are clamped;
- Holders for the slides that hold the diffracting objects;
- A variety of electron beam-etched slides that contain very small patterns or apertures whose diffraction patterns are the object of your study.

3.3 Determination of Wavelength Using Single-slit Diffraction

Combining the small angle approximation with the single-slit diffraction first minimum distance, we arrive at the following:

$$\frac{u}{L} = \frac{\lambda}{a} \text{ or } u = \lambda \frac{L}{a} \quad (11)$$

Question 6.1

What is the propagated error of the distance to the first minimum, u , in Equation 11? That is, what is Δu ? Assume that the error associated with a is negligible.

Locate a slide that contains single slit. The number of slits (**which is not n in the interference formula**) and their widths are generally labeled on the slide. Place the slide in front of the laser so that the beam goes through a selected slit. Observe the diffraction pattern on a screen located a distance L away from the slide (measure and record that distance, you should aim for 2 meters or more in distance).

You can use paper taped on the wall as a screen for the purpose of this experiment. Measure the distance u from the center of the diffraction pattern to the first minimum, and calculate the wavelength using equation 11 (L and u are measured, a is labeled for each slit). You may assume that the error of a is negligible.

In order to cancel effects of misalignment of the slide or the screen with respect to the beam direction, you should measure the distance between the first minima on either sides of the central peak and divide by 2.

Do this for several different slit widths, a . Measure u and L for several different slit widths, and then plot u vs. L/a using ORIGIN. From Equation 11 this should be a linear graph with zero intercept, and its slope will be λ . Compare your estimate for λ with $\lambda_0 = 632.8$ nm.

3.4 Determination of Thin Strands' Diameter

Through the use of Babinet's Principle and the single-slit diffraction pattern, it is a simple matter to measure the diameters of very thin filaments. You will be supplied with samples of two sizes of very fine wire. Place a strand of each of these in the beam near the laser, and by measuring the diffraction pattern, infer its diameter (with uncertainty). Your result can be checked against the wire diameter, which is listed on the slide.

3.5 Determination of Wavelength Using Multi-slit Interference Pattern

Using the result from the derivation above and the same angle approximation the distance, y , between an interference maximum and the n^{th} interference maximum from it is:

$$\frac{y}{L} = \frac{n\lambda}{d} \quad \text{or} \quad y = \frac{n\lambda L}{d} \quad (12)$$

Find the slide labeled 1C on its frame (or any other slide containing slit arrangements with similar parameters), and place it on the magnetic holder. With the labels horizontal, adjust the slide so that it is near the front of the laser, with the laser beam passing approximately through the two slit group labeled 0.04 mm width, 0.125 mm spacing. This slide may not be well labeled but if you examine your supplies you should find a slide with 2, 3, 4 and 5 slits. This is the 1C slide. If you have trouble finding it, ask your TA for assistance.

Question 6.2

What do you expect the distance between consecutive peaks to be for the 2 slit slide with spacing 0.125mm? What about the 5 slit slide?

By moving the slide laterally, and moving the slide vertically if necessary, you should find a location where a horizontal row of spots appears on the screen. Here, the light is passing through a pair of parallel slits. By using the relationship in equation 12 above for spot separation from the screen central peak, you can estimate the value of λ (with uncertainty). The value of d that you will need for this calculation (slit spacing) is marked on the slide (0.125 mm). Compare the experimentally determined value of λ with the actual value of $\lambda_0 = 632.8 \text{ nm}$.

Move the slide laterally so that the beam passes through the slit group labeled “5 slit”. You should once again find a row of spots, but now, the pattern is produced by a set of 5 evenly spaced slits. What difference is there in the character of the maxima? How do you account for a larger number of slits making this difference? Once again, by measuring the displacement of these spots from the screen center, estimate λ , and compare to the actual value of λ_0 .

3.6 Two-Dimensional Array

In this portion of the experiment, we employ a very fine-mesh screen of a kind used in photoengraving. Samples of this screen are mounted in the slides labeled 1D or ones which are provided for you. You will notice that your unaided eye can see through the screen because the wires are so fine.

In order to understand the diffraction pattern of this screen, we will once again consider the pattern from a simple system of two slits with laser light in the first section of this experiment. Suppose the slits are spaced a distance d apart (center-to-center), with each having a width a , where we suppose also that d is much greater than a . As we have learned, the interference pattern from the pair of slits will be an array of spots, separated by an interval:

$$\Delta v = v_{n+1} - v_n = \frac{\lambda L}{d} \quad (13)$$

where L is the slit-to-screen distance.

However, each of the slits also produces its own diffraction pattern, i.e., a series of spots that satisfy

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$$

The solutions of which are:

$$I = 0 \text{ for } \sin \theta = \frac{\lambda}{a}, \frac{2\lambda}{a}, \dots, L \quad (14)$$

$$\frac{u}{L} = \frac{\lambda}{a}, \frac{2\lambda}{a}, \dots, L \quad \text{or} \quad u = \frac{n\lambda L}{a} \quad (15)$$

Thus, when you measure an interference pattern you will probably notice that some of the interference maximums are brighter than others. This is caused by the diffraction pattern which overlays it. The result of superimposing the diffraction patterns of the individual slits and the interference pattern they produce in combination is a composite pattern as represented in Figure 7.

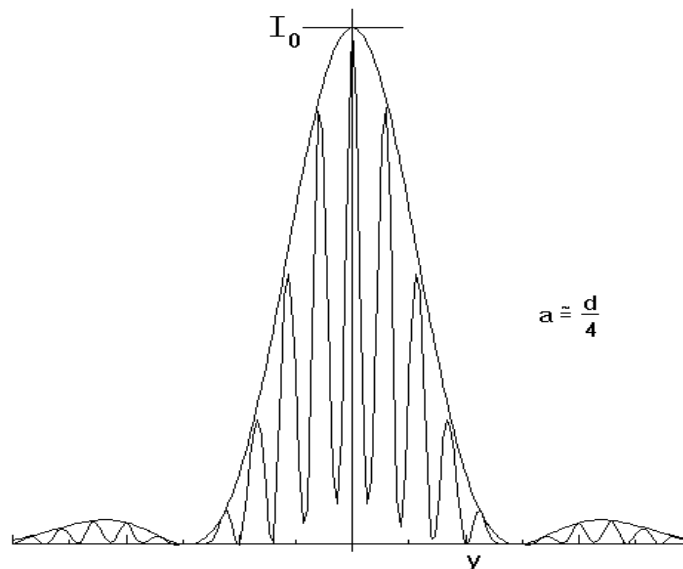


Figure 10 – Light intensity pattern from fine-mesh screen, showing multi-slit interference superimposed on the single-slit diffraction pattern

The closely spaced peaks arise from interference between the two slits, while the broader "envelope function" that modulates the interference maxima is itself the diffraction pattern of a single slit.

As already mentioned before, large features in diffraction or interference patterns correspond to small features in the objects that is probed by the light, and vice versa small diffraction features correspond to large objects. Therefore in a situation such as in Figure 10, the large features - the envelope of the pattern will originate from the smaller features in the sample, namely the slit width (or by Babinet's principle, the strand width), and the small features - the rapid intensity oscillations, originate from the larger features in the sample, namely the slit spacing (or the mesh interval). Measure the locations of the minima of the diffraction pattern to find the strand width, and the location of the interference minima to find the mesh interval.

Question 6.3

What variables do the mesh interval and wire thickness correspond to in the equations given above?

Analysis

Describe the following:

- Parameters in Equation 11;
- Parameters in Equation 12.

Compute the following:

- The strand thickness of the wires provided;
- The wavelength of the laser based on the interference pattern measurements from the 2-slit slide;
- The wavelength of the laser based on the interference pattern measurements from the 5-slit slide;
- The mesh interval and individual wire thickness of the 2 dimensional wire mesh.

Compute t-values for the following:

- The wavelength of the laser based on the first minimum diffraction plots;
- The strand thickness of the wires provided;
- The wavelength of the laser based on the interference pattern measurements from the 2-slit slide;
- The wavelength of the laser based on the interference pattern measurements from the 5-slit slide.

Conclusions

Highlight the themes of the lab and the physics the experiment verifies. You should discuss the errors you encounter in the lab and how you could improve the lab if you had to repeat it. If your results are unexpected or your t-values are high, you should identify possible explanations.

Hints on reports

- Write neatly—if your TA cannot read it, you could lose points.
- Be organized—if your TA cannot find it, you could lose points.
- Report your data, including plots—if your data is not in your report, your TA does know you did it.
- Record uncertainty.
- Propagate uncertainty.
- Write your final answers with proper significant figures.