

Experiment 1: RC Circuits

Introduction

In this laboratory you will examine a simple circuit consisting of only one capacitor and one resistor. By applying a constant¹ voltage (also called DC or direct current) to the circuit, you will determine the capacitor discharge decay time (defined later) and compare this value to that which is expected. Alternately, by applying alternating current (AC) and varying the frequency of the current you will be able to determine the decay time in another, independent way.

You will determine these expected values using formulas derived from Ohm's law, Kirchhoff's Law and the concept of complex impedance. These concepts will be detailed further in the laboratory.

In addition to these new ideas you will need to recall the concepts from last week's lab. Specifically you will use the multimeter and oscilloscope during this lab. Recall that the oscilloscope must be calibrated at the beginning of any experiment in which it is used.

1 Physics

1.1 Electrical Circuit Definitions

Any section of a circuit that is at constant voltage is called a "node." An example is a piece of wire joining two or more resistors. The sum of all the currents flowing into a node must be zero since charge can neither be created nor destroyed in a circuit.

Circuit "elements" are resistors, capacitors, and inductors. One might consider wires connecting these elements to be a fourth circuit element but since idealized wires have no resistance, capacitance or inductance they are represented only by lines in a circuit diagram and do not appear in the equations relating current and voltage in circuits.

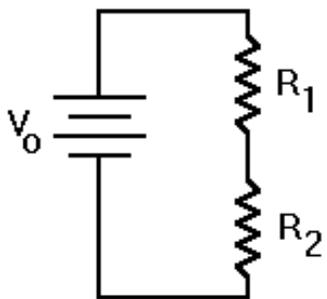


Figure 1 Resistors in series

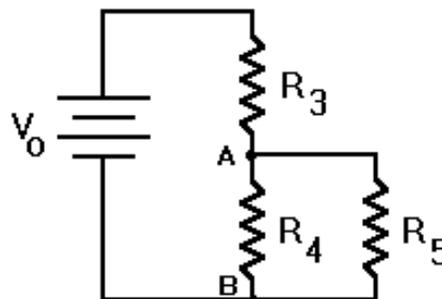


Figure 2 Resistors R_4 and R_5 in parallel

¹ Applying a square wave to the circuit is not exactly applying constant voltage. However, if the period of the square wave is long enough, you will not have to worry about what happens at the end when the voltage switches from one constant voltage to another.

Two circuit elements are in series if all of the current flowing through one also flows through the other. In Figure 1, all of the current flowing from the battery must also flow through the resistors R_1 and R_2 . They are “in series.” In Figure 2, the current flowing through R_4 does not flow through R_5 (and vice versa) so that R_4 and R_5 are not in series.

Two circuit elements are in parallel if they are connected to the same nodes. R_4 and R_5 in Figure 2 are both connected to nodes A and B. This then also requires that the potential difference (voltage drop) across all elements connected in parallel must be the same.

In more complicated circuits you will need to generalize the notions of series and parallel. For example, in Figure 2, the equivalent resistance of R_4 and R_5 in parallel, $(1/R_4 + 1/R_5)^{-1} = R_4 * R_5 / (R_4 + R_5)$, is in series with R_3 .

1.2 Ohm’s Law

Ohm’s Law states that for a resistor the current and voltage are in phase and proportional. That is:

$$V = I * R$$

1.3 Kirchhoff’s Law

This law or rule states: “The algebraic sum of the changes in potential encountered in a complete traversal of any closed circuit must be zero.” The adjective, algebraic, is added to indicate that the sign of the potential change encountered in crossing various parts of the circuit must be accounted for. For example, if two batteries are in the circuit but placed such that their potentials tend to force current in opposite directions, then the potentials must have opposite sign and the “algebraic” sum yields the net potential difference across both. Thus, the elements which cause a change in voltage in any circuit loop (e.g. resistors, capacitors, inductors and batteries) will be signed and will sum to zero.

1.4 Common Grounding

In numerous circuits that you will construct you will apply a voltage to an entire circuit and also measure the voltage over specific current elements. Both the applied voltage and the device which measures the voltage (typically the oscilloscope) will have one end which fixes the voltage to the common ground voltage. The common ground voltage is our standard for zero voltage and the voltage you record is measured relative to that.

Below there is an example of a circuit with two resistors with AC voltage (from the function generator) applied to a resistor and capacitor in series. Figure 3 shows the circuit in which we measure the voltage of the capacitor. Notice that the ground of the function generator and the ground of the oscilloscope are next to each other in the circuit diagram.

Figure 4 measures the voltage over the resistor but in this case one of the grounds is one side of the capacitor and the other is on the opposite side of the capacitor. This will measure the voltage over the resistor, but in effect removes the capacitor from the circuit because the voltage difference over the capacitor is close to zero. This is a short circuit.

Figure 5 also records the voltage over the resistor. However, since we have swapped the resistor and the capacitor, the grounds are at the same point in the circuit. Thus, you are measuring the voltage of the resistor in an RC circuit. Be careful when measuring the voltage of the different elements in your circuit and place the grounds in the appropriate places so that you do not short your circuit (as is the case in Figure 4).

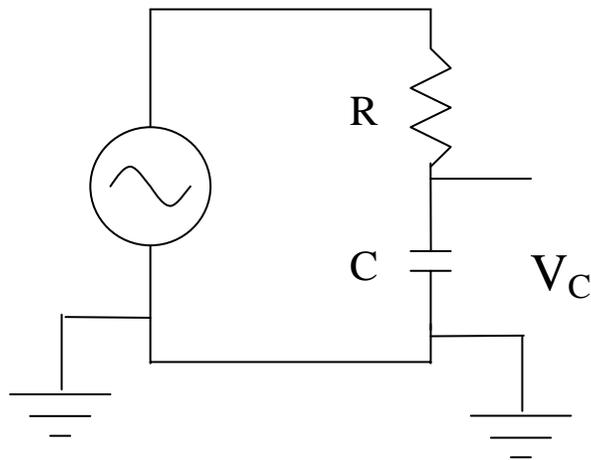


Figure 3 Voltage over the capacitor is recorded.

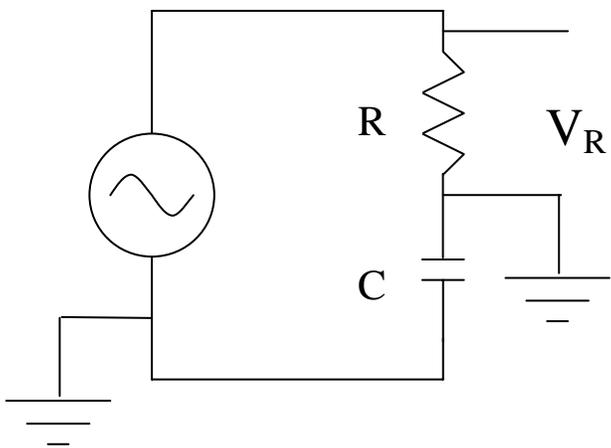


Figure 4 Voltage over resistor is measured while circuit is 'shorted'.

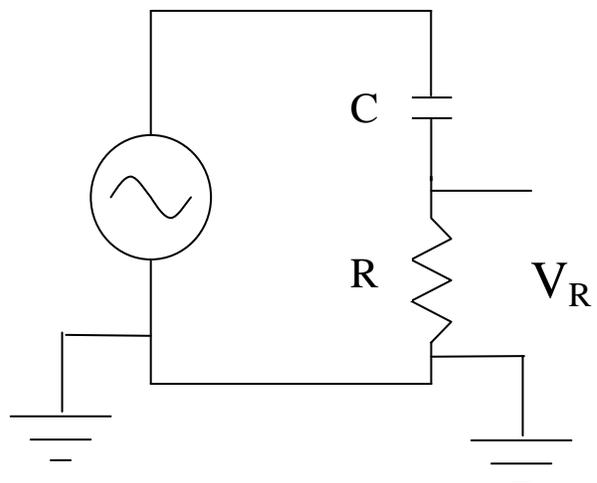


Figure 5 Voltage over resistor is measured properly in RC circuit.

2 Mathematical Applications

2.1 The Discharging (Charging) RC Circuit

A capacitor can be charged by connecting its two terminals to the two terminals of a battery. If the capacitor is then disconnected from the battery it will retain this charge and the potential across the capacitor will remain that of the battery. If a resistor is then connected across the capacitor, charge will flow through the resistor until the potential difference between the two terminals goes to zero. This series RC circuit is shown in Figure 6. The potential will decrease with time according to the relation:

$$V(t) = V_0 e^{-t/\tau}, \text{ where } \tau = RC \quad (1)$$

V_0 represents the voltage at time $t = 0$, and τ represents the “time constant” or time that it takes for the voltage to decrease by a factor of $1/e$.

Question 1.1

What is the propagated error of the time decay, τ , in equation 1? That is, what is $\Delta\tau$?

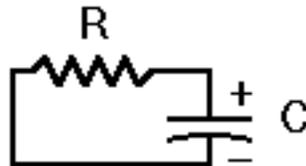


Figure 6 Simple RC circuit

The time dependence of the voltage is derived using Kirchhoff's law and the relations between current and voltage in the resistor and capacitor. Traversing the loop of Figure 6 clockwise, Kirchhoff's law tells us that:

$$V_R + V_C = 0 \quad (2)$$

where V_R and V_C represent voltage drop across the resistor or capacitor, respectively.

It is important to note that if one applies a constant voltage to this circuit (which is what you will be asked to do) then the additional voltage source will be an additional term in the sum. This term is will be a constant. The result of the forthcoming differential equation is the same with the addition of a constant.

These circuit elements are related to their voltages in the following ways:

$$V_R = I_R R \quad (3)$$

$$Q_C = C V_C \quad (4)$$

where Q_C is charge accumulation in the capacitor. Substituting these two equations into the Kirchhoff equation and solving for I_R yields

$$I_R = \frac{-1}{RC} Q_C \quad (5)$$

Since R and C are in series

$$\frac{dQ_C}{dt} = I_R = \frac{-1}{RC} Q_C \quad (6)$$

Using the initial conditions $Q=Q_0$ at $t=0$ the charge Q on the capacitor at some later time t is found by integration

$$\int_{Q_0}^Q \frac{dQ}{Q} = \frac{-1}{RC} \int_0^t dt$$

$$\ln\left(\frac{Q}{Q_0}\right) = \frac{-1}{RC} t \quad (7)$$

$$Q = Q_0 e^{-t/\tau}$$

Since

$$I_R = \frac{dQ}{dt} = Q_0 \left(\frac{-1}{\tau}\right) e^{-t/\tau} \quad (8)$$

we can determine the following:

$$V_R = I_R R = Q_0 R \left(\frac{-1}{\tau}\right) e^{-t/\tau} = \frac{-Q_0}{C} e^{-t/\tau} = V_0 e^{-t/\tau} \quad (9)$$

Equation (9) is the desired equation. Recall that if the voltage is not zero but still constant, then the voltage equation above will be the same with the addition of a constant term.

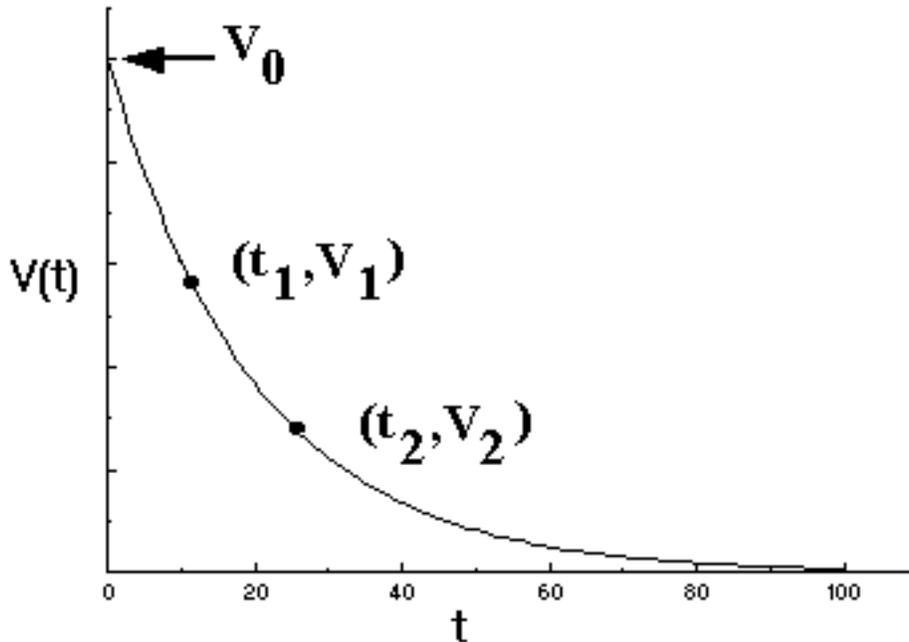


Figure 7 Exponential decay of a capacitor potential

In the lab, you will be asked to determine τ from measurements of $V(t)$, INDEPENDENT of any knowledge of R and C . There are several ways to do this but the properties of the exponential function allow you to do it “simply” by measuring the voltage at only two times, t_1 , and t_2 . According to equation 1, the voltages at these times will be given by:

$$\begin{aligned}
 V_1 &= V_0 e^{-t_1/\tau} \quad \text{and} \quad V_2 = V_0 e^{-t_2/\tau} \\
 \frac{V_1}{V_2} &= e^{(t_2-t_1)/\tau} \\
 \tau &= \frac{t_2 - t_1}{\ln(V_1 / V_2)}
 \end{aligned}
 \tag{10}$$

An especially simple pair of points can be chosen such that the second voltage is equal to the first voltage divided by e . The time between the two points will then be equal to τ . The important property of the exponential is that the ratio of voltages at two different times does not depend on when you begin the measurements. It depends only on the time between them.

Question 1.2

What is the propagated error of the time decay, τ , from the two point method (equation 10)?

2.2 Complex Impedance

When one is interested in finding the voltage of an element in an AC circuit, the method of complex impedance is very useful. In general, the complex impedance is defined as follows:

$$Z = \frac{\tilde{V}}{\tilde{I}} \quad (11)$$

where Z is complex (of the form $a + ib$ where a & b are real numbers) and \tilde{V} & \tilde{I} are of the form $V_0 e^{i\omega t}$ & $I_0 e^{i(\omega t + \phi)}$. The angle ϕ is the amount by which \tilde{V} & \tilde{I} are out of phase. Impedance is the ratio and relative phase of voltage and current. The important point to realize can be thought of as a generalization of resistance in AC circuits.

Taking a quick look at Ohm's Law you might guess that the impedance of the resistor is completely real and equal to the resistance. Other relevant impedances will only be listed here.

$$Z_R = R; \quad Z_C = \frac{1}{i\omega C}; \quad Z_L = i\omega L; \quad (12)$$

The general strategy when using complex impedance is to find first the total impedance of the system in order to find the current. Then one can find the voltage of any particular element by multiplying the current by impedance of that element.

2.3 RC Circuit in the Frequency Domain

In section 2.1 an RC circuit with only a discharging capacitor was examined. Now the same circuit with alternating current (AC) will be examined. Figure 8 shows a schematic for the circuit. Notice that there are three sources of voltage in this picture. Those are the signal generator, the capacitor and the resistor. Using Kirchhoff's Law we have:

$$\tilde{V}_S + \tilde{V}_C + \tilde{V}_R = 0 \quad (13)$$

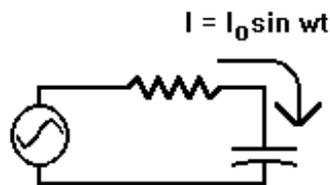


Figure 8 *Basic RC circuit with signal generator as a voltage source*

The current is the same through each element because they are in series. By replacing the voltage with the impedance multiplied by the current the equation becomes:

$$\tilde{V}_s - \tilde{I}(Z_R + Z_C) = 0 \quad (14)$$

Note there is a minus sign because the voltage drops over these two elements. Now the total impedance of this circuit becomes especially important. It is calculated below:

$$\begin{aligned} Z_R + Z_C &= R + \frac{1}{i\omega C} \\ &= R \left(1 - i \frac{1}{\omega RC} \right) \\ &= R \sqrt{1 + \frac{1}{(\omega RC)^2}} \exp \left(i \arctan \left[\frac{-1}{\omega RC} \right] \right) \end{aligned} \quad (15)$$

This expression looks awful, but let us separate the two components. $R \sqrt{1 + \frac{1}{(\omega RC)^2}}$ is the ratio of the amplitude of voltage (V) to current (I) and the exponential contains the phase offset between V and I. Assuming that \tilde{V} & \tilde{I} are of the form $V_0 e^{i\omega t}$ & $I_0 e^{i(\omega t - \phi)}$ and then using equation 14 we have:

$$V_0 = I_0 R \sqrt{1 + \frac{1}{(\omega RC)^2}} \quad (16)$$

Now the magnitude of the voltage over the capacitor is the product of the magnitude of the current and the magnitude of the impedance of the capacitor.

$$\begin{aligned} |V_C| &= |Z_C| I_0 \\ &= \left(\frac{1}{\omega C} \right) \left(\frac{V_0}{R} \sqrt{\frac{(\omega RC)^2}{(\omega RC)^2 + 1}} \right) \\ &= \frac{V_0}{\sqrt{1 + (\omega RC)^2}} \\ &= \frac{V_0}{\sqrt{1 + (\omega \tau)^2}} \end{aligned} \quad (17)$$

3 The Experiment

3.1 Determining the Decay Time Constant in the Time Domain

1. Record the measured values for the $500\ \Omega$ resistor and a $0.1\ \mu\text{F}$ capacitor using the multimeter. Make sure to record the appropriate uncertainty for each measurement.
2. Connect the components in an RC circuit with the signal generator as shown in Figure 8, and then connect the oscilloscope across the capacitor.
 - When you connect both the signal generator and the oscilloscope to your circuit you must be strategic about the placement of the ground of them. If you ground your circuit in two places then you have effectively eliminated the element in between them because there is no voltage difference between these elements. The solution is to place both grounds on the same node in your circuit.
3. Before making any measurements **CALIBRATE THE OSCILLOSCOPE**. Refer back to Experiment 0 for instructions or ask your TA if you have difficulty.
4. Compute the theoretical time constant for the circuit (with uncertainty) from Equation 1 using measured component values for R and C and their respective uncertainties. Make sure to include resistance from the signal generator (this extra resistance is $50\ \Omega$ with an uncertainty of 5%). Always use total resistance ($R_{total} = R_R + R_{SG}$) in these calculations. See Figure 9 below.

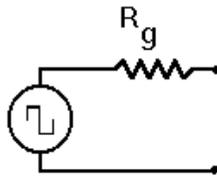


Figure 9 Representation of signal generator with internal resistance

Question 1.3

Based on the resistance and capacitance estimates given in step 1, what is an estimate for the decay time, RC ?

5. Turn on the signal generator, and apply a square wave to the resistor and capacitor in series.
6. The period of the square wave needs to be at least several times the fundamental decay time, τ , or the signal will appear as more like a triangle wave than Figure 10 or 11. The signal needs to decay enough so that you can determine the “baseline”, from which you will measure the voltage. This baseline will be your zero line. The solution to question 1.4 will give you a good first guess as the frequency of the square wave.



Figure 10 *Desired scope trace*



Figure 11 *Checking zero adjustment*

Question 1.4

What frequency should the square wave be if you desire to have the period be approximately ten times the decay time, RC ? (Use the RC you determined in the previous question.)

7. Measure the time constant for the discharge of an RC circuit using the simple two-point method described in section 2.1.
8. Measure and record the voltage level as a function of time. A straightforward way to do this is to adjust the controls such that a single exponential decay fills the screen, and simply record the levels as the trace crosses each of the 11 major vertical lines on the scope screen. For example, a horizontal scale setting of $5 \mu\text{s}/\text{division}$ would provide points at 0, 5, 10, ..., $55 \mu\text{s}$. Remember to include the uncertainties in the measurements. See hints below to get the best view of the decay.
9. Plot a graph of V_C versus t (including error bars) and then fit your data to the functional form of an exponential decay (i.e. $V_C = V_0 e^{-t/\tau}$). From the coefficients of the fit, determine a value (with uncertainty) for the time constant.

Hints:

For the measurement to be fit to an exponential it is best to arrange the horizontal sweep (time scale) such that a single exponential decay fills the screen. Then set the left-right position of the trace such that you can measure the early portion of the decay where the voltage is largest.

Changing the time scaling will not change the position of the baseline and will increase the amount of “good” points available to measure. This will provide the most accurate measurements.

Do not record values of zero. Exponential decay will never theoretically become zero. Fitting zero points will force the fit to decay faster than it actually does (artificially increasing the fitting result for τ).

3.2 Determining the Decay Time Constant in the Frequency Domain

In this section you will investigate how the RC circuit responds to alternating current at various frequencies.

1. Use the same circuit as in Figure 8 for the following measurements.
2. Select the sine wave output from the Signal Generator, and adjust the oscilloscope so as to display several cycles of the sinusoidal voltage across the capacitor.
3. Perform a quick examination of the amplitude of V_C as a function of the frequency to test that the behavior is as expected from equation 17. That is, when you increase the frequency, does the voltage over the capacitor adjust as equation 17 dictates? If it is not, ask the TA for assistance.
4. Measure and record the peak-to-peak voltage V_C across the capacitor as a function of the frequency (a linear frequency range of $\frac{1}{16\tau}$ to $\frac{16}{\tau}$ should be sufficient).
5. Plot a graph of the amplitude (or 1/2 of the peak-to-peak value) of V_C versus frequency (including error bars) using ORIGIN, and then fit your data to the functional form given by equation 17.
6. From the coefficients of the fit, determine a value of RC (with uncertainty), and once again compare it to the theoretical value.

Analysis

Compute the following:

- Theoretical RC from direct measurement of components;
- Experimental RC by 2 point method;
- Comparison of experimental RC's to theoretical RC (3 total).

Conclusions

Highlight the themes of the lab and the physics the experiment verifies. You should discuss the errors you encounter in the lab and how you could improve the lab if you had to repeat it. If your results are unexpected or your t-values are high, you should identify possible explanations.

Hints on reports

- Write neatly—if your TA cannot read it, you could lose points.
- Be organized—if your TA cannot find it, you could lose points.
- Report your data, including plots—if your data is not in your report, your TA does know you did it.
- Record uncertainty.
- Propagate uncertainty.
- Write your final answers with proper significant figures.